

Constellation Optimization for Coherent Optical Channels Distorted by Nonlinear Phase Noise

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Cambridge, April 17, 2013



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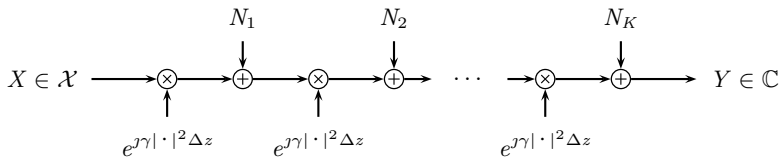
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- Optical channel suffers from distortions that are absent for example in wireless channels
- Focus here: **Nonlinear phase noise**
- **Practical question:** How much can we gain by optimizing the constellation compared to standard QAM?
- **Theoretical question:** How do optimal constellations look like for very strong nonlinearities?

Outline

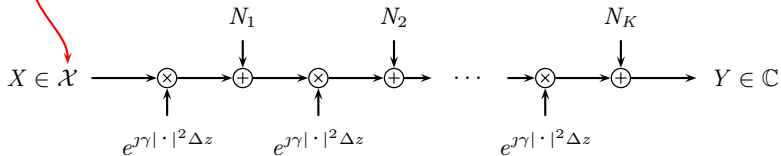
1. Introduction: Channel Model and APSK
2. Detection Methods
3. APSK Optimization
4. Conclusions

Discrete-Time Channel Model

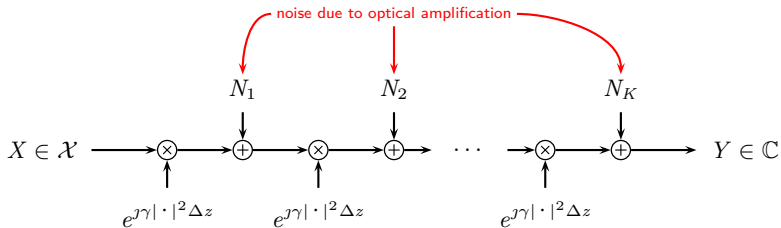


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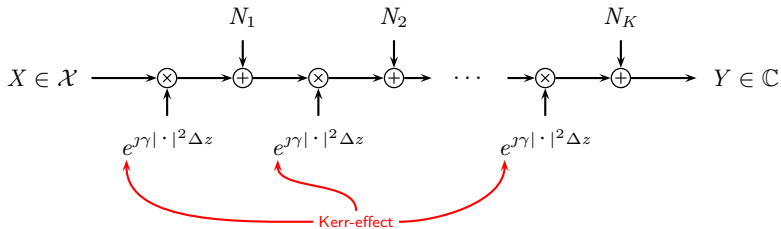
signal constellation, here $|\mathcal{X}| = 16$



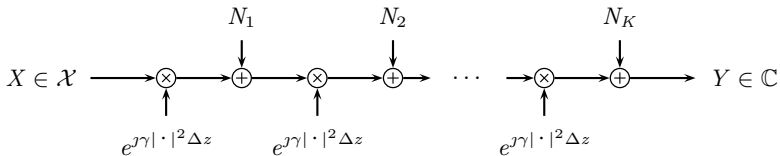
Discrete-Time Channel Model



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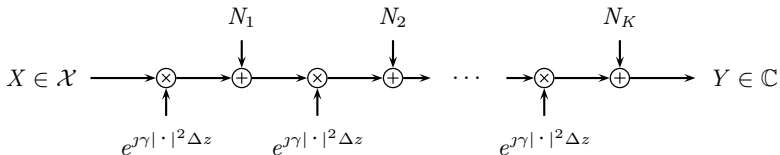


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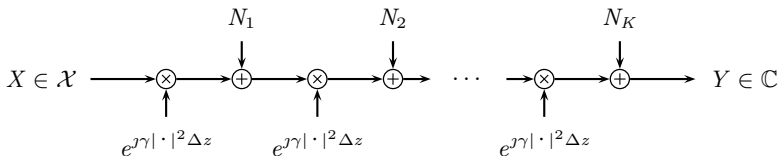
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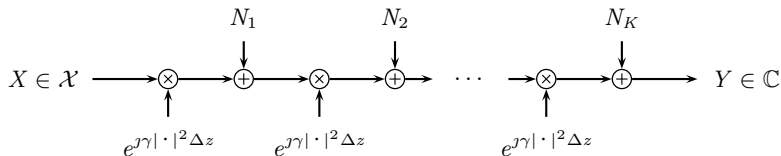
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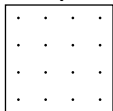


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- $L = 5500$ km, other parameters γ, σ^2 taken from [Lau and Kahn, 2007]

