Relaying Strategies for the Two-Way Gaussian Relay Channel

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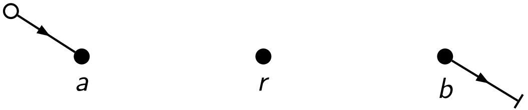
Outline

The Network Model

The Network Model

- Three nodes / devices

The Network Model



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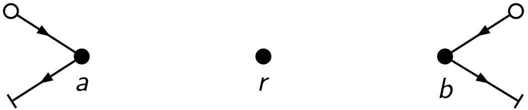

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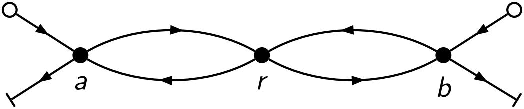

- Three nodes / devices
- One message from a to b
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- No direct link

Start with Toy Problems

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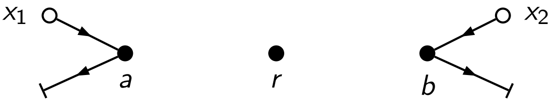
The resulting model can illustrate the basic ideas behind network coding (NC) and physical layer network coding (PLNC).

Routing

Exchanging 2 bits x_1 and x_2 with a routing approach:



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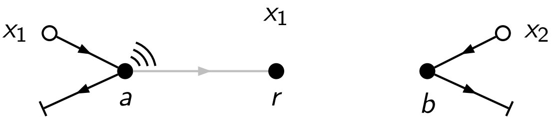

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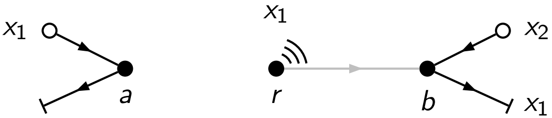

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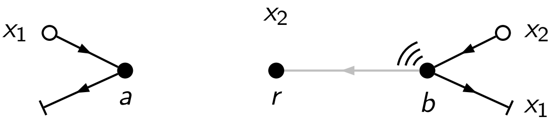

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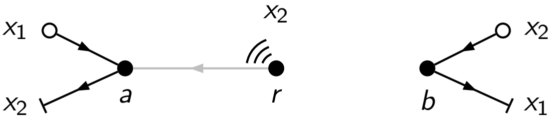

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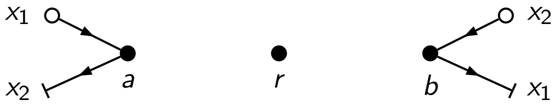

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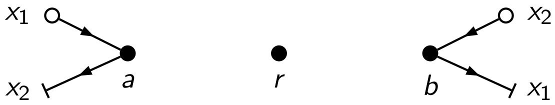
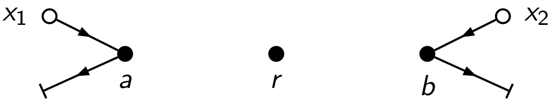


sum rate: 2 bits in 4 transmissions

Network Coding

The relay node r can reach both a and b. Broadcasting saves one transmission.



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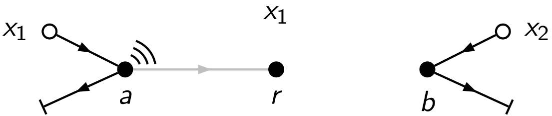

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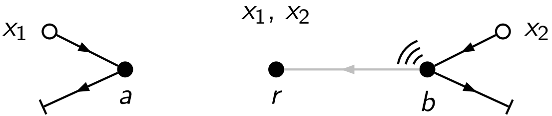

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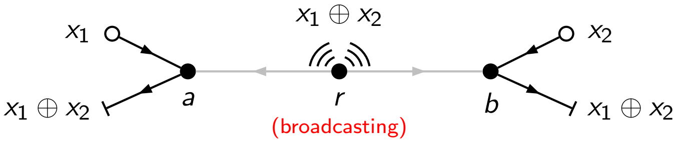

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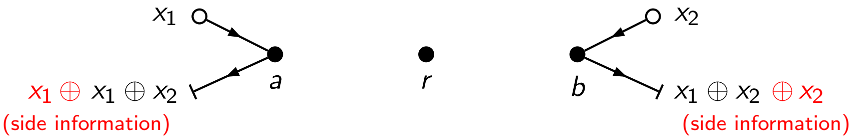

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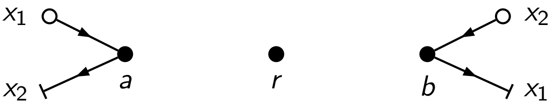

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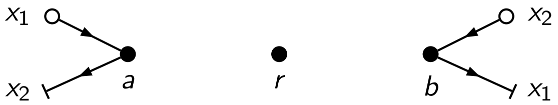
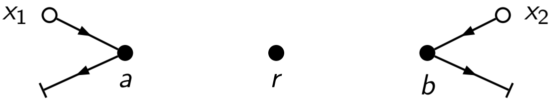


sum rate: 2 bits in 3 transmissions

Physical Layer Network Coding

The relay node doesn't need to know the individual bits.



