Agenda

• Welcome and introduction (Chairman)
• Presentation and mention of
  • Faculty Opponent: Rüdiger Urbanke
  • Evaluation Committee: Michael Lentmaier, Gianluigi Liva, Laurent Schmalen
  • Funding sources
  • Contributors to the thesis work
• Errata List
• Short introduction to the thesis work (Faculty Opponent)
• Presentation (25 min.)
• Discussion (60–90 min.)
• Questions and comments from the Evaluation Committee
• Questions from the audience
• Evaluation Committee meeting, decision and lunch (S2 lunch room)
Analysis and Design of Spatially-Coupled Codes with Application to Fiber-Optical Communications

Christian Häger

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FORCE
Fiber-optic communications research center

PhD Seminar
May 30, 2016
Analysis and Design of Spatially-Coupled Codes with Application to Fiber-Optical Communications

Christian Häger

Many thanks to Alexandre Graell i Amat, Fredrik Brännström, Alex Alvarado, Erik Agrell, and Henry Pfister

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PhD Seminar
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Fiber-Optical Communications
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Cinia opens subsea cable connecting Finland and Germany

Friday 20 May 2016 | 09:48 CET | News

Cinia Group announced the official opening and commercial availability of Cinia C-Lion 1, a new submarine cable system that connects Finland and Germany. The

Designed and commissioned by Cinia Group and built in partnership with Alcatel-Lucent Submarine Networks, the Cinia C-Lion1 cable system totals 1,200 kilometers in length and consists of eight optical fibre pairs.
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- Long distances result in significant signal attenuation
Fiber-Optical Communications

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- Periodic amplification necessary, which leads to random signal distortions or noise
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![Signal transmitted in Rostock](image-url)
Fiber-Optical Communications

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![Waveform diagram](image)
Fiber-Optical Communications

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_signal transmitted in Rostock_
Fiber-Optical Communications

- Long distances result in significant *signal attenuation*
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![Diagram showing signal transmission and amplification in fiber optics](image-url)
Fiber-Optical Communications

- Long distances result in significant **signal attenuation**
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Error-correcting codes are essential in modern fiber-optical communication systems to ensure reliable data transmission.
Error-Correcting Codes

communication channel

repeat several times

amplifier

optical fiber
Error-Correcting Codes

communication channel

repeat several times

amplifier

optical fiber

mathematical description of the transmission medium
Error-Correcting Codes

communication channel

mathematical description of the transmission medium
Error-Correcting Codes

0110

data bits

communication channel
Error-Correcting Codes

Data bits: 0110

communication channel

Received bits: 0100

Binary symmetric channel: each bit flipped with probability $p$. 

Errors
Error-Correcting Codes

0110
data bits

communication channel
Error-Correcting Codes

0110
data bits

encoder

communication channel

decoder
Error-Correcting Codes

- Encoder: 0110
  - Data bits

- Encoder: 011010
  - Data
  - Parity

- Communication channel

- Decoder
Error-Correcting Codes

encoder

communication channel

decoder

data bits

data parity

received bits

errors
Error-Correcting Codes

encoder

communication channel

decoder

0110 data bits

011010 data parity

010010 received bits

0110 restored bits

0110 errors
Error-Correcting Codes

Requirements for Fiber-Optical Communications
Error-Correcting Codes

Requirements for Fiber-Optical Communications

- Very high throughputs (100 Gigabits per second or higher)
- Very high net coding gains (close-to-capacity performance)
- Very low bit error rates (below $10^{-15}$)
Error-Correcting Codes

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Spatially-coupled codes are promising codes that can fulfill these requirements.
Error-Correcting Codes

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Spatially-coupled codes are promising codes that can fulfill these requirements.

In this talk

1. Basics of spatially-coupled codes
2. Asymptotic analysis and design of deterministic codes
3. Designing spectrally-efficient fiber-optical systems
Codes on Graphs
Codes on Graphs

- Parity bits are formed by adding (modulo 2) subsets of data bits:

\[
\begin{array}{cccc}
C_1 & C_2 & C_3 & C_4 \\
0 & 1 & 1 & 0 \\
\end{array}
\]
Codes on Graphs

- Parity bits are formed by adding (modular 2) subsets of data bits:

\[ c_1 + c_2 + c_3 = c_5 \]
\[ c_2 + c_3 + c_4 = c_6 \]
Codes on Graphs

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- Code representation via bipartite Tanner graph, where variable nodes represent code bits and check nodes represent parity-check equations
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- Code rate \( R = \frac{\text{number of data bits}}{\text{code length}} \)
low-density parity-check (LDPC) code
[Gallager, 1962]