Power Dependent Phase Noise

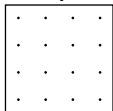
Power Dependent Phase Noise

16-QAM

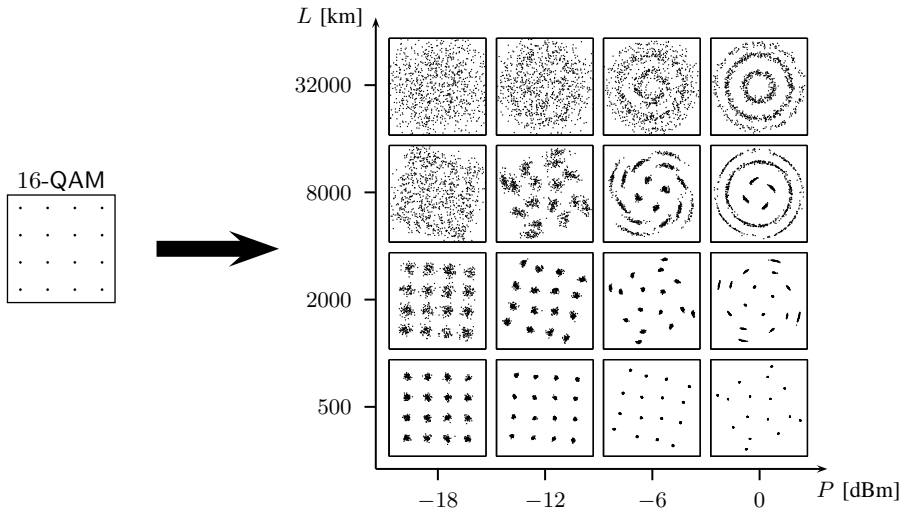


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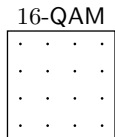
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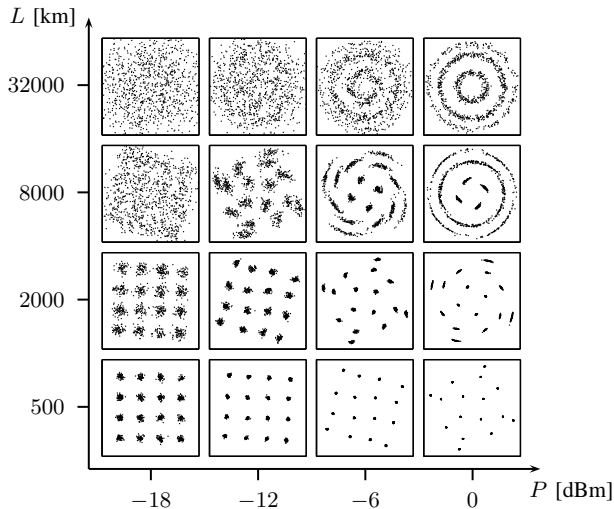
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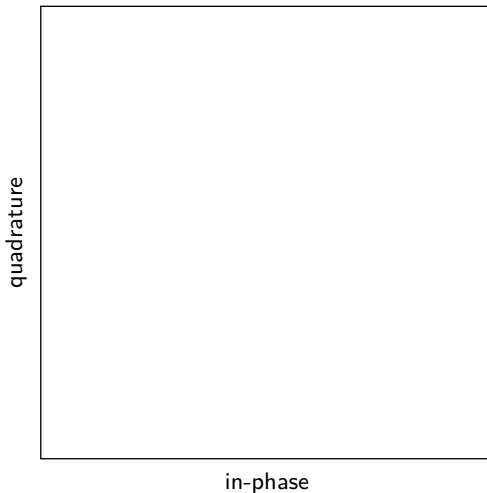
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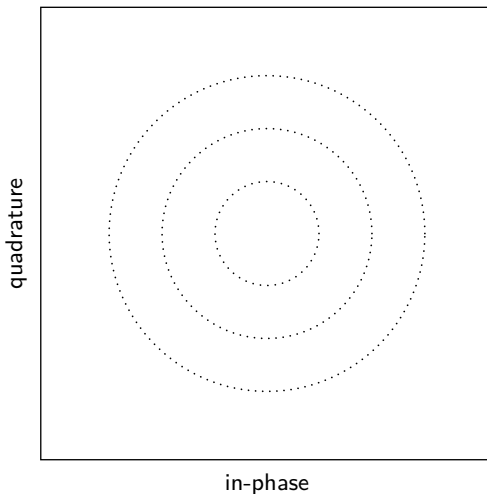
SNR is not sufficient to
characterize the channel!



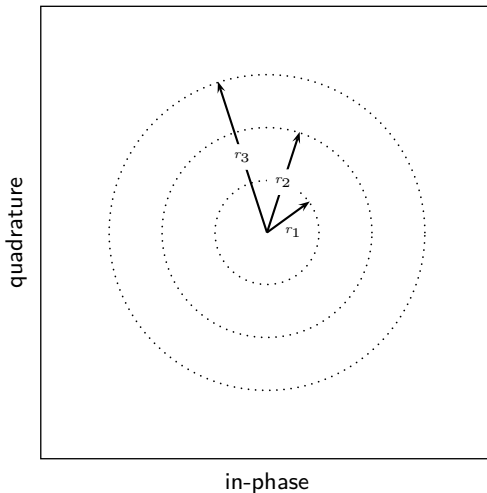
Amplitude Phase-Shift Keying (APSK), Example



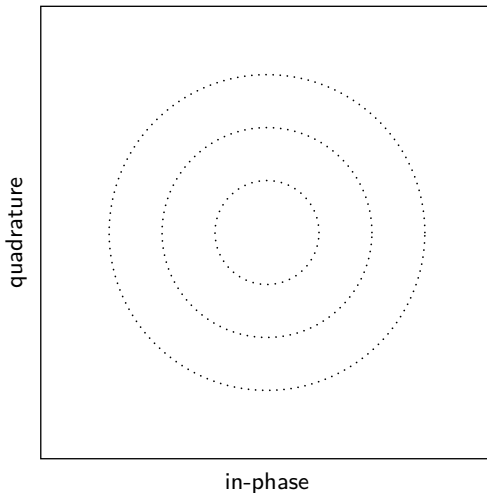
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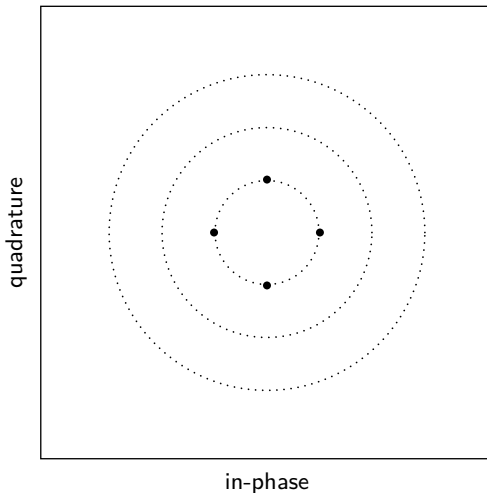


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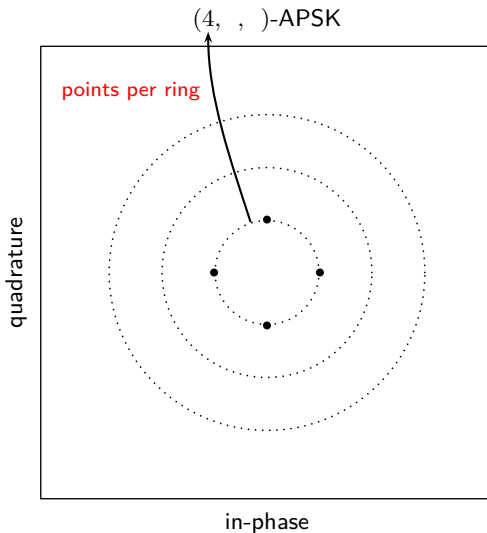


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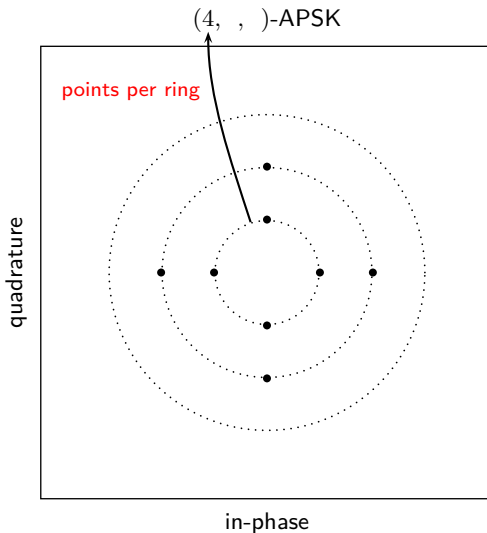
(, ,)-APSK



