Physics-Based Machine Learning for Fiber-Optic Communication Systems

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Department of Electrical Engineering, Chalmers University of Technology, Sweden

Workshop on "Digital Signal Processing in & Optical Fiber Communication"
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CHALMERS

FORCE
FIBER-OPTIC COMMUNICATIONS RESEARCH CENTER

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Duke

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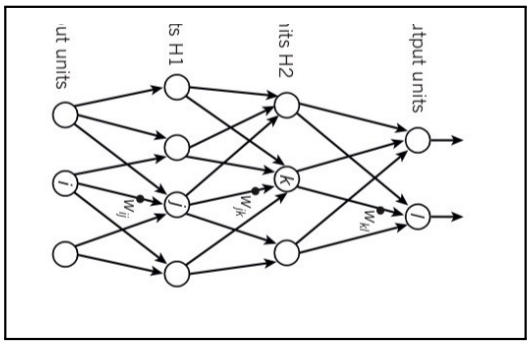
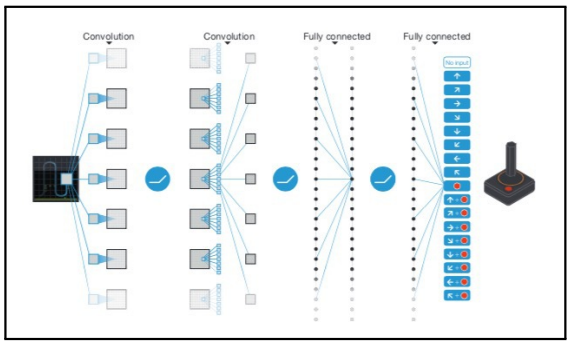
This work started with a simple observation . . .

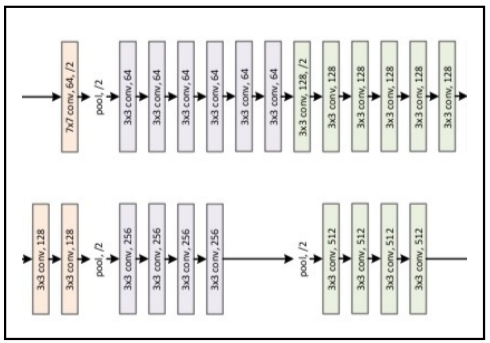
Deep Learning [LeCun et al., 2015]



Deep Q-Learning [Mnih et al., 2015]



ResNet [He et al., 2015]



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Multi-layer neural networks: impressive performance, countless applications

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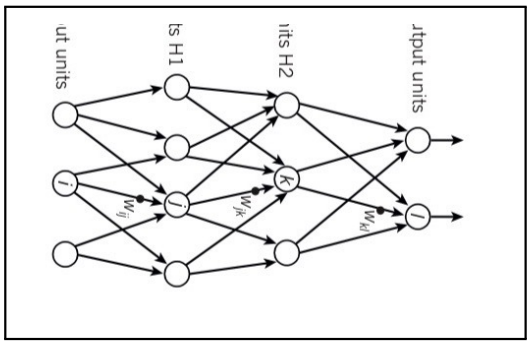

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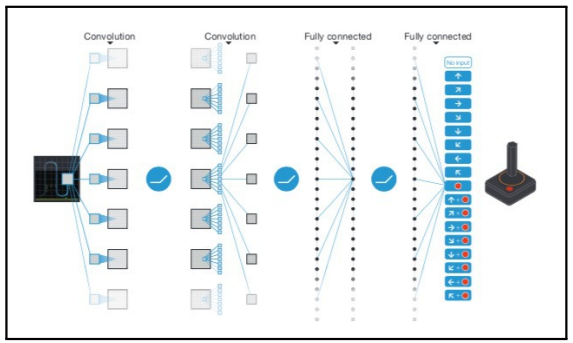

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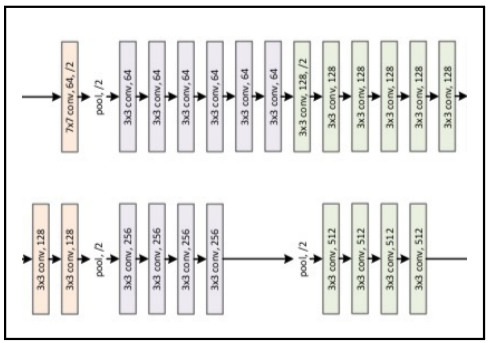