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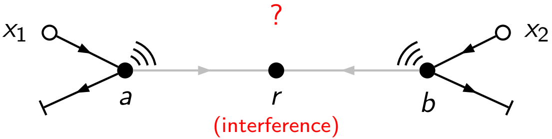

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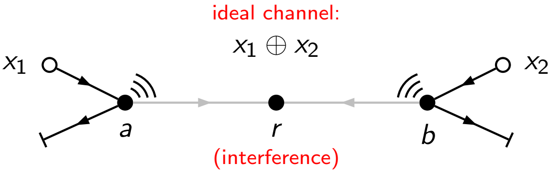

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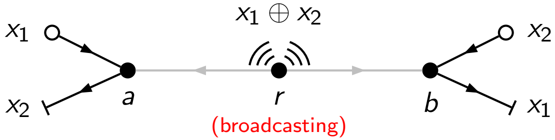

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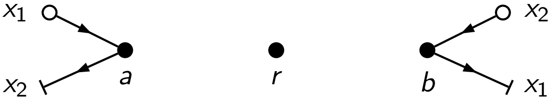

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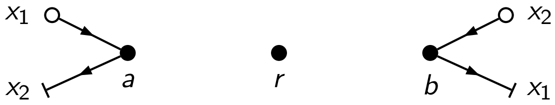
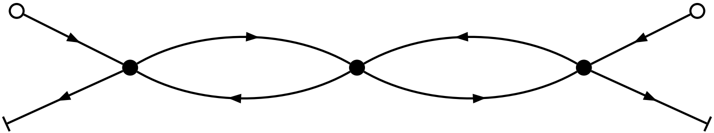


sum rate: 2 bits in 2 transmissions (for ideal channel)

Obvious Question

How does it work for more realistic channel models?

Two-Way Gaussian Relay Channel Model



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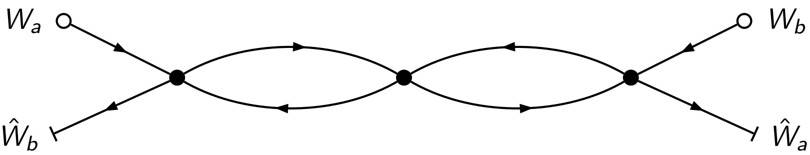

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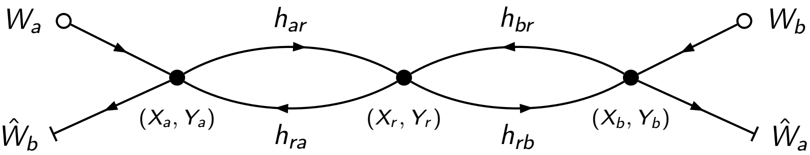

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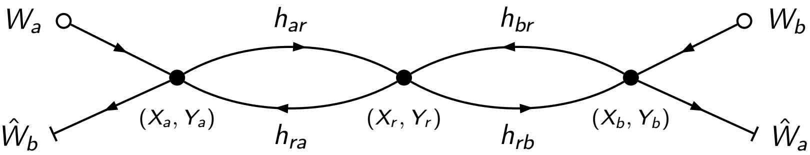

Memoryless additive Gaussian noise:

- $Y_r = h_{ar}X_a + h_{br}X_b + Z_r$
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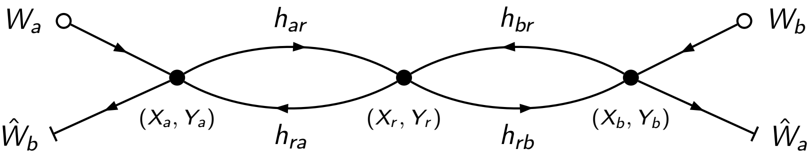

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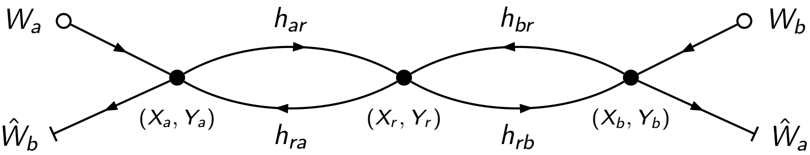

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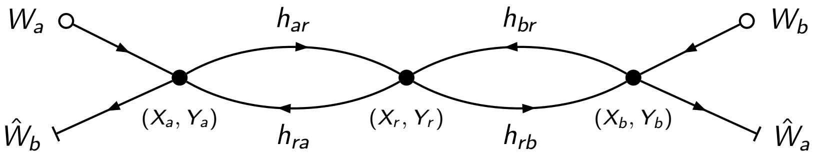

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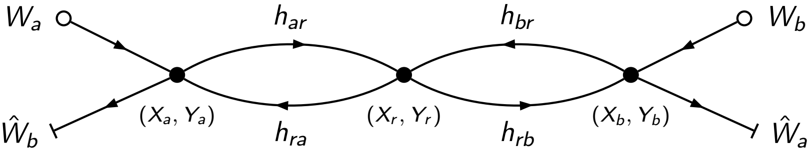

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Channel coefficients h_{ij} are time-invariant. The inputs X_a, X_b and X_r are average power constrained to P. Noise variance is set to $\sigma^2 = 1$. The channel is used n times in total.

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What are achievable rate pairs (R_a, R_b) (in bits per channel use) and achievable sum rates $R = R_a + R_b$?

Amplify-and-Forward (AF)

See [Rankov-Wittneben '05].