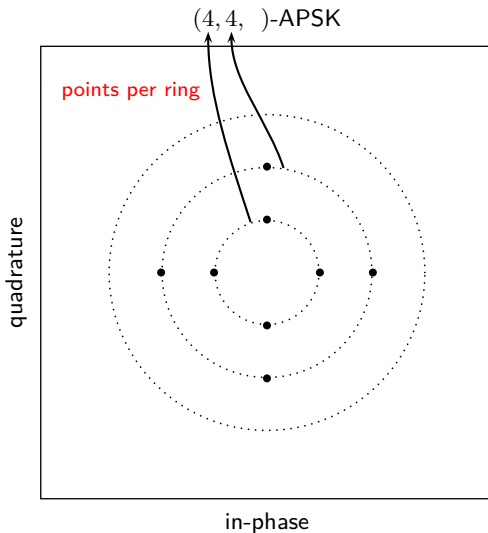
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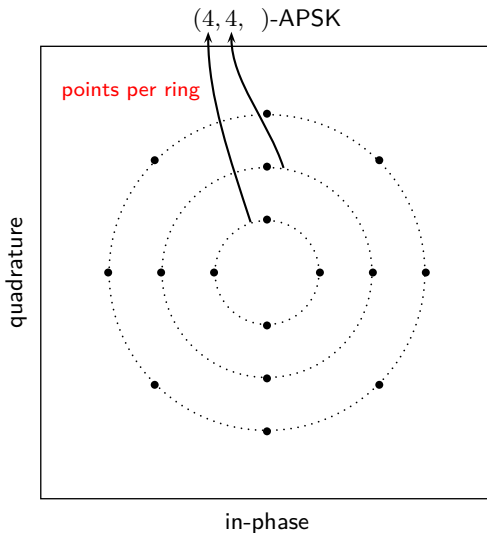
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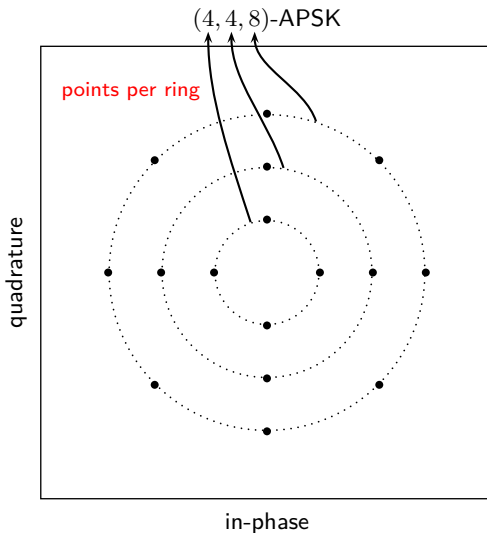
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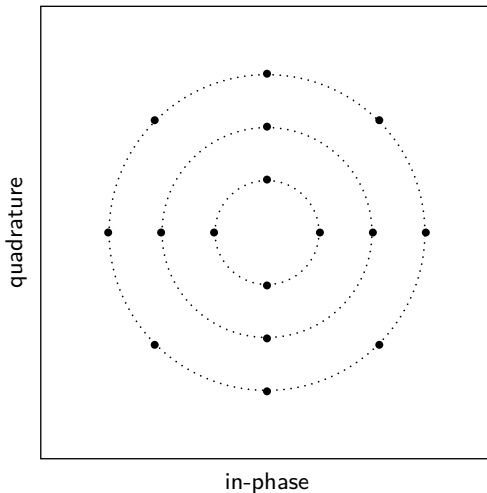


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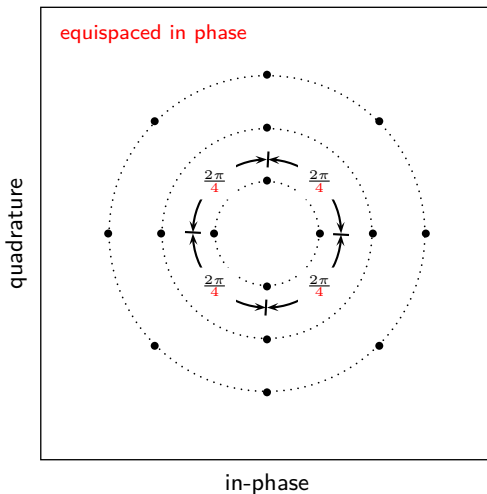
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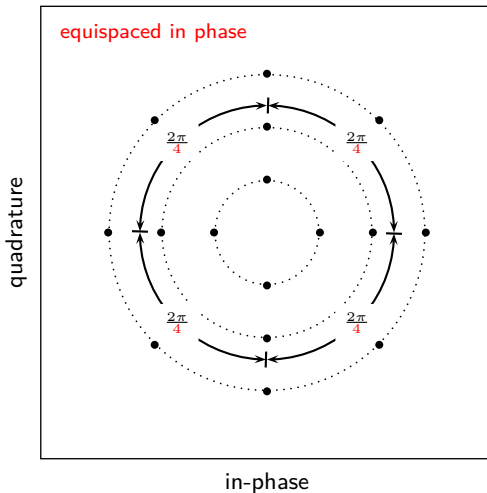
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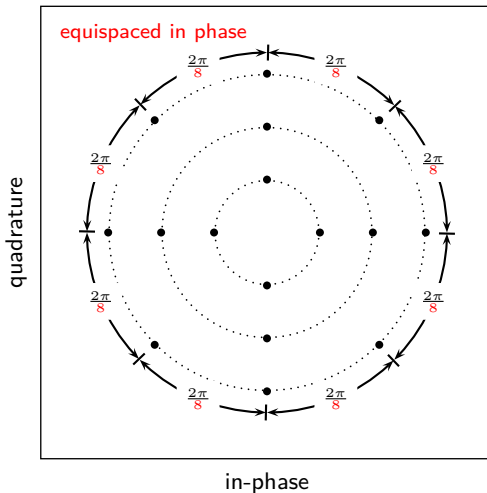
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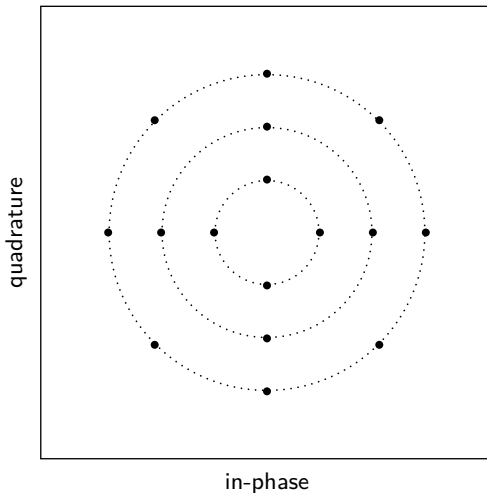
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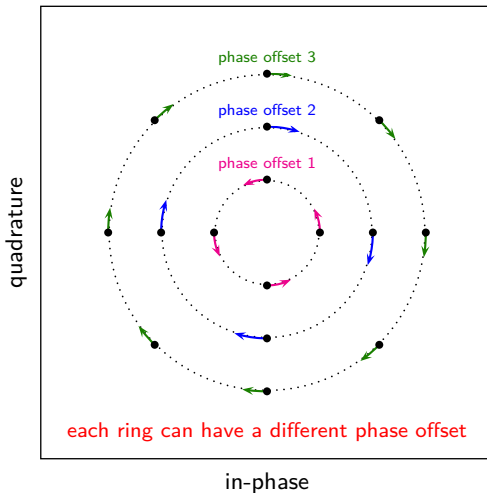
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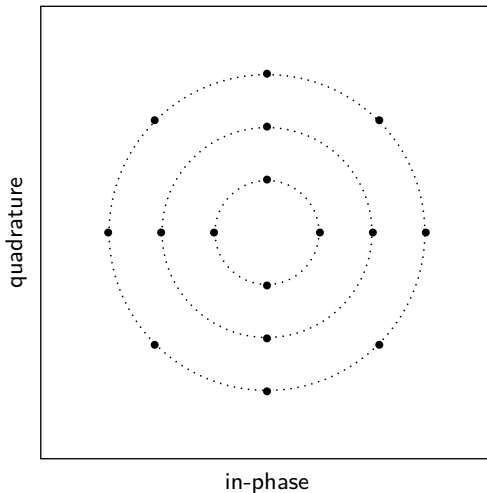


