The Mian Pricinple Benhid Erorr-Corrceiton Techquines

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Gothenburg, Sweden

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CHALMERS

Learning Objectives

Aoccdrnig to a rscheearch at Cmabrigde Uinervtisy, it deosn't mttaer in waht oredr the ltteers in a wrod are, the olny iprmoetnt tihng is taht the frist and lsat ltteer be at the rghit pclae. The rset can be a toatl mses and you can sitll raed it wouthit porbelm.

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After this lecture, you should be able to:

- Apply the error-correction principle and explain why we can read the text.
- Critically evaluate the hypothesis in the second sentence based on what you will learn about the fundamental limitation of error-correction.

Introduction

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written on the CD: . . . 0 1 0 1 1 0 1 0 1 . . .

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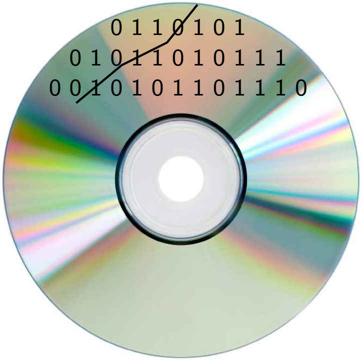

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bit errors

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bit errors

What do we do now?

The engineering goal is to provide (almost) error-free performance, even in the presence of scratches.

Short Assignment (1 Minute)

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$$1\ 0\ 1 \xrightarrow{\text{repeat each bit}} 1\ 1\ 1\ 0\ 0\ 0\ 1\ 1\ 1$$

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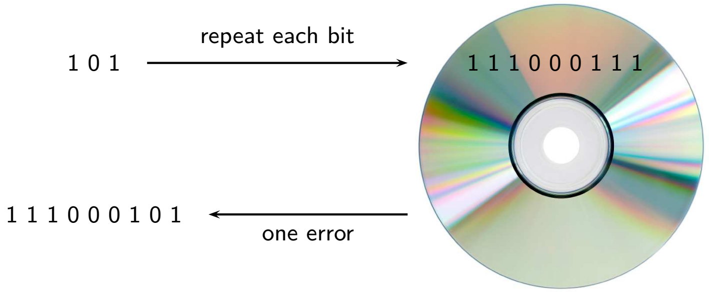

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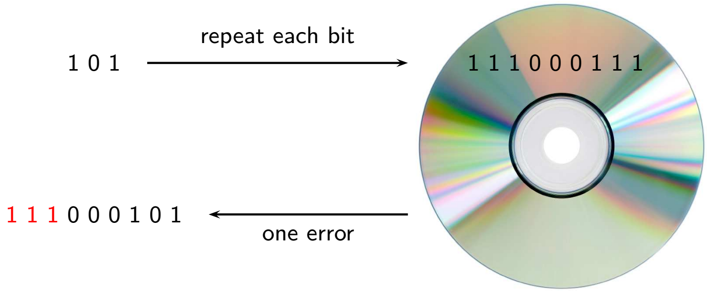

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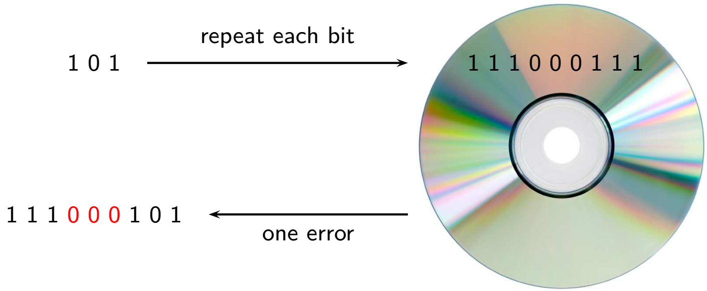

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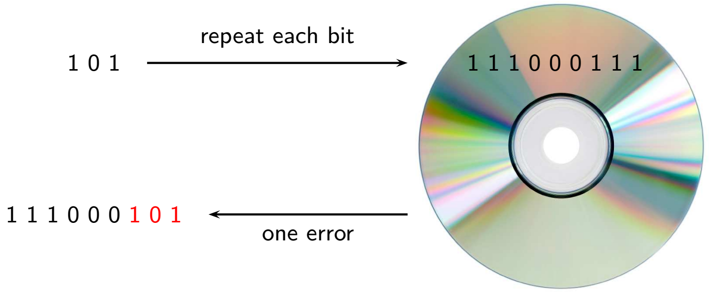

