## Model-Based Machine Learning for Fiber-Optic Communication Systems

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 $\begin{array}{l} \mbox{Joint work with: Henry D. Pfister^{(2)}, Rick M. Bütler^{(3)}, \\ \mbox{Gabriele Liga}^{(3)}, Alex Alvarado^{(3)}, Christoffer Fougstedt^{(4)}, \\ \mbox{Lars Svensson}^{(4)}, \mbox{and Per Larsson-Edefors}^{(4)} \end{array}$ 

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Van der Meulen Seminar, December 13, 2019



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Multi-layer neural networks: impressive performance, countless applications



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#### Multi-step methods for solving the propagation equation in fiber-optics



### In this talk, we ....





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 show that multi-layer neural networks and the so-called split-step method in fiber-optics have the same functional form: both alternate linear and pointwise nonlinear steps





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- show that multi-layer neural networks and the so-called split-step method in fiber-optics have the same functional form: both alternate linear and pointwise nonlinear steps
- propose a model-based machine-learning approach based on parameterizing the split-step method (no black-box neural networks)



#### In this talk, we ...

- show that multi-layer neural networks and the so-called split-step method in fiber-optics have the same functional form: both alternate linear and pointwise nonlinear steps
- propose a model-based machine-learning approach based on parameterizing the split-step method (no black-box neural networks)
- 3. apply the proposed approach by revisiting hardware-efficient nonlinear equalization with deep-learning tools



- 1. Machine Learning and Neural Networks for Communications
- 2. Model-Based Machine Learning for Fiber-Optic Systems
- 3. Nonlinear Equalization: Learned Digital Backpropagation
- 4. Outlook and Future Work
- 5. Conclusions



#### 1. Machine Learning and Neural Networks for Communications

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activation function









channel

shaping, ...



data out



[O'Shea and Hoydis, 2017], An introduction to deep learning for the physical layer, (*IEEE Trans. Cogn. Commun. Netw.*) [Karanov et al., 2018], End-to-end deep learning of optical fiber communications (*J. Lightw. Technol.*) [Jones et al., 2018], Deep learning of geometric constellation shaping including fiber nonlinearities, (*ECOC*) [Li et al., 2018], Achievable information rates for nonlinear fiber communication via end-to-end autoencoder learning, (*ECOC*)

<sup>[</sup>Shen and Lau, 2011], Fiber nonlinearity compensation using extreme learning machine for DSP-based ..., (*OECC*) [Giacoumidis et al., 2015], Fiber nonlinearity-induced penalty reduction in CO-OFDM by ANN-based ..., (*Opt. Lett.*) [Zibar et al., 2016], Machine learning techniques in optical communication, (*J. Lightw. Technol.*) [Kamalov et al., 2018], Evolution from 8qam live traffic to ps 64-qam with neural-network based nonlinearity compensation ..., (*OFC*) ...



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[O'Shea et al., 2018], Approximating the void: Learning stochastic channel models from observation with variational GANs, (arXiv) [Ye et al., 2018], Channel agnostic end-to-end learning based communication systems with conditional GAN, (arXiv) ...



Using neural networks for  $\mathcal{T}_{\theta}, \mathcal{R}_{\theta}, \mathcal{C}_{\theta}$ 

- How to choose network architecture (#layers, activation function)?
- How to initialize parameters?
- How to interpret solutions? Any insight gained?
- . . .



Using neural networks for  $\mathcal{T}_{\theta}, \mathcal{R}_{\theta}, \mathcal{C}_{\theta}$ 

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Model-based learning: sparse signal recovery [Gregor and Lecun, 2010], [Borgerding and Schniter, 2016], neural belief propagation [Nachmani et al., 2016], radio transformer networks [O'Shea and Hoydis, 2017], ...



# Outline

- 1. Machine Learning and Neural Networks for Communications
- 2. Model-Based Machine Learning for Fiber-Optic Systems
- 3. Nonlinear Equalization: Learned Digital Backpropagation
- 4. Outlook and Future Work
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# Fiber-Optic Communications



Fiber-optic systems enable data traffic over very long distances connecting cities, countries, and continents.

## Fiber-Optic Communications



Fiber-optic systems enable data traffic over very long distances connecting cities, countries, and continents.