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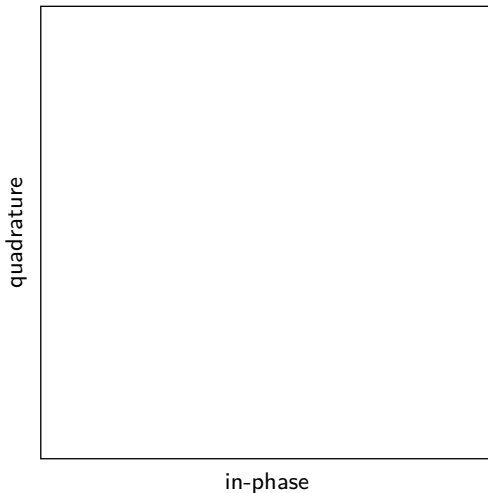
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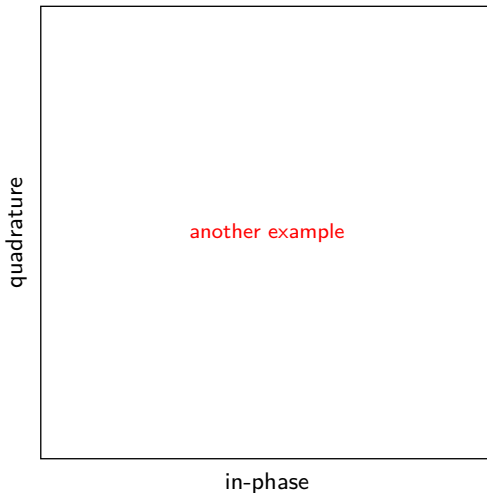
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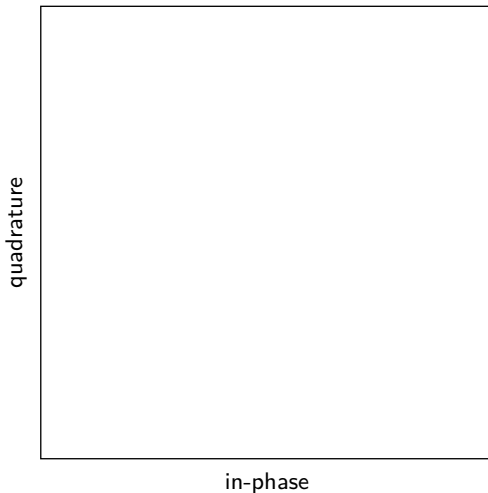


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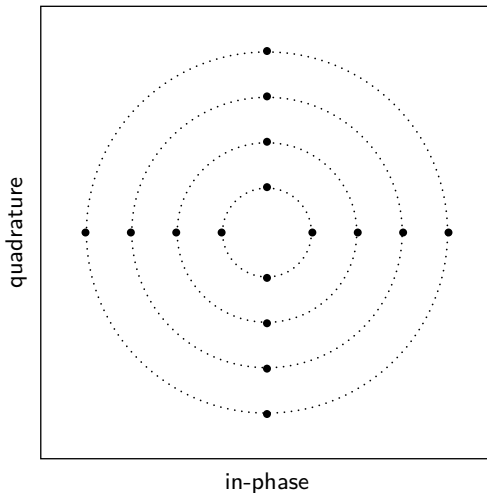
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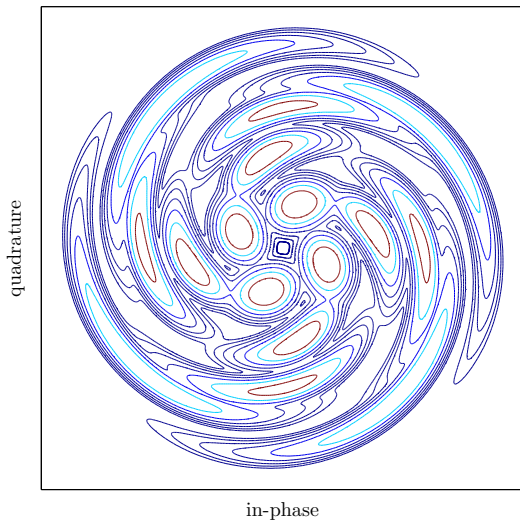
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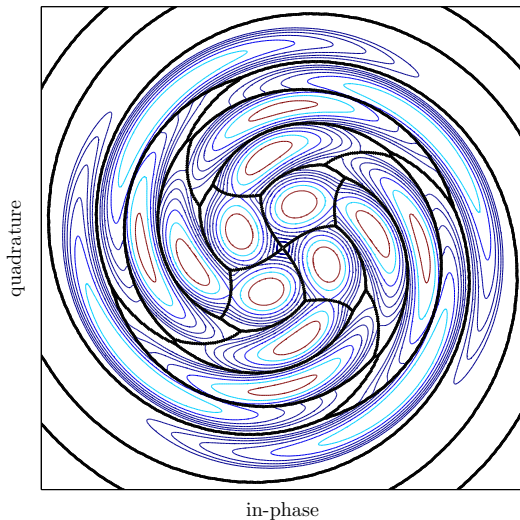
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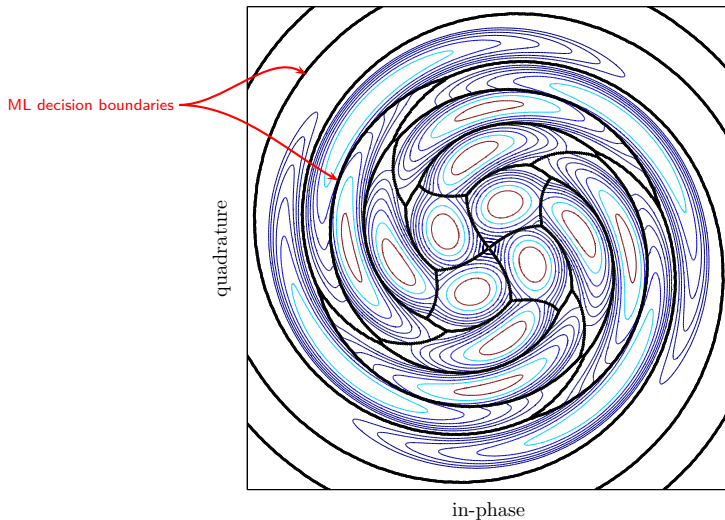
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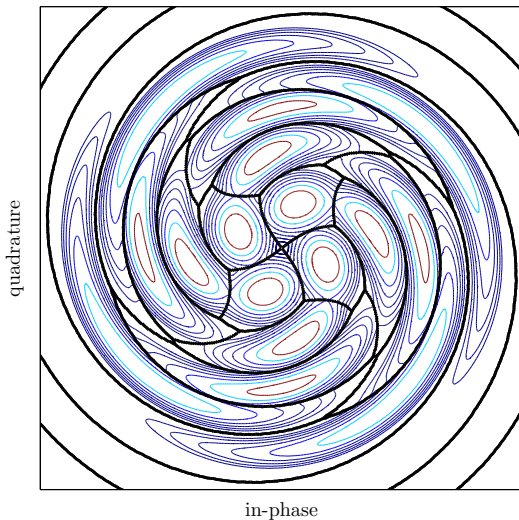
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- Probability density function (PDF) $f_{Y|X=x_i}(y)$ **known**