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$$1\ 0\ 1 \xrightarrow{\text{repeat each bit}} 1\ 1\ 1\ 0\ 0\ 0\ 1\ 1\ 1$$

looks more like 1 1 1 than 0 0 0

$$1\ 1\ 1\ 0\ 0\ 0\ 1\ 0\ 1 \xleftarrow[\text{one error}]{}$$

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- $\mathsf{d}(001, 011) = 1$
- $\mathsf{d}(000, 111) = 3$

Graphical Interpretation

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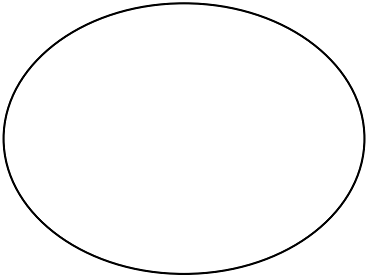

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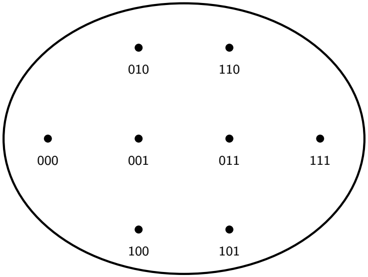

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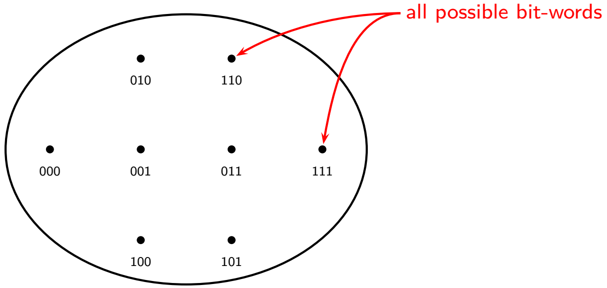

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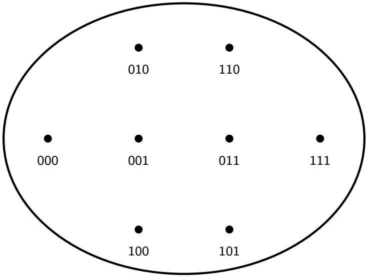

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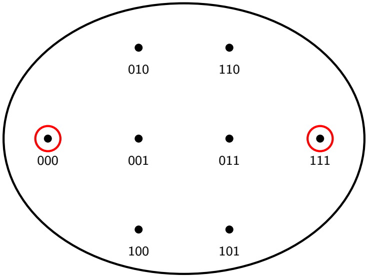

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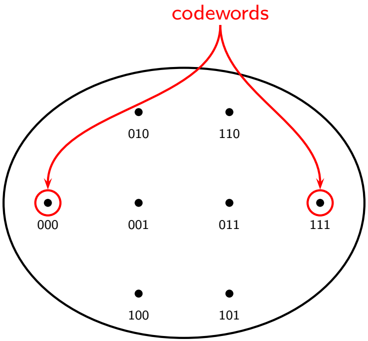

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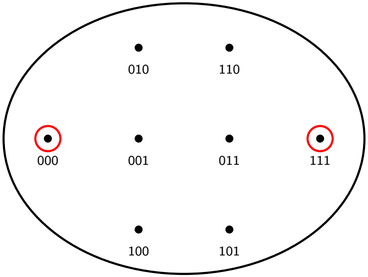

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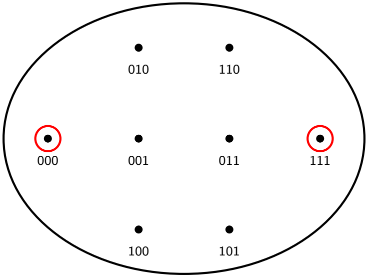

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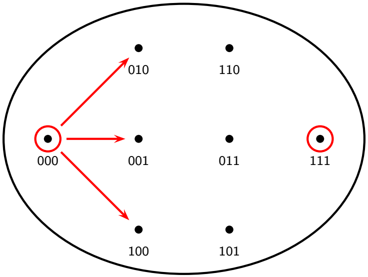

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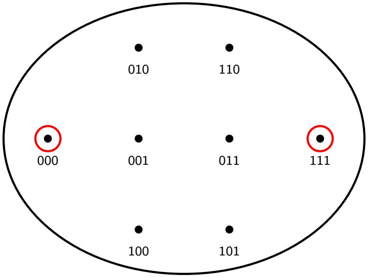