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- Introduce constraint nodes (or generalized check nodes)

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Spatially-Coupled Codes

Deterministic Codes

Spectrally-Efficient Systems

Conclusion

Codes on Graphs

low-density parity-check (LDPC) code

[Gallager, 1962]

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Spatially-Coupled Codes

[Felström and Zigangirov, 1999], [Lentmaier et al., 2005], [Kudekar et al., 2011], ...
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[Diagram of spatial coupling]
Spatially-Coupled Codes

[Felström and Zigangirov, 1999], [Lentmaier et al., 2005], [Kudekar et al., 2011], …

Spatially position

1 2 3 4 5
Spatially-Coupled Codes

[Felström and Zigangirov, 1999], [Lentmaier et al., 2005], [Kudekar et al., 2011], ...
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Spatially-Coupled Codes

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known variable nodes $\implies$ slight graph irregularity at the boundaries $\implies$ better protection

Spatially position

---

[Diagram showing spatially-coupled codes with variable nodes and graph irregularities]
Decoding Wave Effect
Decoding Wave Effect

- Apply (suboptimal) **iterative** decoding, exchanging messages between variable and constraint nodes

![Graph](chart.png)  
**predicted bit error rate per spatial position**  
**spatial position**
Decoding Wave Effect

- Apply (suboptimal) **iterative** decoding, exchanging messages between variable and constraint nodes

![Graph](image_url)
Decoding Wave Effect

- Apply (suboptimal) **iterative** decoding, exchanging messages between variable and constraint nodes

predicted bit error rate per spatial position

\[ l = 15 \]

spatial position
Decoding Wave Effect

- Apply (suboptimal) **iterative** decoding, exchanging messages between variable and constraint nodes

![Graph showing predicted bit error rate per spatial position](image)
Decoding Wave Effect

- Apply (suboptimal) **iterative** decoding, exchanging messages between variable and constraint nodes

predicted bit error rate per spatial position

spatial position

$l = 45$
Decoding Wave Effect

- Apply (suboptimal) **iterative** decoding, exchanging messages between variable and constraint nodes
Decoding Wave Effect

- Apply (suboptimal) iterative decoding, exchanging messages between variable and constraint nodes

predicted bit error rate per spatial position

spatial position

$l = 75$
Decoding Wave Effect

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Decoding Wave Effect

- Apply (suboptimal) iterative decoding, exchanging messages between variable and constraint nodes
Decoding Wave Effect

- Apply (suboptimal) **iterative** decoding, exchanging messages between variable and constraint nodes
- **Successful decoding**

![Graph showing predicted bit error rate per spatial position](image-url)
**Decoding Wave Effect**

- Apply (suboptimal) *iterative* decoding, exchanging messages between variable and constraint nodes
- **Successful decoding** even for cases where decoding of “uncoupled” regular codes fails

![Graph showing predicted bit error rate per spatial position](chart.png)
Decoding Wave Effect

- Apply (suboptimal) **iterative** decoding, exchanging messages between variable and constraint nodes
- Successful decoding even for cases where decoding of “uncoupled” regular codes fails
- Performance can be as good as under **optimal** decoding [Kudekar et al., 2011], [Yedla et al., 2014]
Spatial coupling is a tool to construct codes on graphs that have excellent performance under iterative decoding.
Introduction: Product Codes and Staircase Codes

Code proposals for fiber-optical communication systems are often very structured (i.e., deterministic) and not random-like (for example [Justesen et al., 2010], [Smith et al., 2012], [Jian et al., 2013]).
Introduction: Product Codes and Staircase Codes

rectangular array [Elias, 1954]
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each row/column is a codeword in some component code
Introduction: Product Codes and Staircase Codes

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Tanner graph

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Tanner graph

degree-2 variable node
Introduction: Product Codes and Staircase Codes

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Tanner graph
Introduction: Product Codes and Staircase Codes

- **Rectangular array** [Elias, 1954]
- **Staircase array** [Smith et al., 2012]

Tanner graph
Introduction: Product Codes and Staircase Codes

rectangular array [Elias, 1954]  

Tanner graph

classical product code

staircase array [Smith et al., 2012]  
generalized product code (GPC)

...
Introduction: Product Codes and Staircase Codes

rectangular array [Elias, 1954]  staircase array [Smith et al., 2012]