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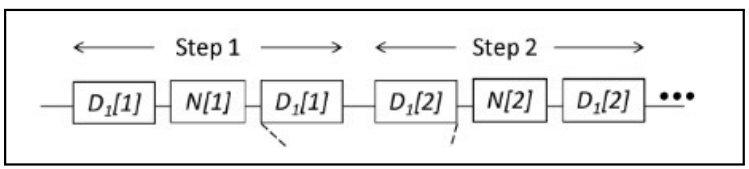
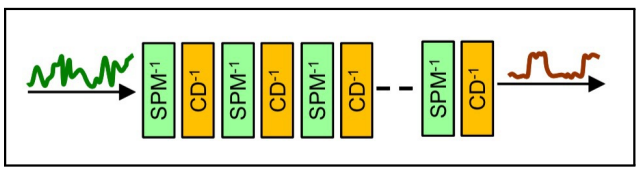


[Du and Lowery, 2010]



[Nakashima et al., 2017]

Split-step methods for solving the propagation equation in fiber-optics

Agenda

In this talk, we . . .

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In this talk, we . . .

1. show that multi-layer neural networks and the split-step method have the same functional form: both alternate linear and pointwise nonlinear steps
2. propose a physics-based machine-learning approach based on parameterizing the split-step method (no black-box neural networks)
3. revisit hardware-efficient nonlinear equalization via learned digital backpropagation

Outline

1. Machine Learning and Neural Networks for Communications
2. Physics-Based Machine Learning for Fiber-Optic Communications
3. Learned Digital Backpropagation
4. Polarization-Dependent Effects
5. Conclusions

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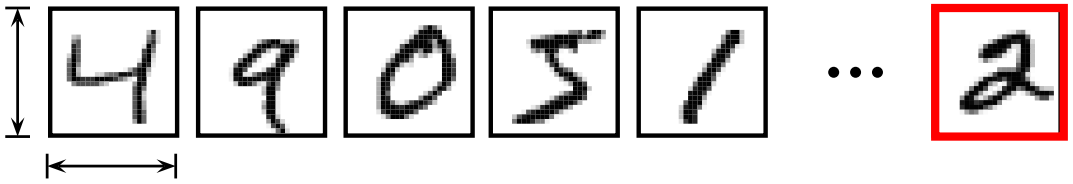
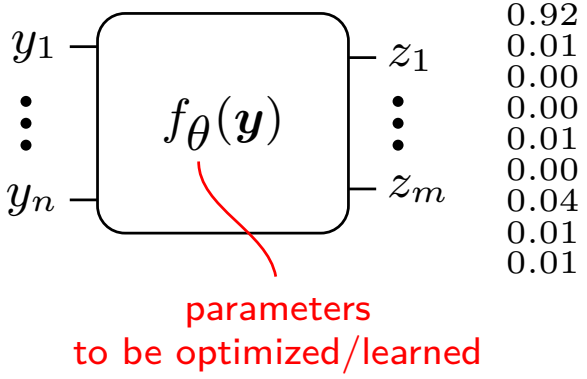
Supervised Learning

handwritten digit recognition (MNIST: 70,000 images)



28×28 pixels $\implies n = 784$



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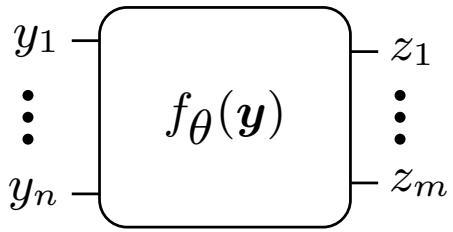
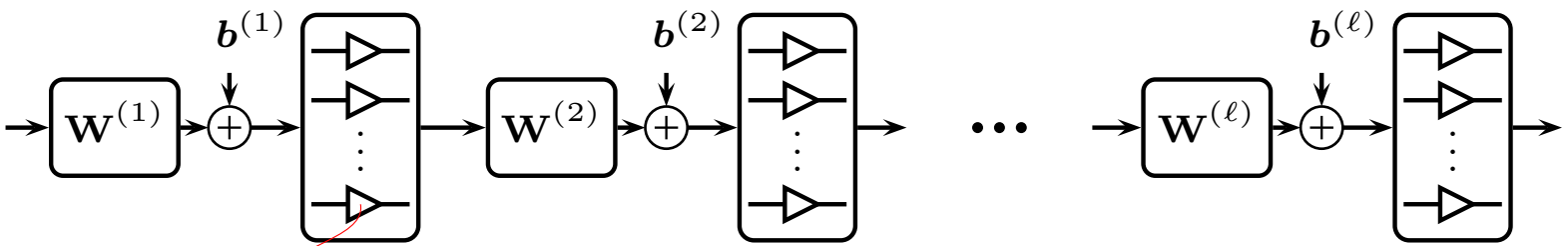
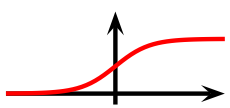