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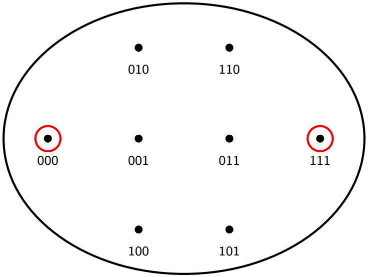

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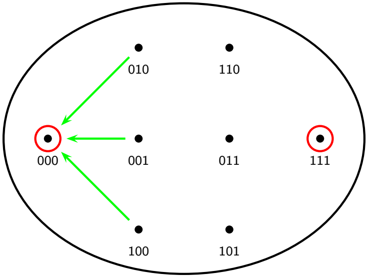

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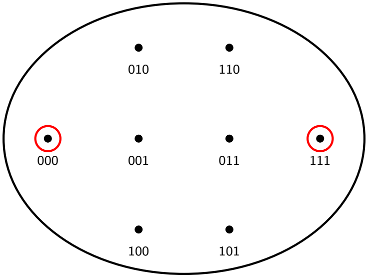

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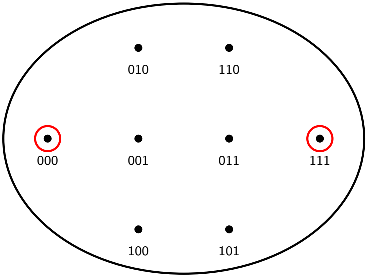

Fundamental Limitation of Error-Correction

An error cannot be corrected if it is so large that one moves too far away from the true codeword and is now, in fact, closer to another codeword.

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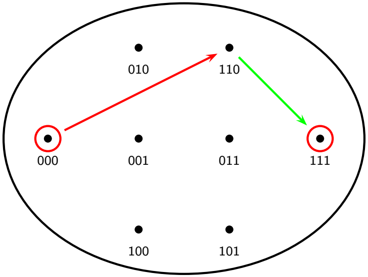

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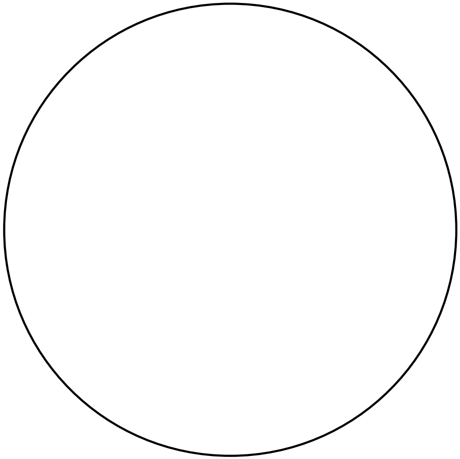
- A distance function exists (but may be hard to define rigorously).

The Space of All 5-Letter Combinations



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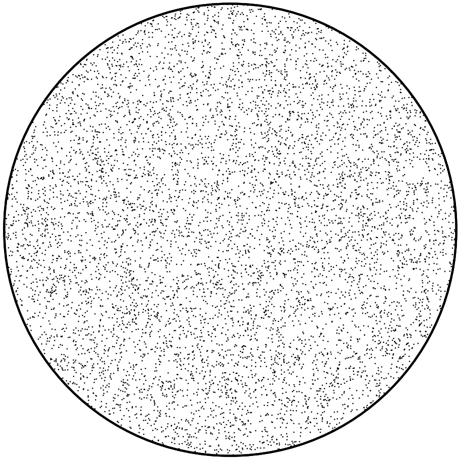