Tanner graph  generalized product code (GPC)
Introduction: Product Codes and Staircase Codes

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generalized product code (GPC)

spatially-coupled code

positions: 1 2 3 4 5
Introduction: Product Codes and Staircase Codes

- **Deterministic codes** with fixed and structured Tanner graph

  - **rectangular array** [Elias, 1954]
  - **staircase array** [Smith et al., 2012]

  - Generalized product code (GPC)
  - Spatially-coupled code

  - Positions: 1 2 3 4 5
Introduction: Product Codes and Staircase Codes

- **Deterministic** codes with fixed and structured Tanner graph
- **GPCs** with iterative bounded-distance decoding are very appealing due to low-complexity hardware implementation
Iterative Bounded-Distance Decoding
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Iterative Bounded-Distance Decoding

- Codeword transmission over *binary erasure channel* with erasure probability $p$
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- $\ell$ iterations of **bounded-distance decoding** = peeling of vertices with degree $\leq t$ (in parallel)
Iterative Bounded-Distance Decoding

1st iteration \((t = 2)\)

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2nd iteration ($t = 2$)

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Staircase Code Optimization
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Problem Formulation

For staircase code with fixed code rate $R$, find “good” component codes
Staircase Code Optimization

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- [Zhang and Kschischang, 2014] use simulations to predict performance → computationally intensive
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- Approach in Paper C based on a connection between staircase codes and random-like spatially-coupled codes from [Jian et al., 2012]
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- Works well, however, only heuristically motivated
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Fundamental question

Is it possible to directly analyze staircase codes (and other deterministic GPCs) without the detour to random-like codes? Papers D–F
Parametrized Construction of Generalized Product Codes

Product codes

Staircase codes

Positions: 1 2 3 4 5
Parametrized Construction of Generalized Product Codes

- Product codes
- Staircase codes

```
positions: 1 2
```

```
positions: 1 2 3 4 5
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Parametrized Construction of Generalized Product Codes

- **Product codes**
- **Staircase codes**