PDF for (4,4,4,4)-APSK at $P = -4$ dBm

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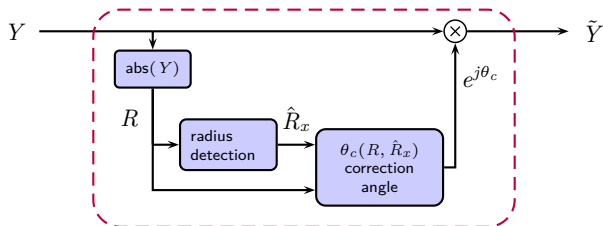
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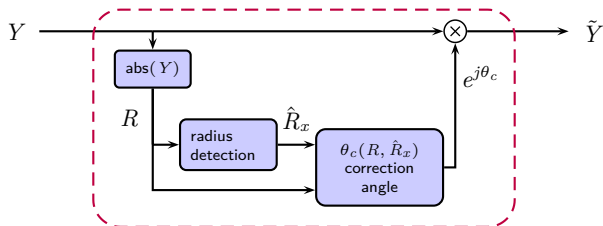
ML detection possible,
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Nonlinear Phase Postcompensation

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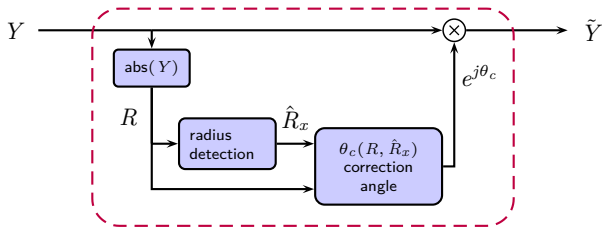


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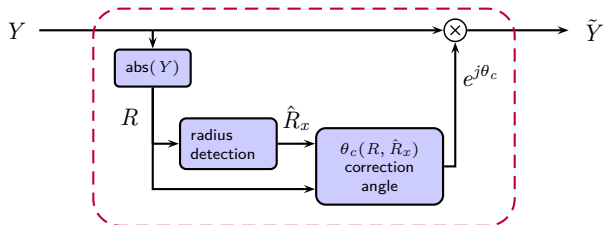
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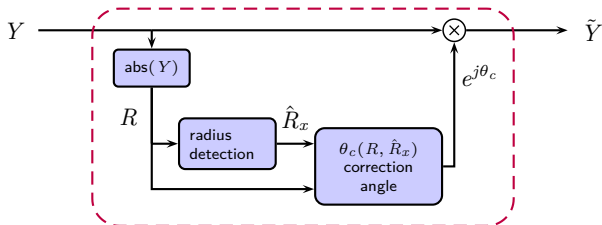
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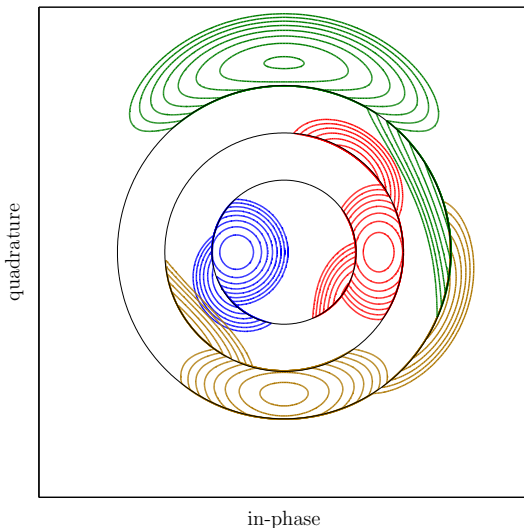
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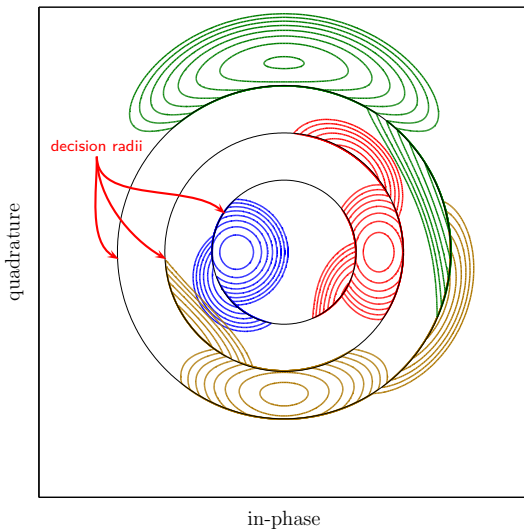
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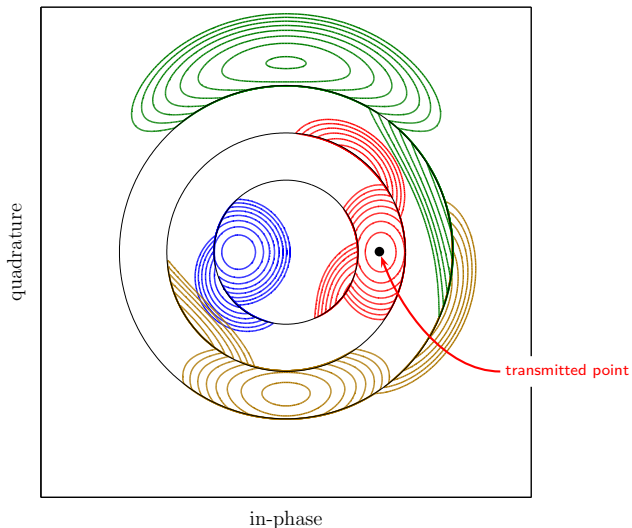


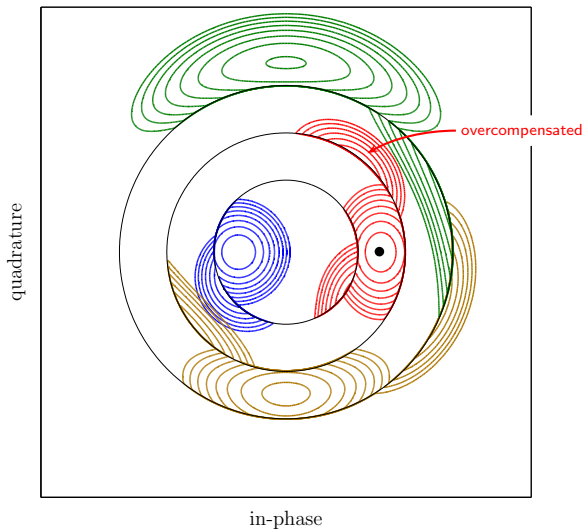
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- Note: the PDF of \tilde{Y} is defined piecewise