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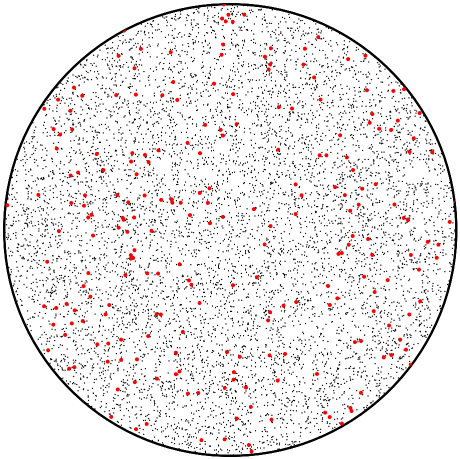
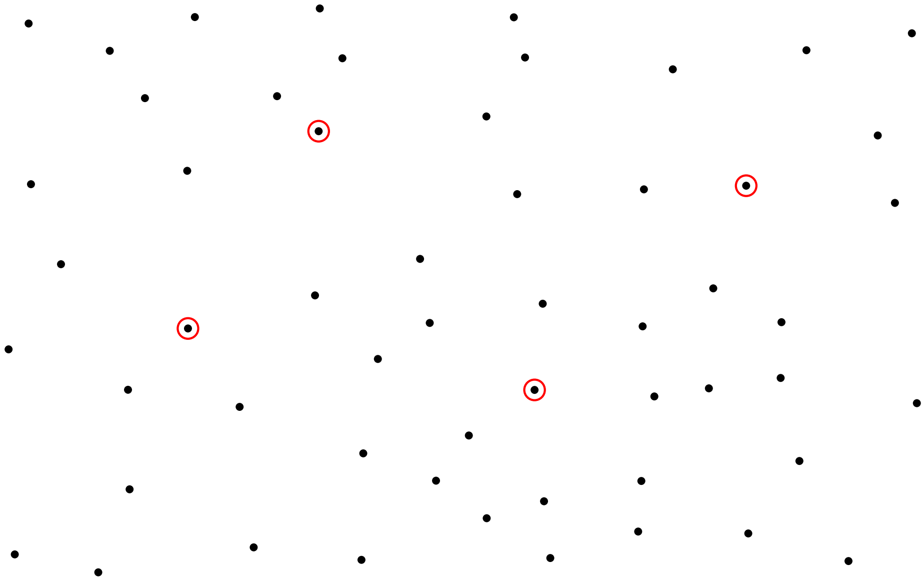


Close-Up View



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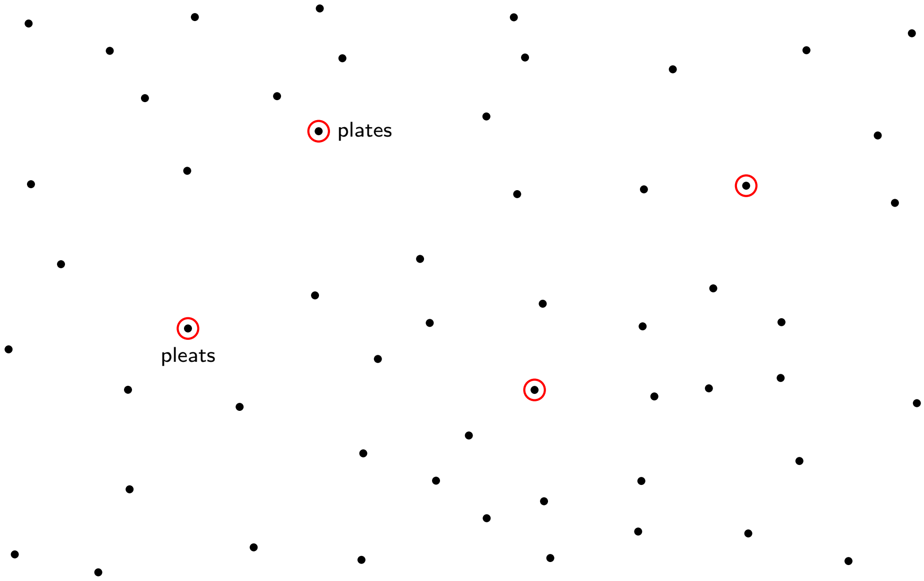

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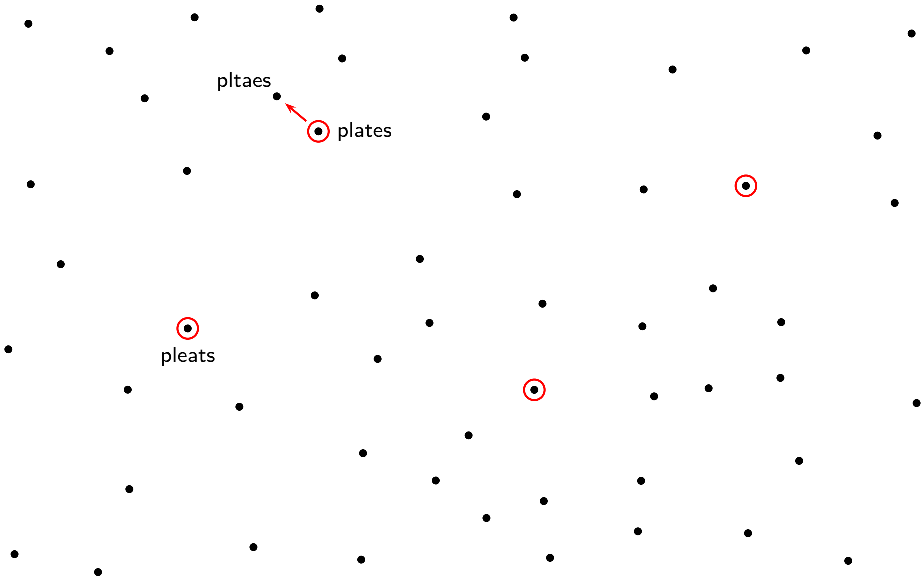