```
positions: 1 2 3 4
L = 2
```

```
positions: 1 2 3 4 5
L = 5
```
Parametrized Construction of Generalized Product Codes

- **Product Codes**
  - $L = 2$
  - $L = 5$

- **Staircase Codes**
  - $L = 2$
  - $L = 5$

**$\eta$:** symmetric $L \times L$ matrix that defines graph connectivity
Parametrized Construction of Generalized Product Codes

\[ \eta = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix} \]

\( \eta \): symmetric \( L \times L \) matrix that defines graph connectivity
Parametrized Construction of Generalized Product Codes

product codes

staircase codes

η = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}

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Parametrized Construction of Generalized Product Codes

product codes

staircase codes
Parametrized Construction of Generalized Product Codes

product codes

staircase codes

$n$: “problem size”, proportional to the number of constraint nodes
Parametrized Construction of Generalized Product Codes

product codes

staircase codes

$n$: “problem size”, proportional to the number of constraint nodes

increasing $n$
Parametrized Construction of Generalized Product Codes

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Density Evolution
Density Evolution

• What happens asymptotically for $n \to \infty$?
Density Evolution

- What happens asymptotically for $n \to \infty$?
- Let $p = c/n$ for $c > 0$, where $c$ is the effective channel quality
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$$x^{(\ell)} = \Psi_{\geq t}(cBx^{(\ell-1)})$$
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\[
B \triangleq \gamma \eta \quad \text{initial condition} \quad x^{(0)} = (1, \ldots, 1)
\]

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x^{(\ell)} = \Psi_{\geq t}(cBx^{(\ell-1)})
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Density Evolution

- What happens asymptotically for $n \to \infty$?
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\begin{align*}
B &\triangleq \gamma \eta \\
x^{(0)} &\triangleq (1, \ldots, 1) \\
x^{(\ell)} &= \Psi_{\geq t}(cBx^{(\ell-1)}) \\
\Psi_{\geq t}(x) &\triangleq 1 - \sum_{i=0}^{t-1} \frac{x^i}{i!} e^{-x}
\end{align*}
\]
Density Evolution

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- What happens asymptotically for $n \to \infty$?
- Let $p = c/n$ for $c > 0$, where $c$ is the effective channel quality.

\[ B \overset{\Delta}{=} \gamma \eta \quad \text{initial condition} \quad x^{(0)} = (1, \ldots, 1) \]

\[ x^{(\ell)} = \Psi_{\geq t}(cBx^{(\ell-1)}) \]

Element-wise application of

\[ \Psi_{\geq t}(x) \overset{\Delta}{=} 1 - \sum_{i=0}^{t-1} \frac{x^i}{i!} e^{-x} \]
Comparison of Deterministic and Random-Like Codes
Comparison of Deterministic and Random-Like Codes

Deterministic

\[ x^{(\ell)} = \Psi_{\geq t}(cBx^{(\ell-1)}) \]

\((B = \gamma\eta)\)

\[
\frac{1}{2} \begin{pmatrix}
0 & 1 & 0 & 0 & 0 & 0 \\
1 & 0 & 1 & 0 & 0 & 0 \\
0 & 1 & 0 & 1 & 0 & 0 \\
0 & 1 & 0 & 1 & 0 & 0 \\
0 & 0 & 1 & 0 & 1 & 0 \\
0 & 0 & 0 & 1 & 0 & 1 \\
0 & 0 & 0 & 0 & 1 & 0
\end{pmatrix}
\]
Comparison of Deterministic and Random-Like Codes

Deterministic

\[ x^{(\ell)} = \Psi \ge t(cBx^{(\ell-1)}) \]

\[(B = \gamma \eta)\]

\[ \frac{1}{2} \begin{pmatrix}
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0 & 0 & 1 & 0 & 1 & 0 \\
0 & 0 & 0 & 1 & 0 & 1 \\
0 & 0 & 0 & 0 & 1 & 0
\end{pmatrix} \]
Comparison of Deterministic and Random-Like Codes

**Deterministic**

\[
x^{(\ell)} = \Psi_{\geq t}(cBx^{(\ell-1)})
\]

\[
(B = \gamma \eta)
\]

\[
\frac{1}{2} \begin{pmatrix}
0 & 1 & 0 & 0 & 0 & 0 \\
1 & 0 & 1 & 0 & 0 & 0 \\
0 & 1 & 0 & 1 & 0 & 0 \\
0 & 0 & 1 & 0 & 1 & 0 \\
0 & 0 & 0 & 1 & 0 & 1 \\
0 & 0 & 0 & 0 & 1 & 0
\end{pmatrix}
\]

**Random-Like [Jian et al., 2012]**

\[
x^{(\ell)} = \Psi_{\geq t}(c\tilde{B}x^{(\ell-1)})
\]

\[
\tilde{B} = \begin{pmatrix}
1 & 1 & 0 & 0 & 0 & 0 \\
1 & 2 & 1 & 0 & 0 & 0 \\
0 & 1 & 2 & 1 & 0 & 0 \\
0 & 0 & 1 & 2 & 1 & 0 \\
0 & 0 & 0 & 1 & 2 & 1 \\
0 & 0 & 0 & 0 & 1 & 1
\end{pmatrix}
\]
Comparison of Deterministic and Random-Like Codes

**Deterministic**

\[ x^{(\ell)} = \Psi_{\geq t}(cBx^{(\ell-1)}) \]

\((B = \gamma \eta)\)

\[ \frac{1}{2} \begin{pmatrix} 0 & 1 & 0 & 0 & 0 & 0 \\ 1 & 0 & 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 & 0 & 1 \\ 0 & 0 & 0 & 0 & 1 & 0 \end{pmatrix} \]


**Random-Like [Jian et al., 2012]**

\[ x^{(\ell)} = \Psi_{\geq t}(c\tilde{B}x^{(\ell-1)}) \]

\[ \tilde{B} = \begin{pmatrix} 1 & 1 & 0 & 0 & 0 & 0 \\ 1 & 2 & 1 & 0 & 0 & 0 \\ 0 & 1 & 2 & 1 & 0 & 0 \\ 0 & 0 & 1 & 2 & 1 & 0 \\ 0 & 0 & 0 & 1 & 2 & 1 \\ 0 & 0 & 0 & 0 & 1 & 1 \end{pmatrix} \]