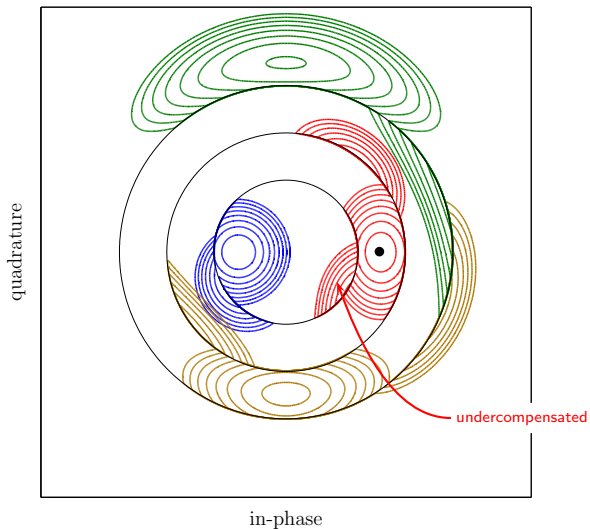
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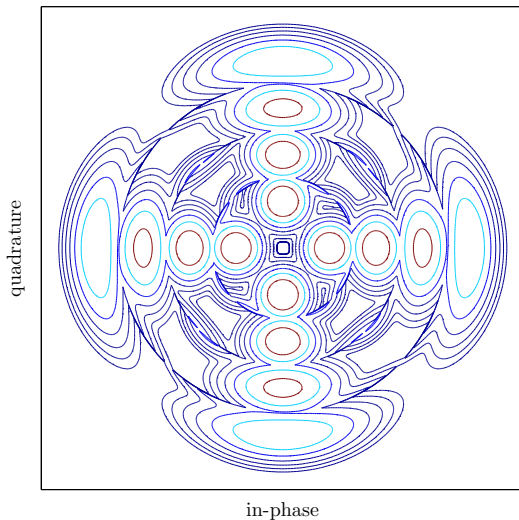
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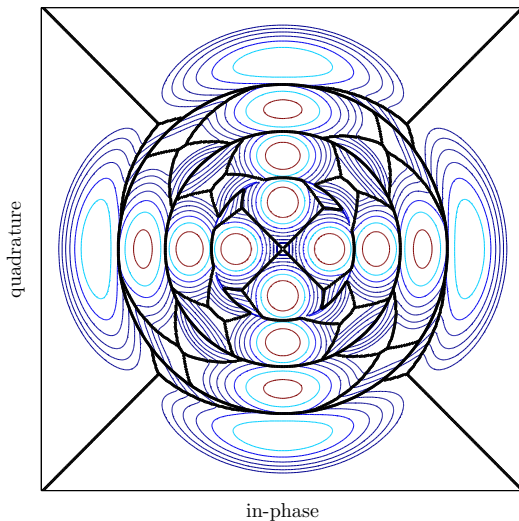
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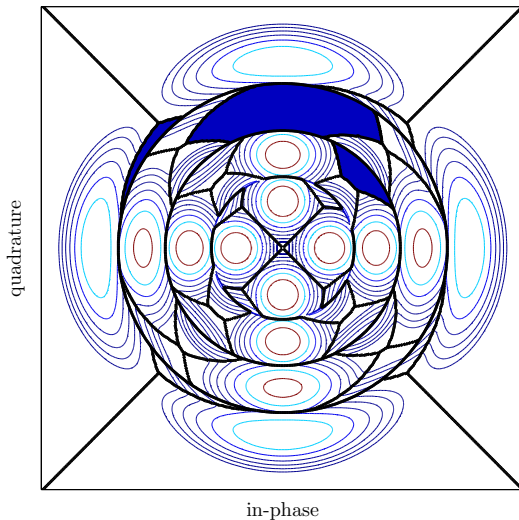
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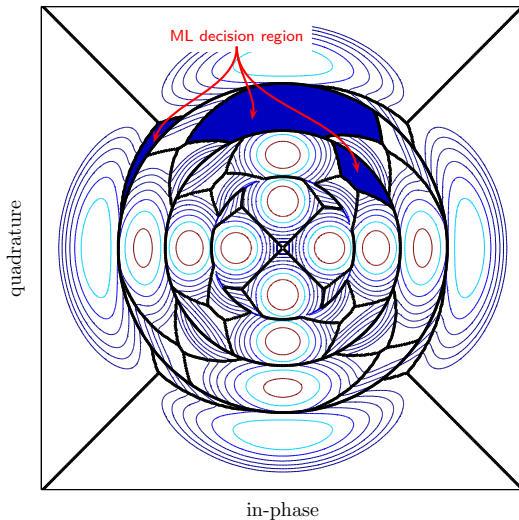
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ML Detection based on \tilde{Y}

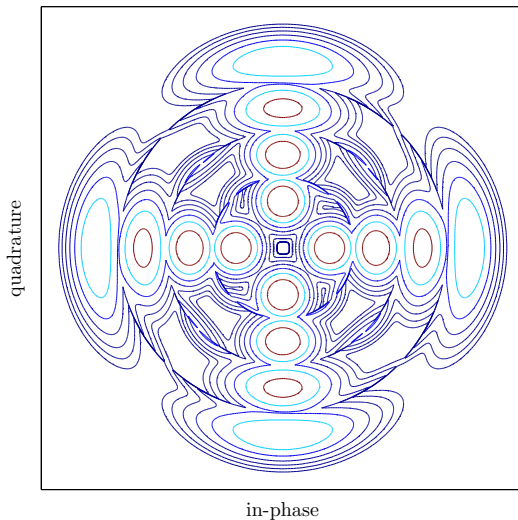
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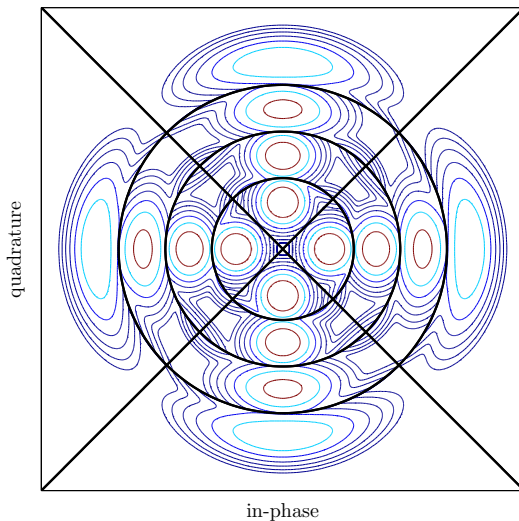
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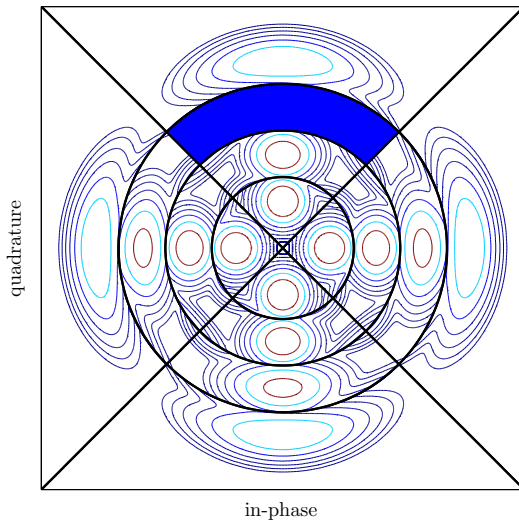
Two-Stage Detection