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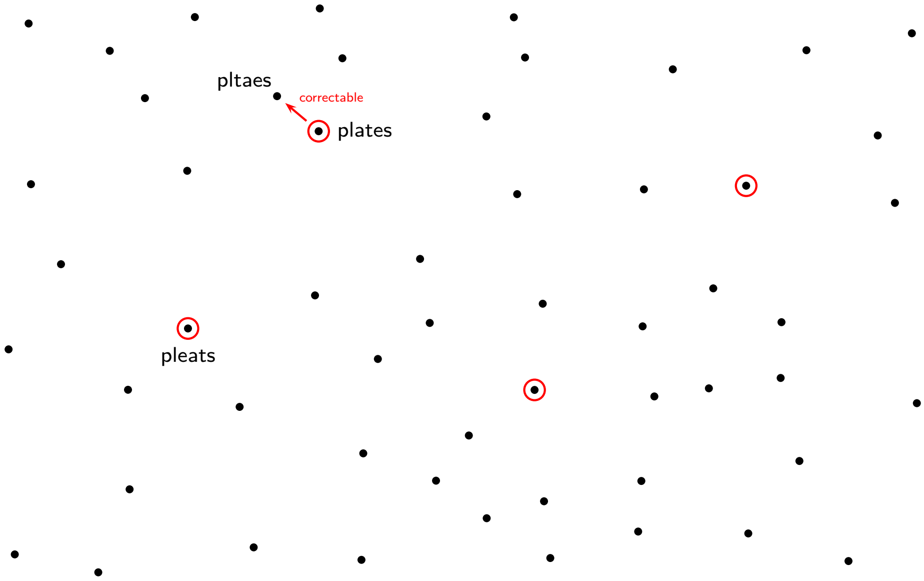

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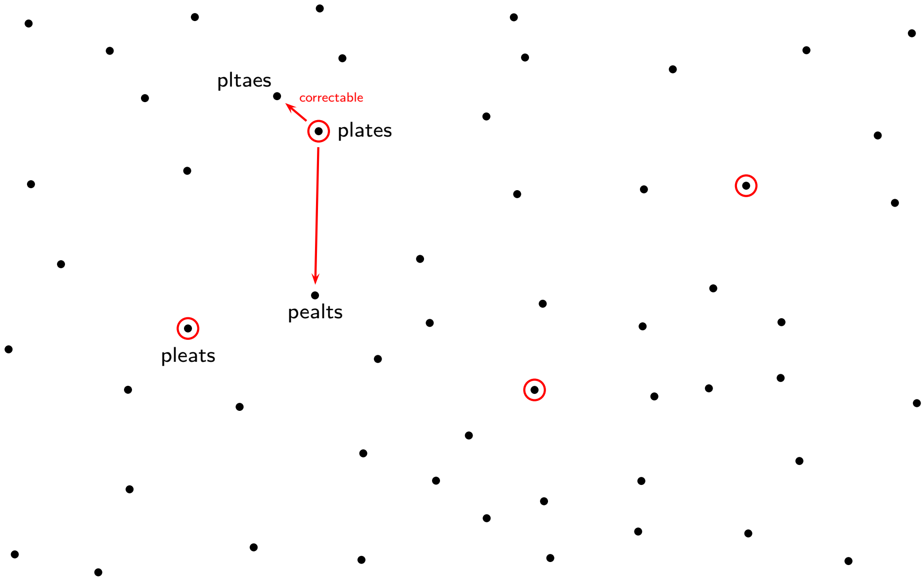