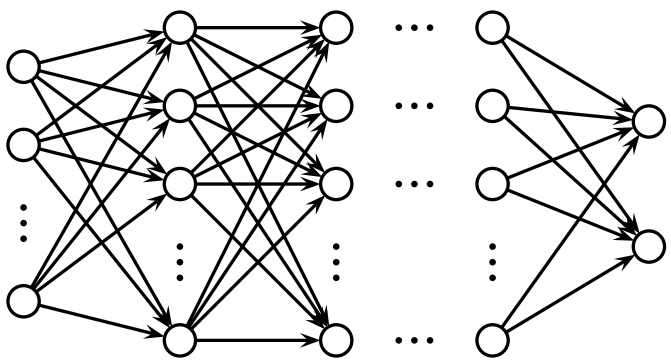
$\boldsymbol{z}$
0.01
0.92
0.01
0.00
0.00
0.01
0.00
0.04
0.01
0.01

How to choose $f_\theta(\boldsymbol{y})$? Deep feed-forward neural networks



activation function





equivalent graph representation

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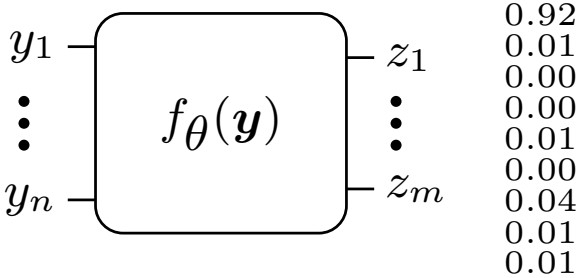

How to optimize $\theta = \{\boldsymbol{W}^{(1)}, \ldots, \boldsymbol{W}^{(\ell)}, \boldsymbol{b}^{(1)}, \ldots, \boldsymbol{b}^{(\ell)}\}$?

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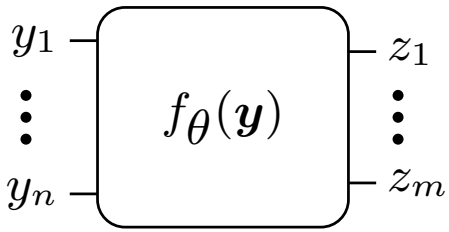


$\boldsymbol{z}$	$\boldsymbol{x}$
0.01	0
0.92	1
0.01	0
0.00	0
0.00	0
0.01	0
0.00	0
0.04	0
0.01	0
0.01	0

How to optimize $\theta = \{\boldsymbol{W}^{(1)}, \ldots, \boldsymbol{W}^{(\ell)}, \boldsymbol{b}^{(1)}, \ldots, \boldsymbol{b}^{(\ell)}\}$?

Given a data set $\mathcal{D} = \{(\boldsymbol{y}^{(i)}, \boldsymbol{x}^{(i)})\}_{i=1}^{N}$, where $\boldsymbol{y}^{(i)}$ are model inputs and $\boldsymbol{x}^{(i)}$ are labels, we iteratively minimize

$$\frac{1}{|\mathcal{B}_k|} \sum_{(\boldsymbol{y},\boldsymbol{x}) \in \mathcal{B}_k} \mathcal{L}(f_\theta(\boldsymbol{y}), \boldsymbol{x}) \triangleq g(\theta) \qquad \text{using} \quad \theta_{k+1} = \theta_k - \lambda \nabla_\theta g(\theta_k)$$