- Dispersion: different wavelengths travel at different speeds (linear)
- Kerr effect: refractive index changes with signal intensity (nonlinear)







• Sampling over a fixed time interval  $\implies \mathcal{F}: \mathbb{C}^n \to \mathbb{C}^n$ 

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- Sampling over a fixed time interval  $\implies \mathcal{F}:\mathbb{C}^n \to \mathbb{C}^n$
- Split-step method with M steps ( $\delta = L/M$ ):



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Model-Based Learning	Learned Digital Backpropagation	Outlook and Future Work	
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Deep Learning [LeCun et al., 2015]





ResNet [He et al., 2015]







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<sup>[</sup>Häger & Pfister, 2018], Nonlinear Interference Mitigation via Deep Neural Networks, (OFC)



• Parameterized model  $f_{\theta}$  with  $\theta = \{\mathbf{A}^{(1)}, \dots, \mathbf{A}^{(M)}\}$


- Parameterized model  $f_{\theta}$  with  $\theta = {\mathbf{A}^{(1)}, \dots, \mathbf{A}^{(M)}}$
- Includes as special cases: step-size optimization, "placement" of nonlinear operator, higher-order dispersion, matched filtering ...



# **Possible Applications**









### Model-based learning approaches

- How to choose network architecture (#layers, activation function)?  $\checkmark$
- How to initialize parameters? ✓
- How to interpret solutions? Any insight gained?  $\checkmark$



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- Digital backpropagation: invert a partial differential equation in real time [Essiambre and Winzer, 2005], [Roberts et al., 2006], [Li et al., 2008], [Ip and Kahn, 2008]



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- Digital backpropagation: invert a partial differential equation in real time [Essiambre and Winzer, 2005], [Roberts et al., 2006], [Li et al., 2008], [Ip and Kahn, 2008]
- Widely considered to be impractical (too complex): linear equalization is already one of the most power-hungry DSP blocks in coherent receivers

Model-Based Learning	Learned Digital Backpropagation	Outlook and Future Work	
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# Real-Time Digital Backpropagation







Our approach: deep learning and model compression

- Joint optimization,
- pruning, and
- quantization

of all linear steps  $\implies$  hardware-efficient digital backpropagation

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# Learned Digital Backpropagation



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**TensorFlow implementation** of the computation graph  $f_{\theta}(\boldsymbol{y})$ :







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Deep learning of parameters 
$$heta=\{oldsymbol{h}^{(1)},\ldots,oldsymbol{h}^{(M)}\}$$
:

$$\min_{\boldsymbol{\theta}} \sum_{i=1}^{N} \mathsf{Loss}(f_{\boldsymbol{\theta}}(\boldsymbol{y}^{(i)}), \boldsymbol{x}^{(i)}) \triangleq g(\boldsymbol{\theta})$$
mean squared error

using  $\theta_{k+1} = \theta_k - \lambda \nabla_{\theta} g(\theta_k)$ Adam optimizer, fixed learning rate

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Iteratively prune (set to 0) outermost filter taps during gradient descent





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- $\gg 1000$  total taps (70 taps/step)  $\implies > 100 \times$  complexity of EDC
- Learned approach uses only 77 total taps: alternate 5 and 3 taps/step and use different filter coefficients in all steps [Häger and Pfister, 2018a]



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- Learned approach uses only 77 total taps: alternate 5 and 3 taps/step and use different filter coefficients in all steps [Häger and Pfister, 2018a]
- Can outperform "ideal DBP" in the nonlinear regime [Häger and Pfister, 2018b]





<sup>[</sup>Fougstedt et al., 2017], Time-domain digital back propagation: Algorithm and finite-precision implementation aspects, (OFC) [Fougstedt et al., 2018], ASIC implementation of time-domain digital back propagation for coherent receivers, (PTL) [Sherborne et al., 2018], On the impact of fixed point hardware for optical fiber nonlinearity compensation algorithms, (JLT)



• Our linear steps are very short symmetric FIR filters (as few as 3 taps)



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- 28-nm ASIC at 416.7 MHz clock speed (40 GHz signal)
  - Only 5-6 bit filter coefficients via learned quantization
  - Hardware-friendly nonlinear steps (Taylor expansion)
  - All FIR filters are fully reconfigurable

<sup>[</sup>Fougstedt et al., 2018], ASIC implementation of time-domain digital backpropagation with deep-learned chromatic dispersion filters, (ECOC)



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Machine Learning	Model-Based Learning	Learned Digital Backpropagation	Outlook and Future Work	Conclusions	
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# Real-Time ASIC Implementation



- [Crivelli et al., 2014]
  - Our linear steps are very short symmetric FIR filters (as few as 3 taps)
  - 28-nm ASIC at 416.7 MHz clock speed (40 GHz signal)
    - Only 5-6 bit filter coefficients via learned quantization
    - Hardware-friendly nonlinear steps (Taylor expansion)
    - All FIR filters are fully reconfigurable
  - $< 2 \times$  power compared to EDC [Crivelli et al., 2014, Pillai et al., 2014]