- Equations have the same form \(\implies\) similar performance
Comparison of Deterministic and Random-Like Codes

Deterministic

Random-Like [Jian et al., 2012]

capacity-achieving at high rates over the binary symmetric channel

\[ x^{(\ell)} = \Psi_\geq t (cBx^{(\ell-1)}) \]

\[ (B = \gamma \eta) \]

\[
\frac{1}{2} \begin{pmatrix}
0 & 1 & 0 & 0 & 0 & 0 \\
1 & 0 & 1 & 0 & 0 & 0 \\
0 & 1 & 0 & 1 & 0 & 0 \\
0 & 0 & 1 & 0 & 1 & 0 \\
0 & 0 & 0 & 1 & 0 & 1 \\
0 & 0 & 0 & 0 & 1 & 0
\end{pmatrix}
\]

\[
\frac{1}{4} \begin{pmatrix}
1 & 1 & 0 & 0 & 0 & 0 \\
1 & 2 & 1 & 0 & 0 & 0 \\
0 & 1 & 2 & 1 & 0 & 0 \\
0 & 0 & 1 & 2 & 1 & 0 \\
0 & 0 & 0 & 1 & 2 & 1 \\
0 & 0 & 0 & 0 & 1 & 1
\end{pmatrix}
\]

- Equations have the same form \( \Longrightarrow \) similar performance
Comparison of Deterministic and Random-Like Codes

Deterministic

\[ x^{(\ell)} = \Psi_{\geq t}(cBx^{(\ell-1)}) \]

\[ (B = \gamma \eta) \]

\[ \frac{1}{2} \]

\[
\begin{pmatrix}
0 & 1 & 0 & 0 & 0 & 0 \\
1 & 0 & 1 & 0 & 0 & 0 \\
0 & 1 & 0 & 1 & 0 & 0 \\
0 & 0 & 1 & 0 & 1 & 0 \\
0 & 0 & 0 & 1 & 0 & 1 \\
0 & 0 & 0 & 0 & 1 & 0 \\
\end{pmatrix}
\]

Random-Like [Jian et al., 2012]

\[ \frac{1}{4} \]

\[
\begin{pmatrix}
1 & 1 & 0 & 0 & 0 & 0 \\
1 & 2 & 1 & 0 & 0 & 0 \\
0 & 1 & 2 & 1 & 0 & 0 \\
0 & 0 & 1 & 2 & 1 & 0 \\
0 & 0 & 0 & 1 & 2 & 1 \\
0 & 0 & 0 & 0 & 1 & 1 \\
\end{pmatrix}
\]

capacity-achieving at high rates over the binary symmetric channel

- Equations have the same form \( \implies \) similar performance
- The performance of random-like codes (over the binary erasure channel) can be “emulated” with deterministic codes [Paper F]
Design and Analysis of Deterministic Codes

Summary
Design and Analysis of Deterministic Codes

Summary

- Several deterministic codes (including spatially-coupled versions) have been proposed for fiber-optical communications
Design and Analysis of Deterministic Codes

Summary

- Several deterministic codes (including spatially-coupled versions) have been proposed for fiber-optical communications.
- Rigorous asymptotic performance analysis over the binary erasure channel under iterative bounded-distance decoding possible.
Design and Analysis of Deterministic Codes

Summary

- Several deterministic codes (including spatially-coupled versions) have been proposed for fiber-optical communications.
- Rigorous asymptotic performance analysis over the binary erasure channel under iterative bounded-distance decoding possible.
- Future work: extension to binary symmetric channel.
Large interest in analyzing and designing spectrally-efficient fiber-optical systems ([Essiambre et al., 2010], [Smith and Kschischang, 2010], [Schmalen et al., 2013], [Beygi et al., 2014], . . . )
Spectrally-Efficient Communication

![Diagram showing communication channel](image)
Spectrally-Efficient Communication

communication channel

multilevel signal constellation
Spectrally-Efficient Communication

\[ \Phi \]

\[ b_1, b_2, \ldots, b_m \]

communication channel

000 001 011 010 110 111 101 100
Spectrally-Efficient Communication

\[ \Phi \rightarrow \text{communication channel} \rightarrow \Phi^{-1} \]

- modulator: \( b_1, \ldots, b_m \)
- demodulator: log-likelihood ratios (LLR) ("soft" information)

\[ l_1, \ldots, l_m \]

000 001 011 010 110 111 101 100
Spectrally-Efficient Communication

\[
\begin{align*}
\Phi & \quad \quad \quad \text{communication channel} \quad \quad \quad \Phi^{-1} \\
\Phi^{-1} & \quad \quad \quad \text{communication channel} \quad \quad \quad \Phi
\end{align*}
\]

- \( b_1 \)
- \( \vdots \)
- \( b_m \)
- \( l_1 \)
- \( \vdots \)
- \( l_m \)
Spectrally-Efficient Communication

- Approximate setup: parallel channels with different qualities (constellation size determines the number of channels)
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- Fix one binary encoder/decoder pair
Spectrally-Efficient Communication

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- Fix one binary encoder/decoder pair
- Bit mapper determines the allocation of coded bits to the channels
Spectrally-Efficient Communication

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- Fix one binary encoder/decoder pair
- Bit mapper determines the allocation of coded bits to the channels

Problem Formulation ([Richter et al., 2007], [Cheng et al., 2012], ...)
Optimize the bit mapper for a given code and signal constellation
Protograph LDPC Codes
Protograph LDPC Codes

- Compact representation of a large random-like graph [Thorpe, 2005]