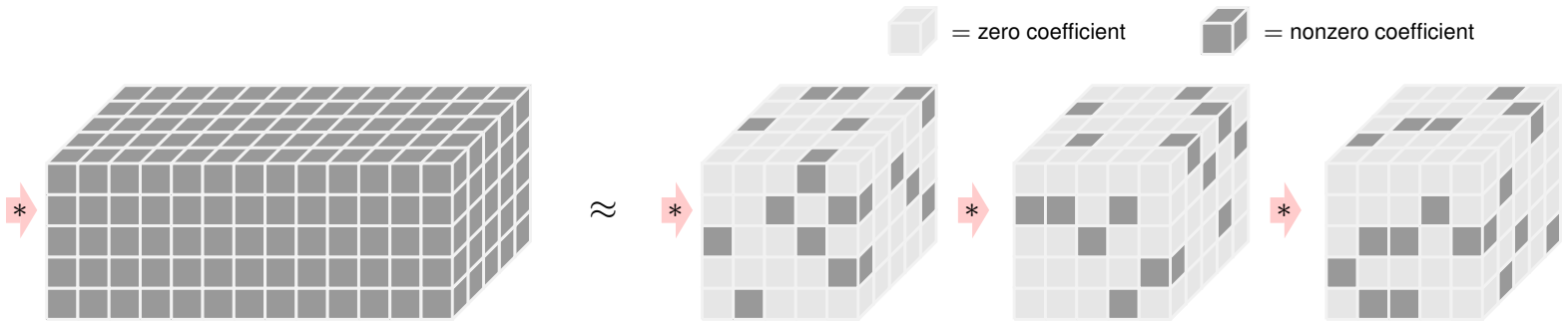
stochastic gradient descent

- $\mathcal{B}_k \subset \mathcal{D}$ and $|\mathcal{B}_k|$ is called the batch (or minibatch) size
- Typical loss function: mean squared error $\mathcal{L}(\boldsymbol{a}, \boldsymbol{b}) = \|\boldsymbol{a} - \boldsymbol{b}\|^2$ (regression)
- λ is called the step size or learning rate

Why Deep Models?

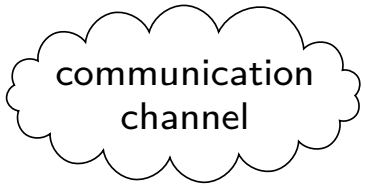
Many possible answers

One advantage is complexity: deep computation graphs tend to be more parameter efficient than shallow graphs [Lin et al., 2017]



- Sparsity can emerge due to (approximate) factorization (even for linear models, e.g., FFT)
- Deep computation graphs allow for very simple elementary steps
- Deep models typically have many "good" parameter configurations that are close to each other $\implies$ robustness to, e.g., quantization noise

Physical-Layer Design: Conventional vs. Machine Learning



- Conventional: handcrafted DSP blocks based on mathematical modeling

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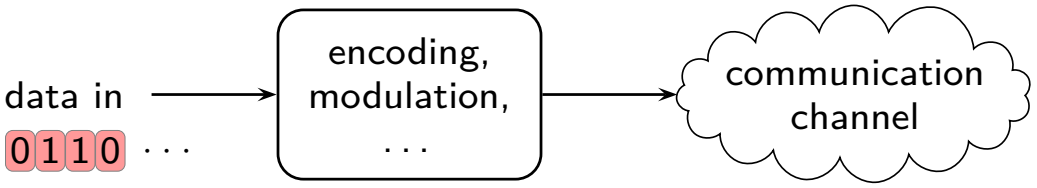

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- Use function approximators and learn parameter configurations θ from data

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[Shen and Lau, 2011], Fiber nonlinearity compensation using extreme learning machine for DSP-based . . . , (*OECC*)
[Giacoumidis et al., 2015], Fiber nonlinearity-induced penalty reduction in CO-OFDM by ANN-based . . . , (*Opt. Lett.*)
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- Joint transmitter–receiver learning via autoencoder [O'Shea and Hoydis, 2017]