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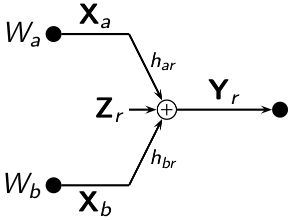
1. MAC phase



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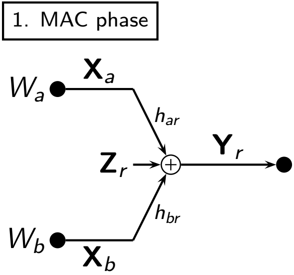

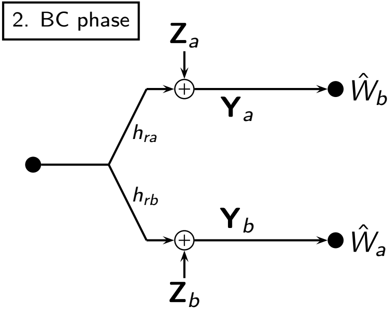

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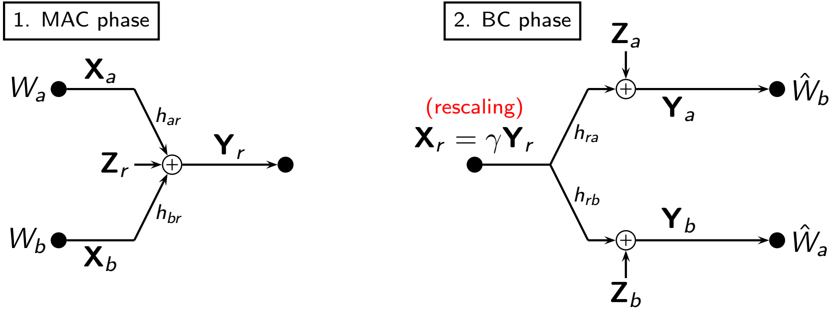

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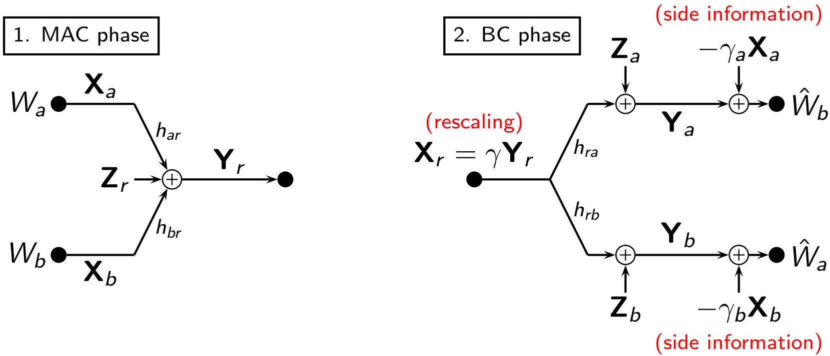

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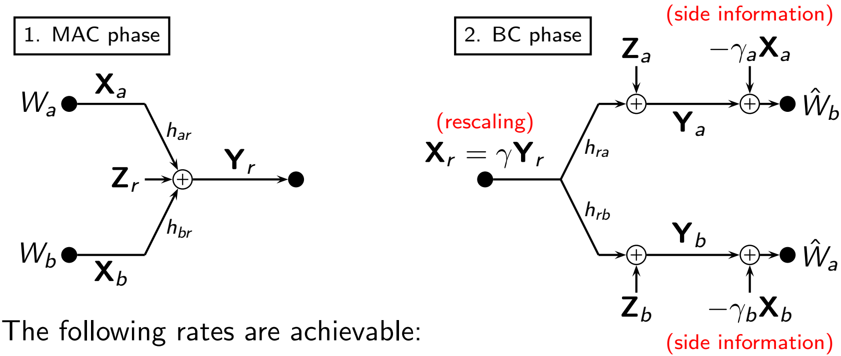

The following rates are achievable:

$$R_a < \frac{1}{4} \log_2 \left(1 + \frac{h_{ra}^2 h_{br}^2 P}{h_{ra}^2 P + h_{ar}^2 P + h_{br}^2 P + 1} \right)$$

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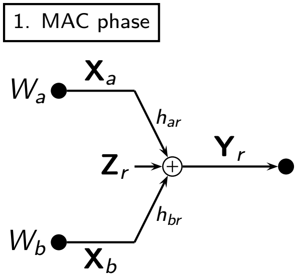

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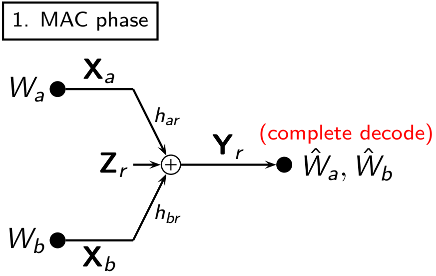

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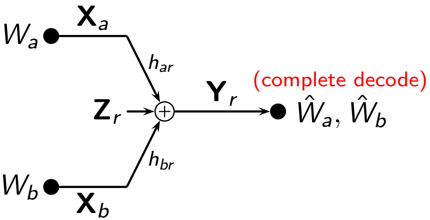

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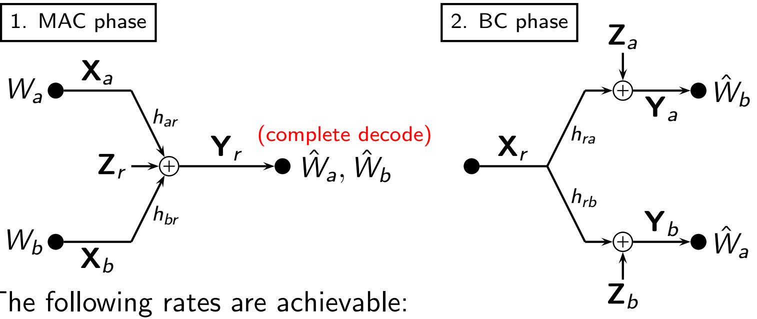

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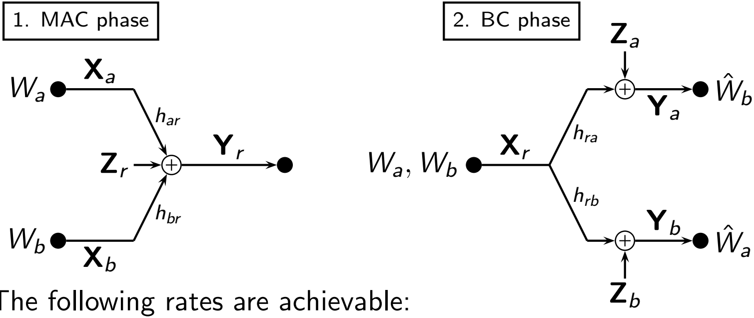