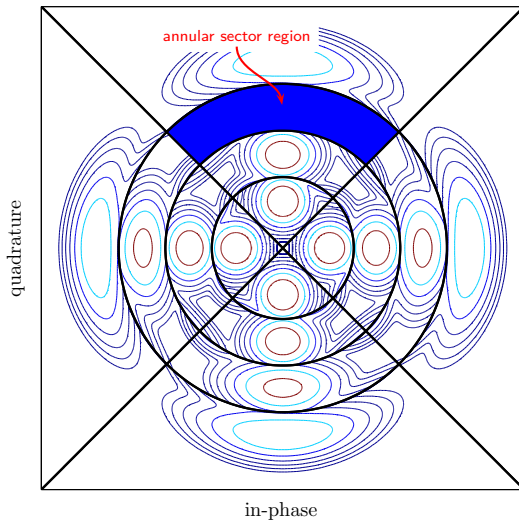
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Optimization Opportunities for APSK

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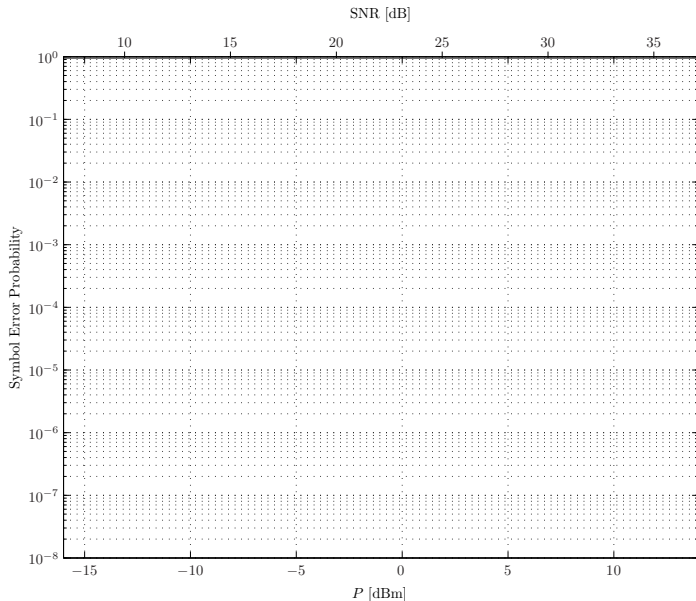
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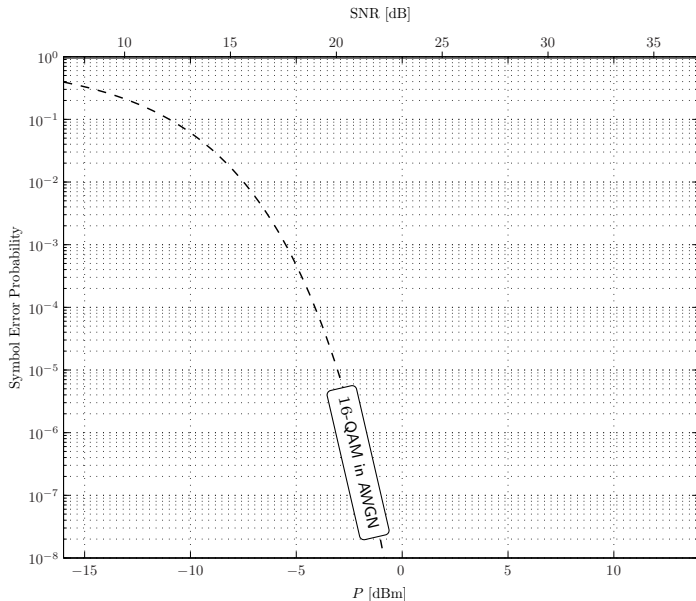
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- What phase offset? **Two-stage detection is insensitive to a phase offset.**