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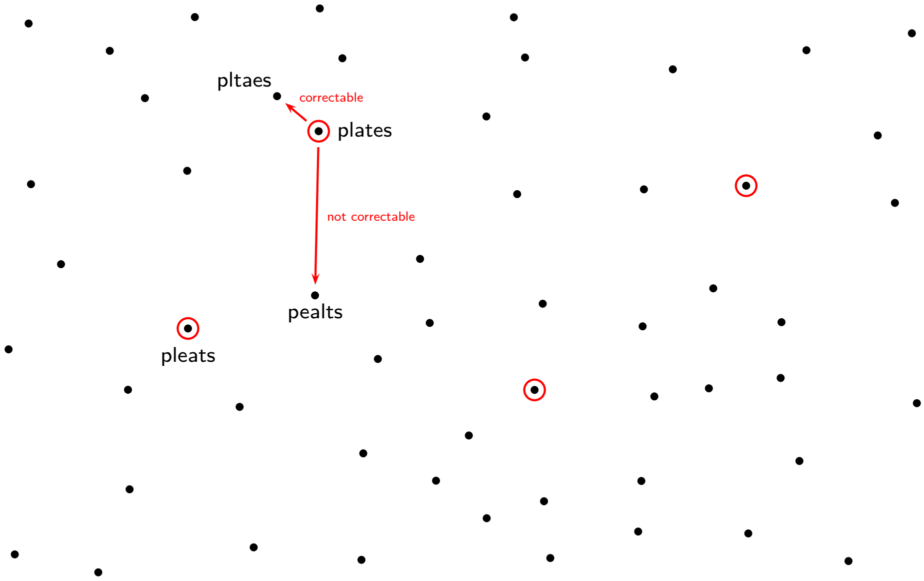

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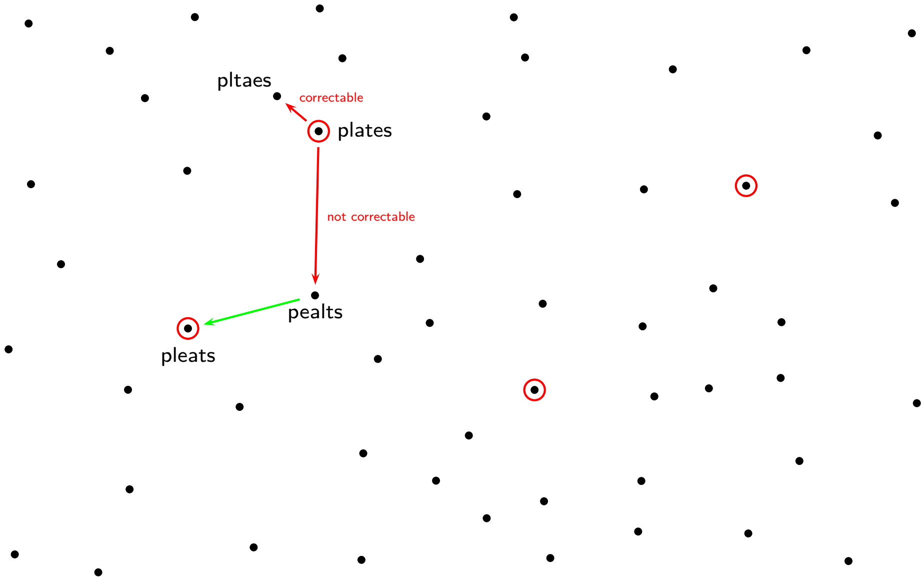

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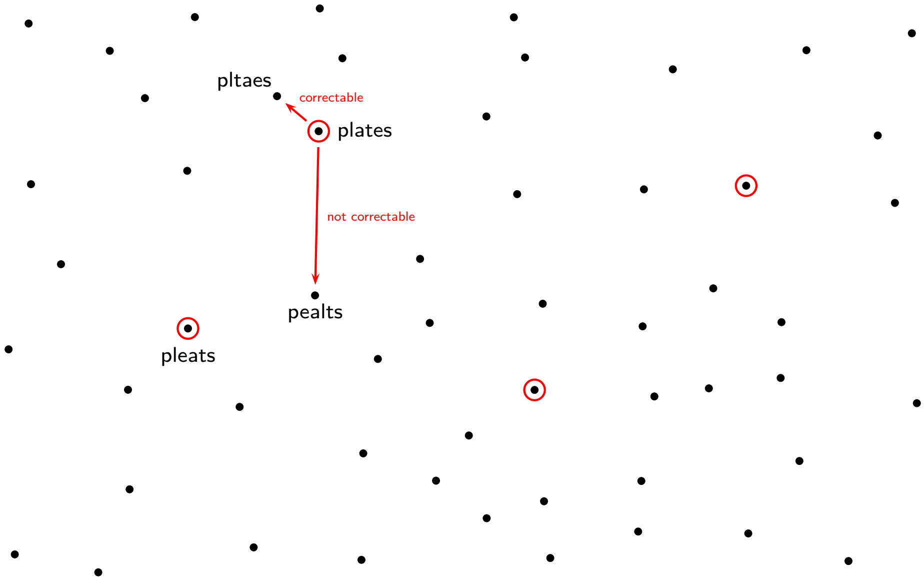