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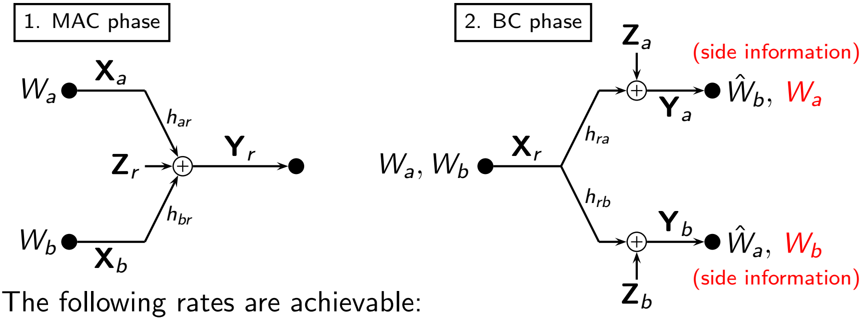

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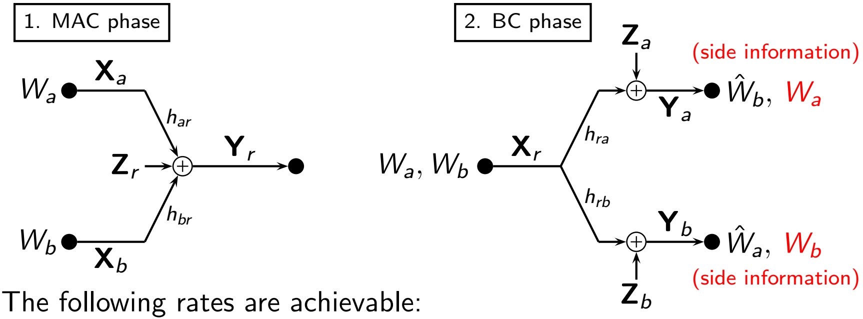

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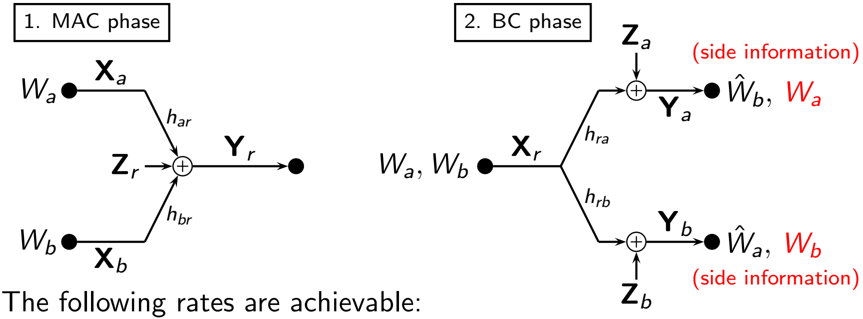

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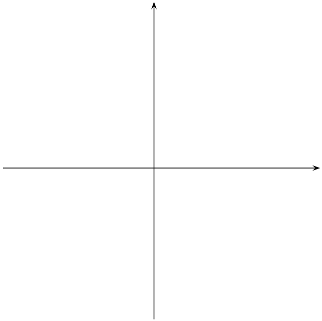
- PLNC idea: the relay should only try to understand a linear combination of the transmitted messages.
- Translation of this linear combination of messages to the signals (physical layer) can be done with nested lattice codes.

Lattices, Lattice Codes and Nested Lattice Codes



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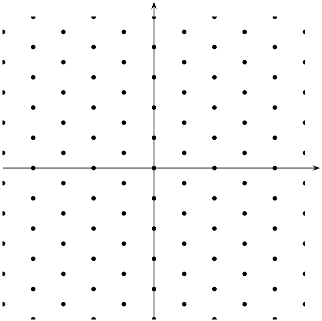

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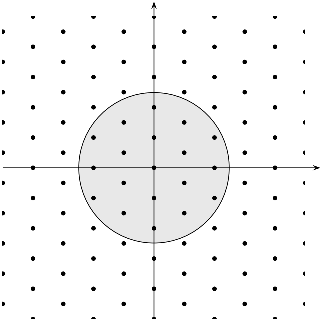

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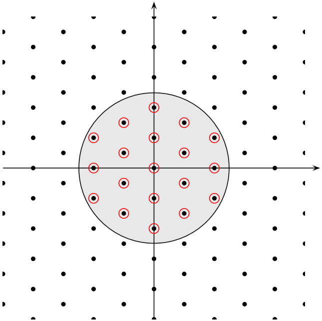
- Lattice Λ_f
- Region $\mathcal{R}$
- Lattice code $(\Lambda_f + \mathbf{t}) \cap \mathcal{R}$



Lattices, Lattice Codes and Nested Lattice Codes

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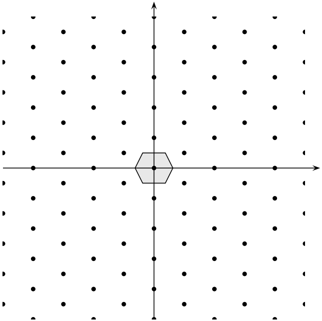