Optimizing the Number of Rings and Points per Ring

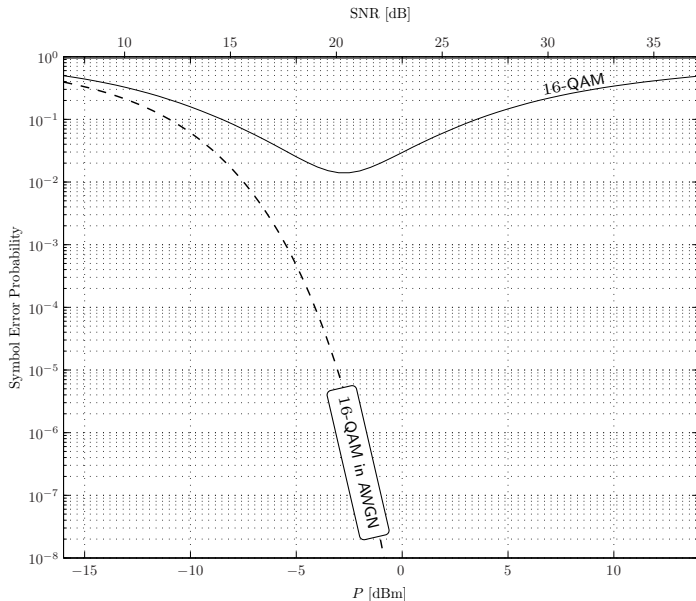
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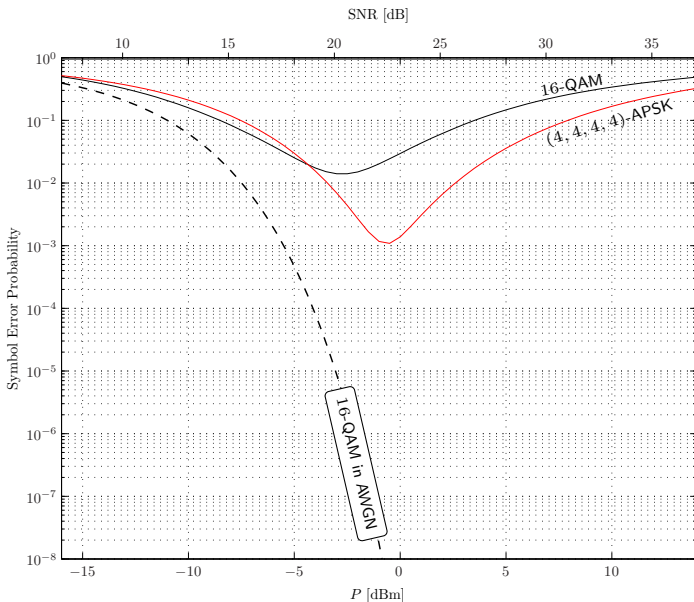
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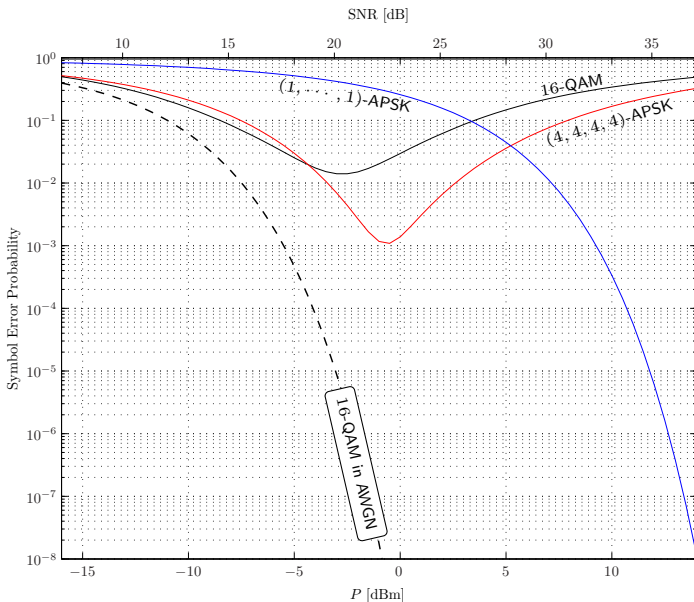
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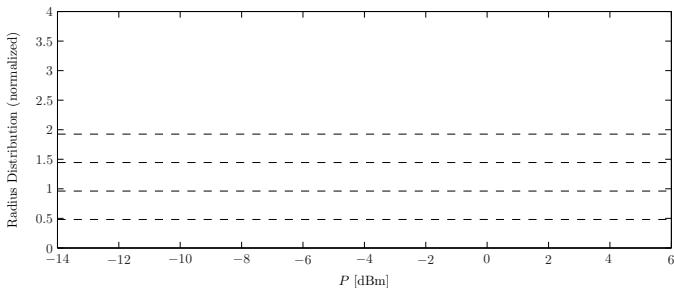
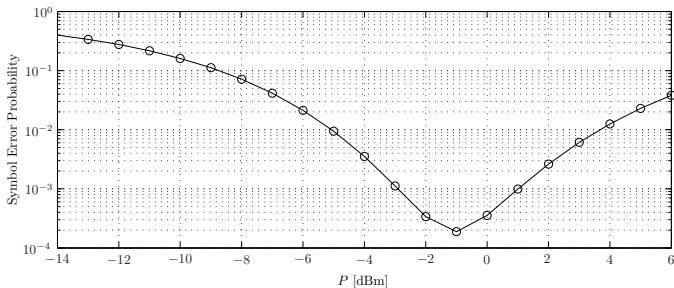
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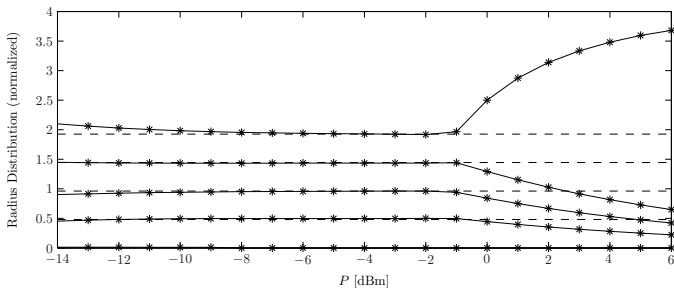
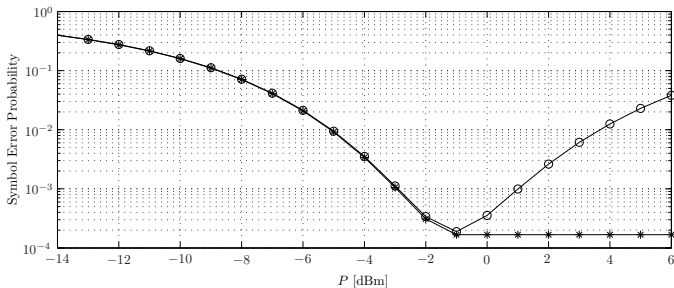
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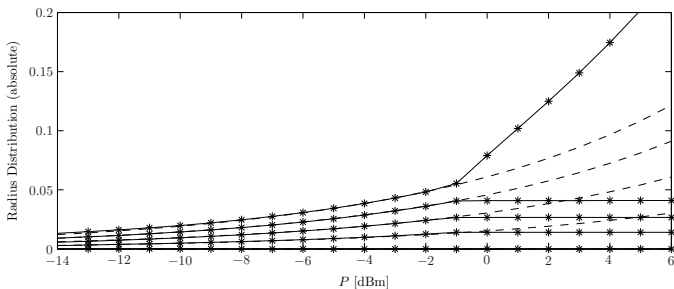
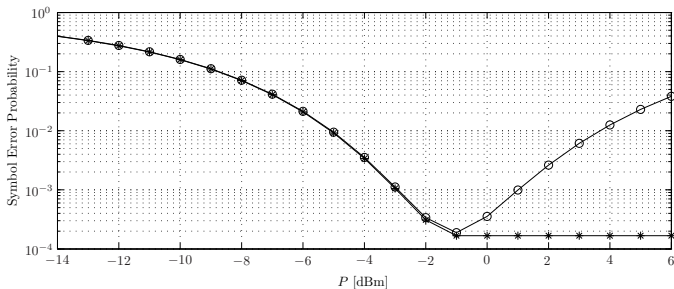
Radius Optimization for (1,6,5,3,1)-APSK



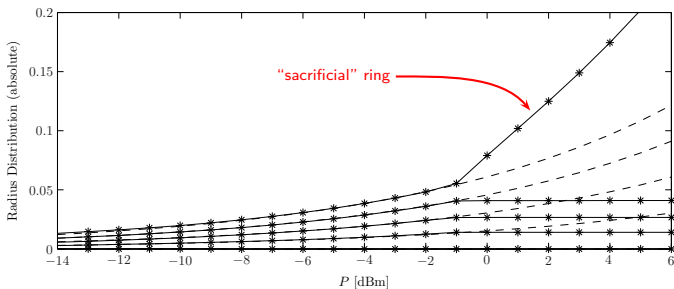
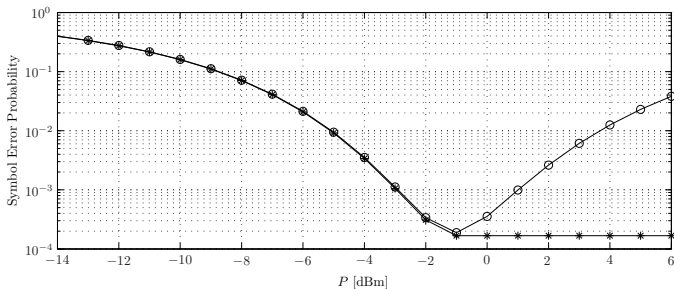
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Thank you!



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