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- We propose a bit mapper optimization technique that is more flexible than previous approaches in [Divsalar and Jones, 2005], [Jin et al., 2010], [Van Nguyen et al., 2011]
Protograph LDPC Codes

- Compact representation of a large random-like graph [Thorpe, 2005]
- We propose a bit mapper optimization technique that is more flexible than previous approaches in [Divsalar and Jones, 2005], [Jin et al., 2010], [Van Nguyen et al., 2011]

AR4JA codes [Divsalar et al., 2005]
Protograph LDPC Codes

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AR4JA codes [Divsalar et al., 2005]

spatially-coupled LDPC codes
Terminated
Terminated

protograph
Terminated

protograph

graph irregularity: yes (boundaries)
Terminated

protograph

- graph irregularity: yes (boundaries)
- wave effect: yes (capacity-approaching)
Terminated

- protograph
- graph irregularity: yes (boundaries)
- wave effect: yes (capacity-approaching)
- rate loss: yes
Terminated

- protograph

Tail-biting

- graph irregularity: yes (boundaries)
- wave effect: yes (capacity-approaching)
- rate loss: yes
Terminated

- protograph

- graph irregularity: yes (boundaries)

- wave effect: yes (capacity-approaching)

- rate loss: yes

Tail-biting

- no
**Terminated**

- Protograph
- Graph irregularity: yes (boundaries)
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- Rate loss: yes

**Tail-biting**

- Protograph
- Graph irregularity: no
- Wave effect: no (comparable to regular LDPC)
- Rate loss: yes
**Terminated**

- Protograph
- Graph irregularity: yes (boundaries)
- Wave effect: yes (capacity-approaching)
- Rate loss: yes

**Tail-biting**

- Graph irregularity: no
- Wave effect: no (comparable to regular LDPC)
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Terminated

Terminated

Tail-biting

protograph

graph irregularity

yes (boundaries)

wave effect

yes

(rate-approaching)

rate loss

yes

no

Idea: Use unequal error protection of a multilevel signal constellation to induce wave-like decoding behavior for tail-biting codes.
Optimization Result: Decoding Behavior

predicted BER per spatial position (baseline)

window decoder

predicted BER per spatial position (optimized)
Optimization Result: Decoding Behavior

predicted BER per spatial position (baseline)

predicted BER per spatial position (optimized)
Optimization Result: Decoding Behavior

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Optimization Result: Decoding Behavior

predicted BER per spatial position (baseline)

predicted BER per spatial position (optimized)
Optimization Result: Decoding Behavior

Locally improved decoding convergence in the first spatial positions leads to wave-like decoding behavior, similar to terminated spatially-coupled codes.
Design of Spectrally-Efficient Fiber-Optical Systems

Summary
Design of Spectrally-Efficient Fiber-Optical Systems

Summary

- Spectrally-efficient communication with binary codes leads to the problem of bit mapper optimization
Summary

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- Optimized bit mapper can offer significant performance improvements
Summary

- Spectrally-efficient communication with binary codes leads to the problem of bit mapper optimization
- Optimized bit mapper can offer significant performance improvements
- For tail-biting spatially-coupled codes, unequal error protection of a nonbinary signal constellation can be exploited to induce wave-like decoding behavior
Conclusions
## Conclusions

- **Spatially-coupled codes** have excellent performance using practical iterative decoding algorithms.
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- Certain **deterministic** codes (including spatially-coupled codes) can be analyzed rigorously with **density evolution** over the binary erasure channel.
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- Certain **deterministic codes** (including spatially-coupled codes) can be analyzed rigorously with **density evolution** over the binary erasure channel.
- Optimizing bit mappers can offer significant performance improvements, in particular for **tail-biting spatially-coupled codes**.
Conclusions

- Spatially-coupled codes have excellent performance using practical iterative decoding algorithms.
- Certain deterministic codes (including spatially-coupled codes) can be analyzed rigorously with density evolution over the binary erasure channel.
- Optimizing bit mappers can offer significant performance improvements, in particular for tail-biting spatially-coupled codes.

Thank you!
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