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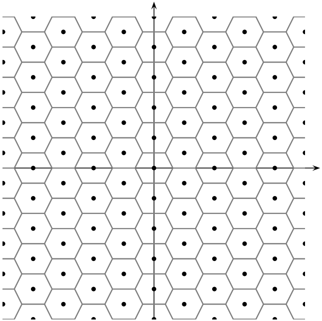

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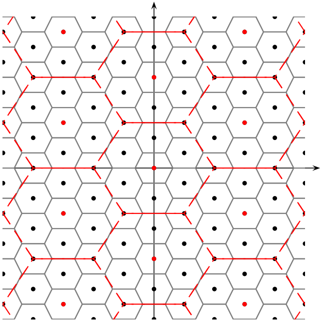

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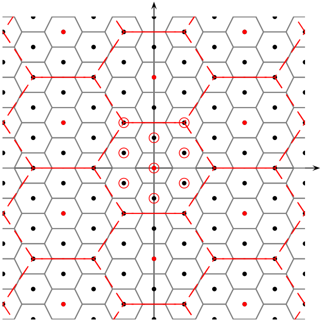
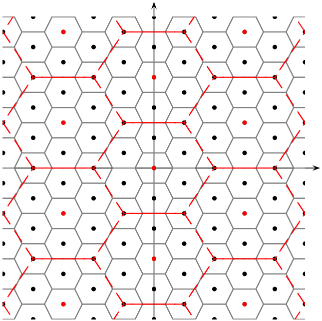


Modulo-Lattice (Without Noise)



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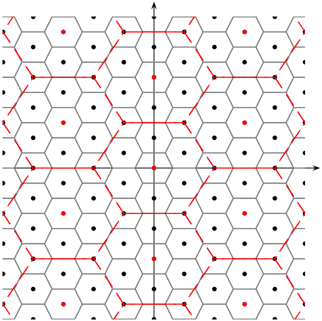

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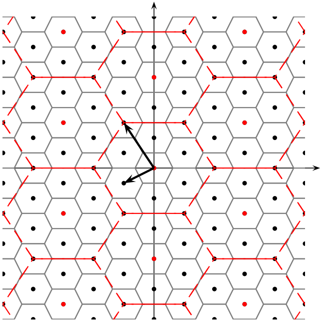

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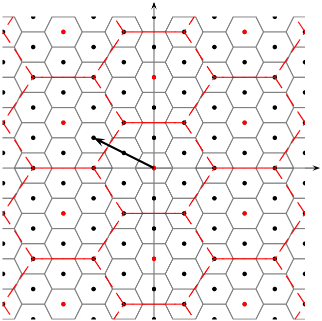

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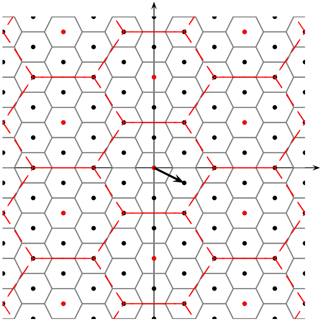

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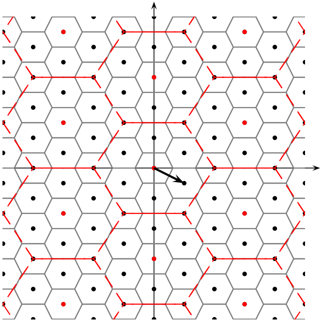
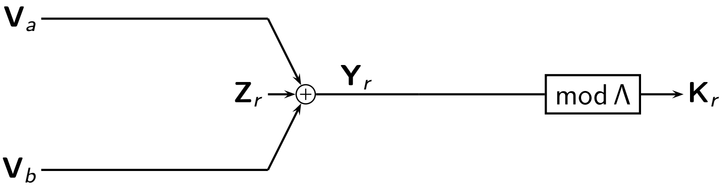
- Side information allows each user to recover the lattice point of the other user.



Modulo-Lattice-and-Forward (MF)

See [Baik-Chung '08].



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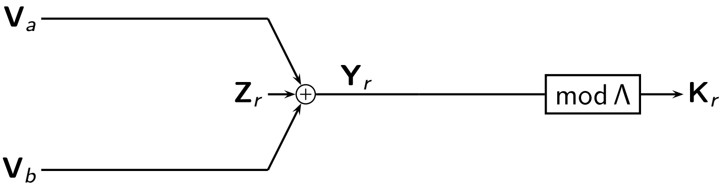

For easier illustration the channel gains are all assumed to be 1.

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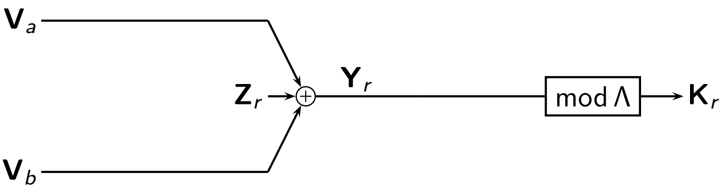

The number of lattice points that can be reliably distinguished at the relay is bounded by $M < \frac{\mathrm{Vol}(\mathcal{V}(\Lambda))}{\mathrm{Vol}_{\mathrm{noise}}} \approx P^{n/4}$ for very large n and ideal shaping gain of Λ. Then the rate is limited by

$$R < \frac{1}{n}\log_2 M = \frac{1}{4}\log_2(P).$$

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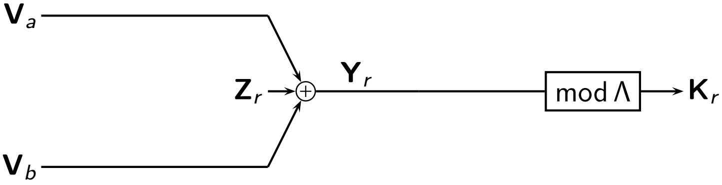

We can do better!