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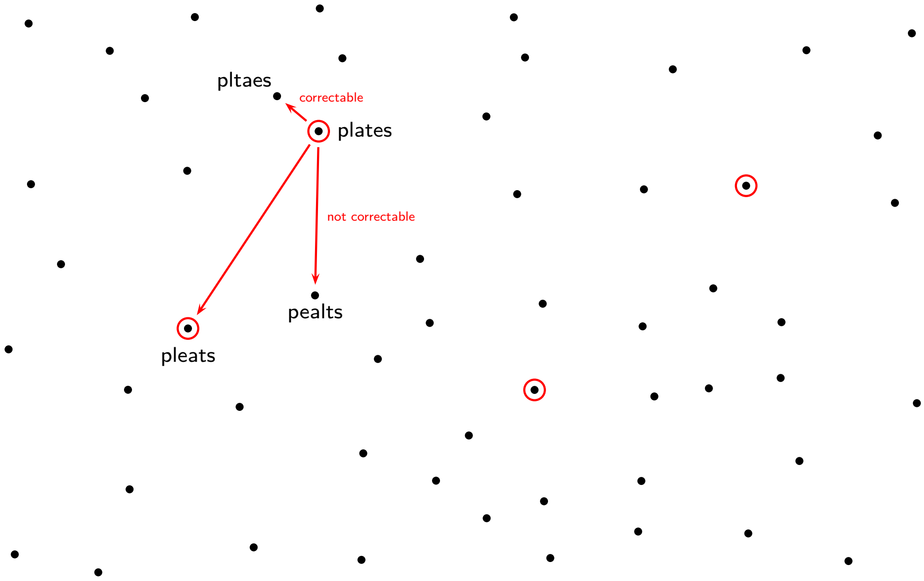

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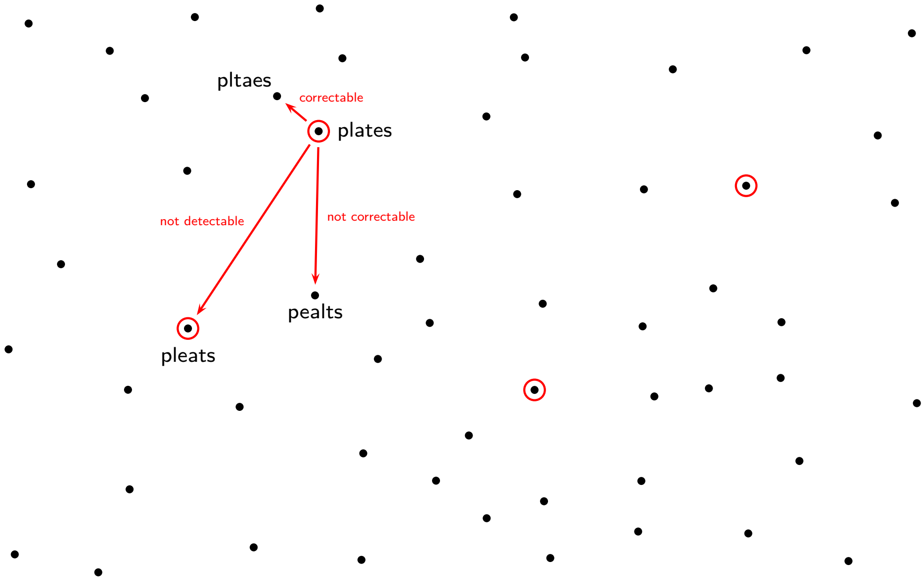

Summary

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- Error-correction has limits based on the properties of the code.
- Language can be interpreted as a code.

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