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<sup>[</sup>Fougstedt et al., 2018], ASIC implementation of time-domain digital backpropagation with deep-learned chromatic dispersion filters, (ECOC)





From [Ip and Kahn, 2009]:

- "We also note that [...] 70 taps, is much larger than expected"
- "This is due to amplitude ringing in the frequency domain"
- "Since backpropagation requires multiple iterations of the linear filter, amplitude distortion due to ringing accumulates (Goldfarb & Li, 2009)"





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- "This is due to amplitude ringing in the frequency domain"
- "Since backpropagation requires multiple iterations of the linear filter, amplitude distortion due to ringing accumulates (Goldfarb & Li, 2009)"

#### The learning approach uncovered that there is no such requirement! [Lian, Häger, Pfister, 2018]. What can machine learning teach us about communications? (*ITW*)

Previous work: design a single filter or filter pair and use it repeatedly.  $\implies$  Good overall response only possible with very long filters.



Sacrifice individual filter accuracy, but different response per step.

 $\Rightarrow$  Good overall response even with very short filters by joint optimization.







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## Wideband Signals and Subband Processing





• Subband processing: split received signal into N parallel signals

<sup>[</sup>Taylor, 2008], Compact digital dispersion compensation algorithms, (OFC)

<sup>[</sup>Ho, 2009], Subband equaliser for chromatic dispersion of optical fibre, (Electronics Lett.)

<sup>[</sup>Slim et al., 2013], Delayed single-tap frequency-domain chromatic-dispersion compensation, (PTL)

<sup>[</sup>Nazarathy and Tolmachev, 2014], Subbanded DSP architectures based on underdecimated filter banks ..., (Signal Proc. Mag.)

<sup>[</sup>Mateo et al., 2010], Efficient compensation of inter-channel nonlinear effects via digital backward ..., (Opt. Express)

<sup>[</sup>Ip et al., 2011], Complexity versus performance tradeoff for fiber nonlinearity compensation ... (OFC)

<sup>[</sup>Oyama et al., 2015], Complexity reduction of perturbation-based nonlinear compensator by sub-band processing, (OFC)

<sup>. . .</sup> 



- Subband processing: split received signal into N parallel signals
- Parameterizing the split-step method for coupled Schrödinger equations [Leibrich and Rosenkranz, 2003] ⇒ low-complexity candidate for wideband processing [Häger and Pfister, 2018c]
- Similar structure as popular convolutional neural networks (alternating filter banks and nonlinearities)

[Nazarathy and Tolmachev, 2014], Subbanded DSP architectures based on underdecimated filter banks ..., (Signal Proc. Mag.)

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## Polarization-Dependent Impairments






 Combining digital backpropagation with compensation of polarization-mode dispersion

<sup>[</sup>Goroshko et al., 2016], Overcoming performance limitations of digital back propagation due to polarization mode dispersion, (CTON) [Czegledi et al., 2017], Digital backpropagation accounting for polarization-mode dispersion, (Opt. Express) [Liga et al., 2018], A PMD-adaptive DBP receiver based on SNR optimization, (OPC)



- Combining digital backpropagation with compensation of polarization-mode dispersion
- Promising performance-complexity tradeoff using model-based factorization approach and machine learning [Häger et al., 2020]

[Goroshko et al., 2016], Overcoming performance limitations of digital back propagation due to polarization mode dispersion, (CTON) [Czegledi et al., 2017], Digital backpropagation accounting for polarization-mode dispersion, (Opt. Express) [Liga et al., 2018], A PMD-adaptive DBP receiver based on SNR optimization, (OFC) [Häger et al., 2020], Model-based machine learning for joint digital backpropagation and PMD compensation, (OFC)



# Ongoing and Future Work

- Experimental Demonstrations: stay tuned ....
- How to integrate into a standard coherent receiver DSP chain?
- How to successfully train in the presence of practical impairments (laser phase noise, transceiver noise, ...)
- How realistic is online learning in custom DSP? (We only have "hundreds" of parameters, not "thousands" or "millions" like neural networks)





## Conclusions

### neural-network-based ML

universal function approximators

good designs require experience and fine-tuning

black boxes, difficult to "open"

	Machine Learnii 000	ng Model-Based Learning 000000	Learned Digital Backpropagation	Outlook and Future Work	Conclusions •	CHALMERS
		Conclusions				
		neural-network-based ML		model-based ML		
		universal function approximators good designs require experience and fine-tuning black boxes, difficult to "open"		application-tailored relies on domain knowledge (algorithms, physics,) familiar building blocks (e.g., FIR filters) can enable interpretability		
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