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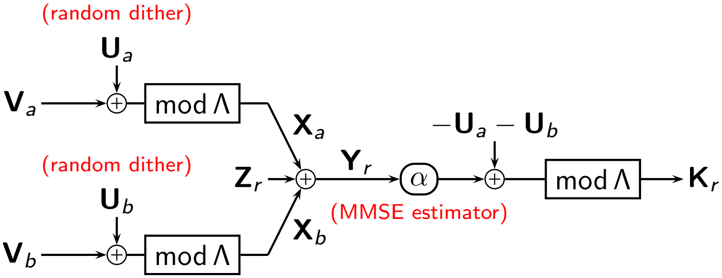

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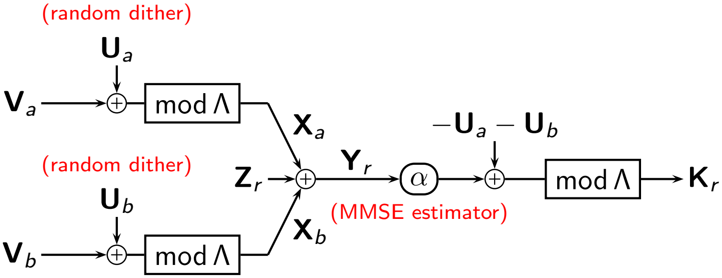

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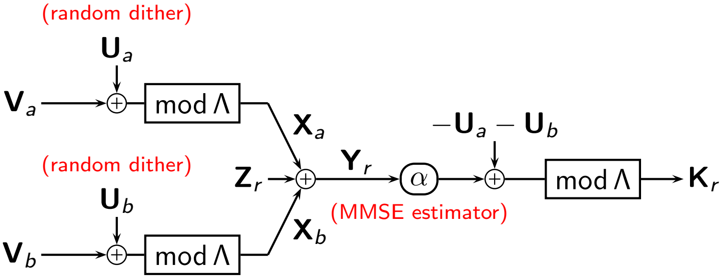

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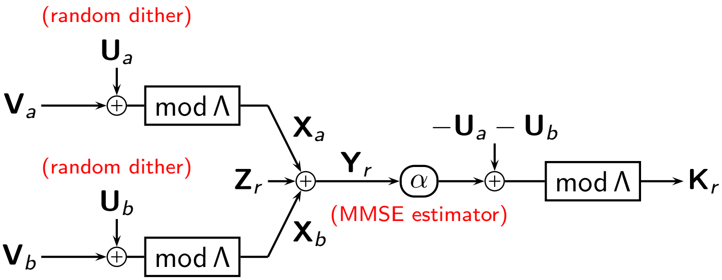

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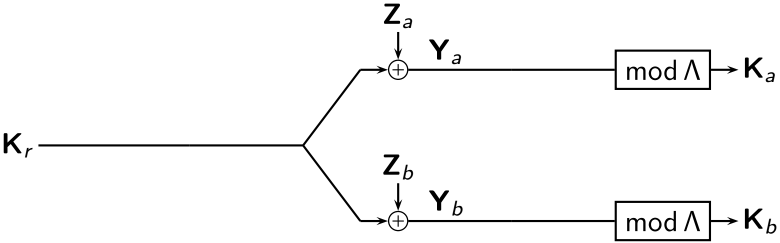

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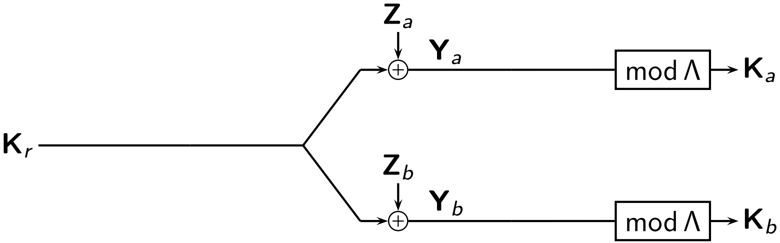

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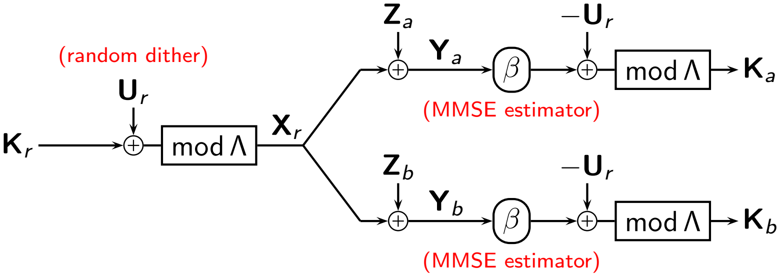

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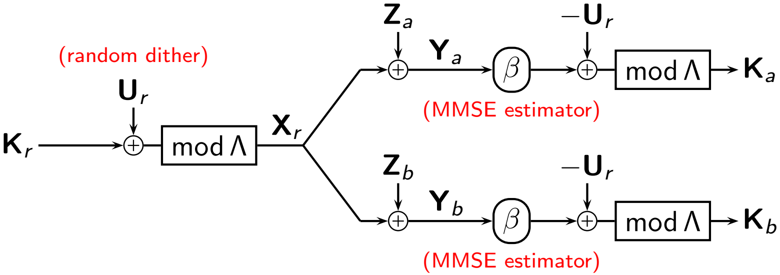

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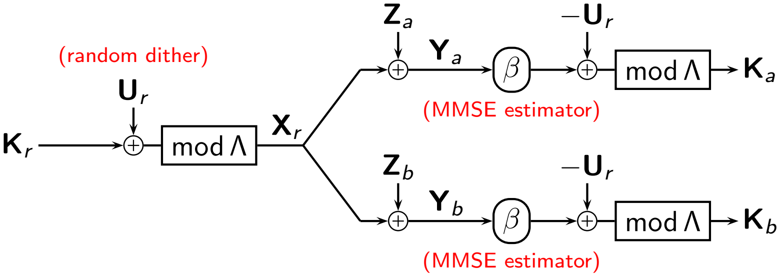

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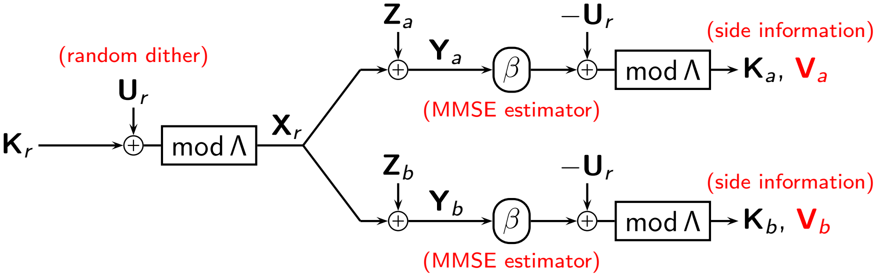

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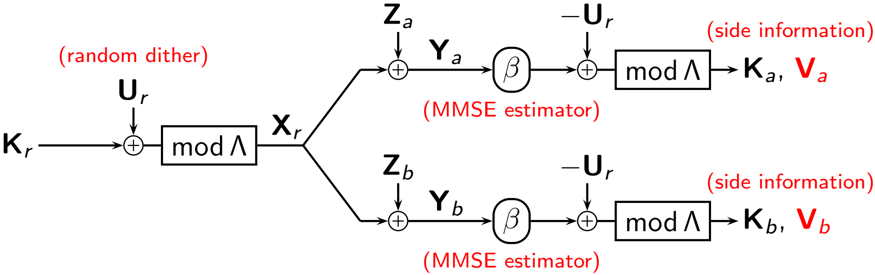

Remark: For general channel conditions superposition coding can be employed to improve the achievable rates.

Sum Rate Comparison

Comparison of achievable sum rates $R = R_a + R_b$ when all channel gains are assumed to be 1.

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$$\begin{aligned}
\text{Upper bound:} \quad & R < \frac{1}{2}\log_2(1+P) \\
\text{AF:} \quad & R < \frac{1}{2}\log_2\left(1+P\frac{P}{3P+1}\right) \\
\text{DF:} \quad & R < \frac{1}{4}\log_2(1+2P) \\
\text{MF:} \quad & R < \frac{1}{2}\log_2\left(\frac{1}{2}+\frac{P}{2}\right)
\end{aligned}$$

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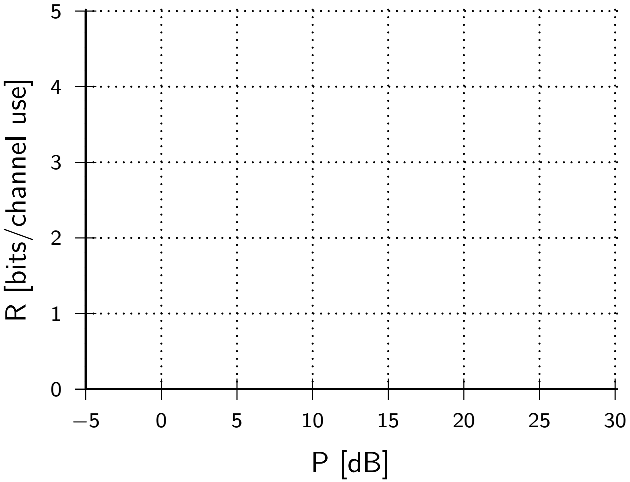

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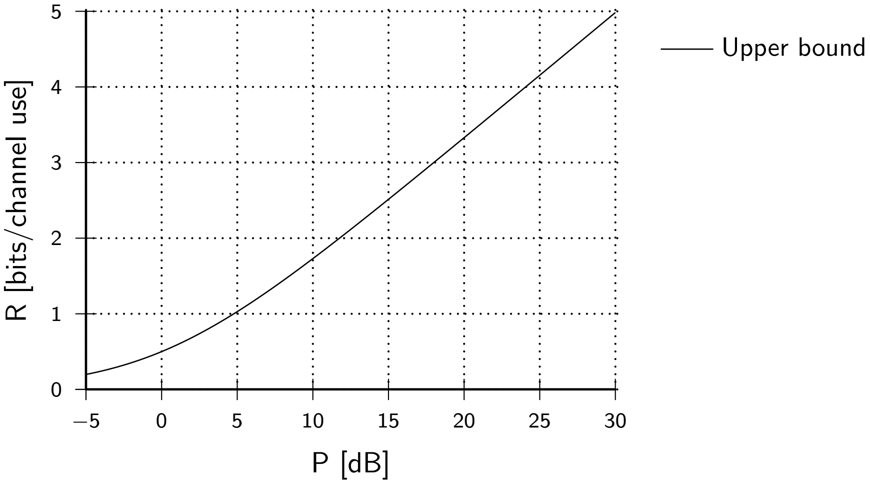

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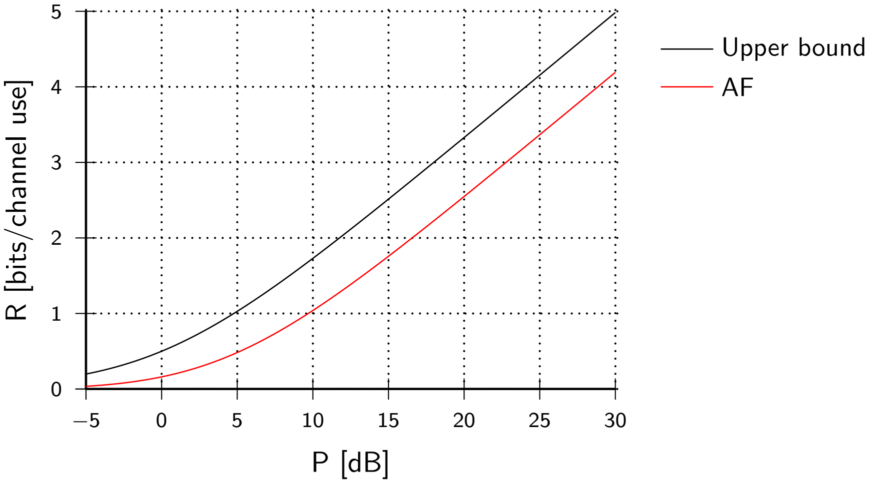

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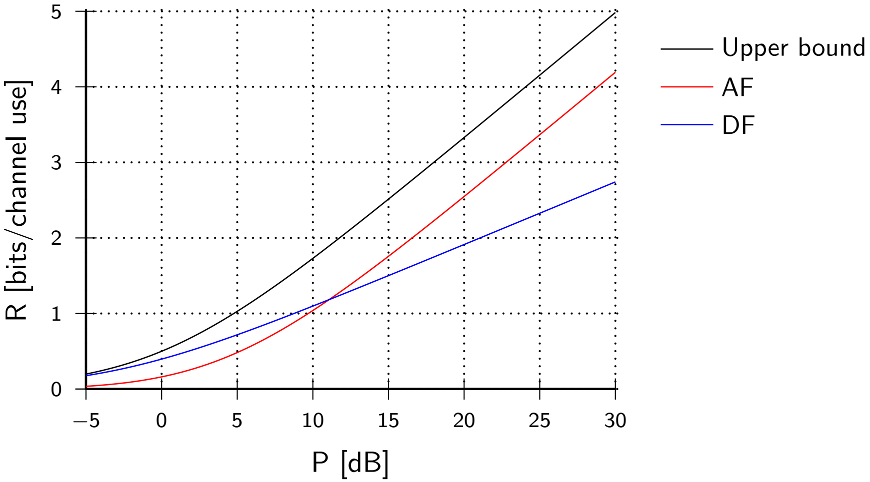

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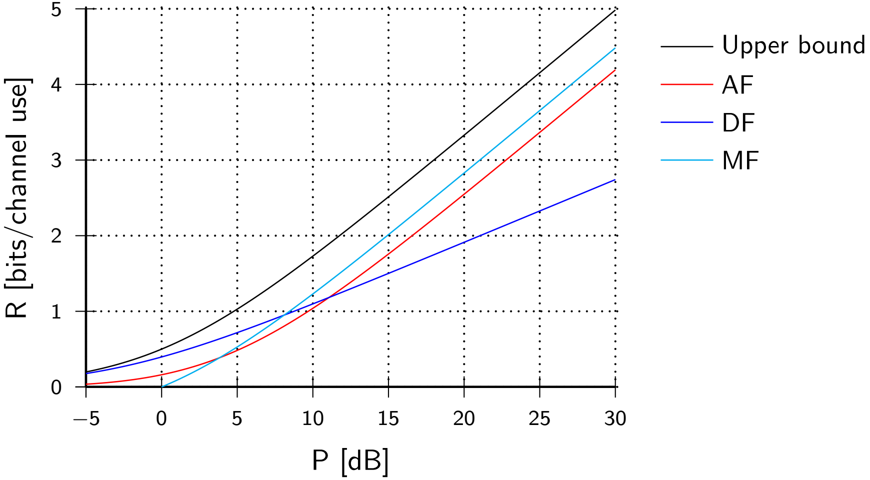

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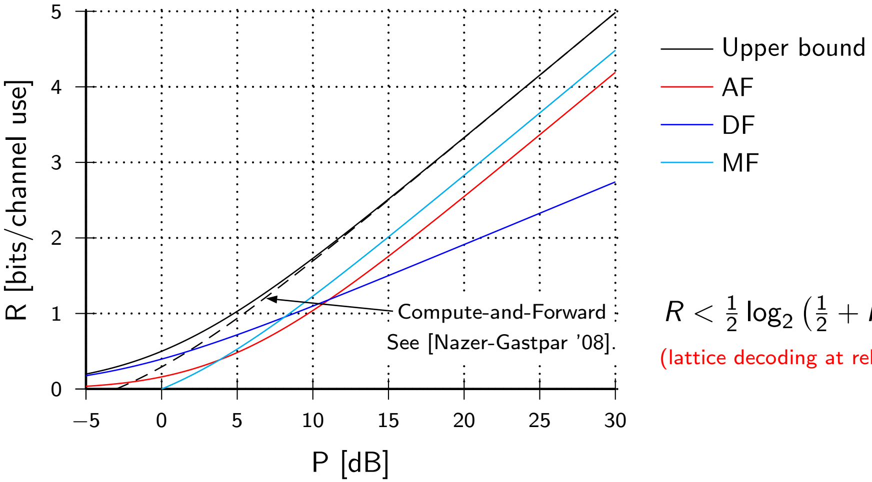

$$R < \frac{1}{2} \log_2 \left(\frac{1}{2} + P \right)$$

(lattice decoding at relay)

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- Involved principles: physical layer network coding, broadcasting, using side information
- In certain networks the interference of users can be harnessed.
- However a strategy to show achievability of the upper bound for this network for all channel conditions is still not available.

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