[Karanov et al., 2018], End-to-end deep learning of optical fiber communications (*J. Lightw. Technol.*)
[Li et al., 2018], Achievable information rates for nonlinear fiber communication via end-to-end autoencoder learning, (*ECOC*)
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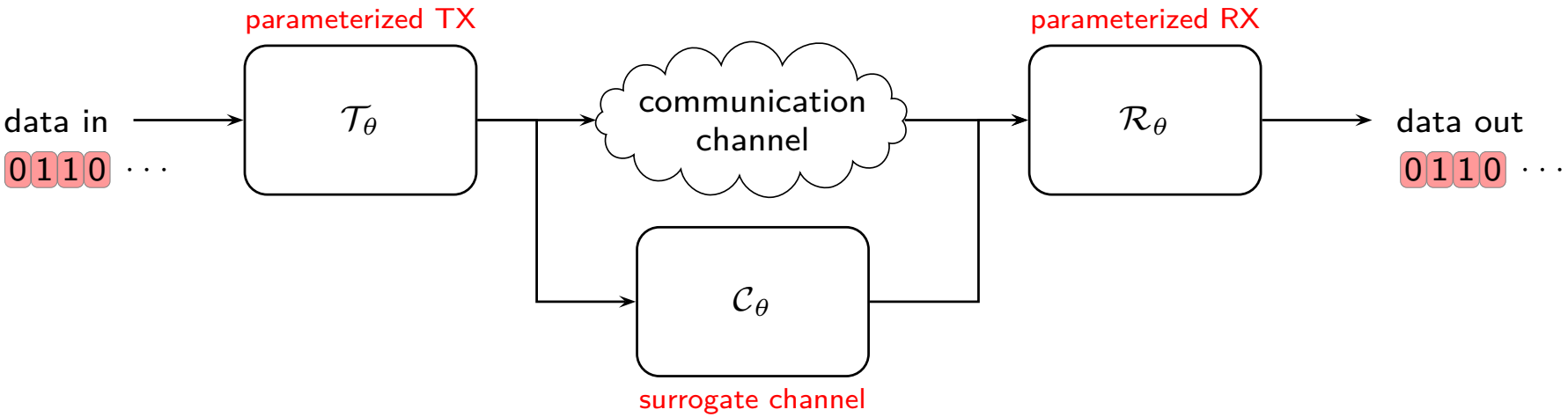

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- Surrogate channel models for gradient-based TX training

[O'Shea et al., 2018], Approximating the void: Learning stochastic channel models from observation with variational GANs, (*arXiv*)
[Ye et al., 2018], Channel agnostic end-to-end learning based communication systems with conditional GAN, (*arXiv*)
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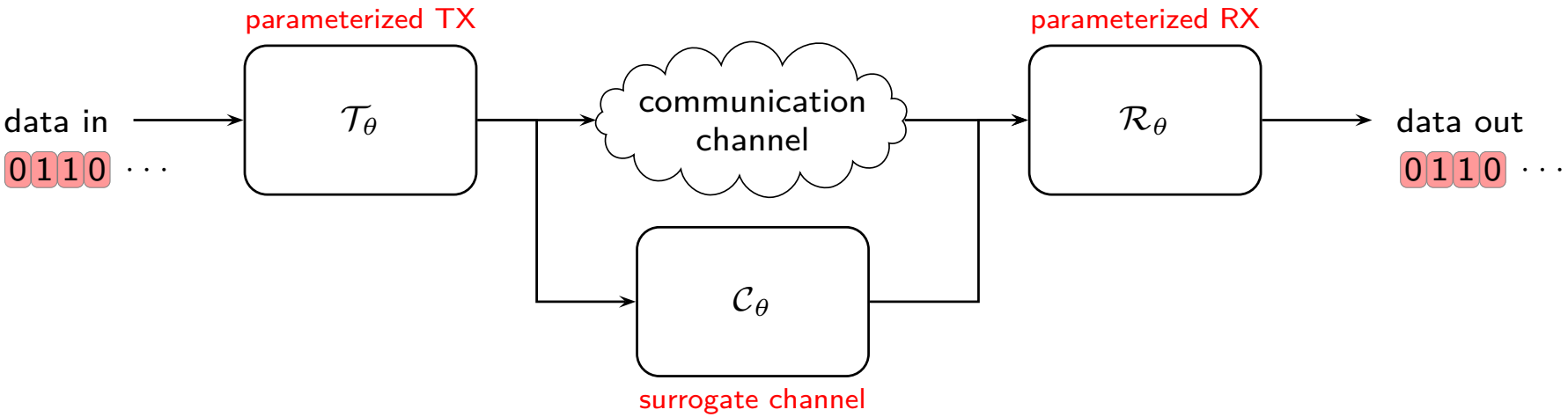

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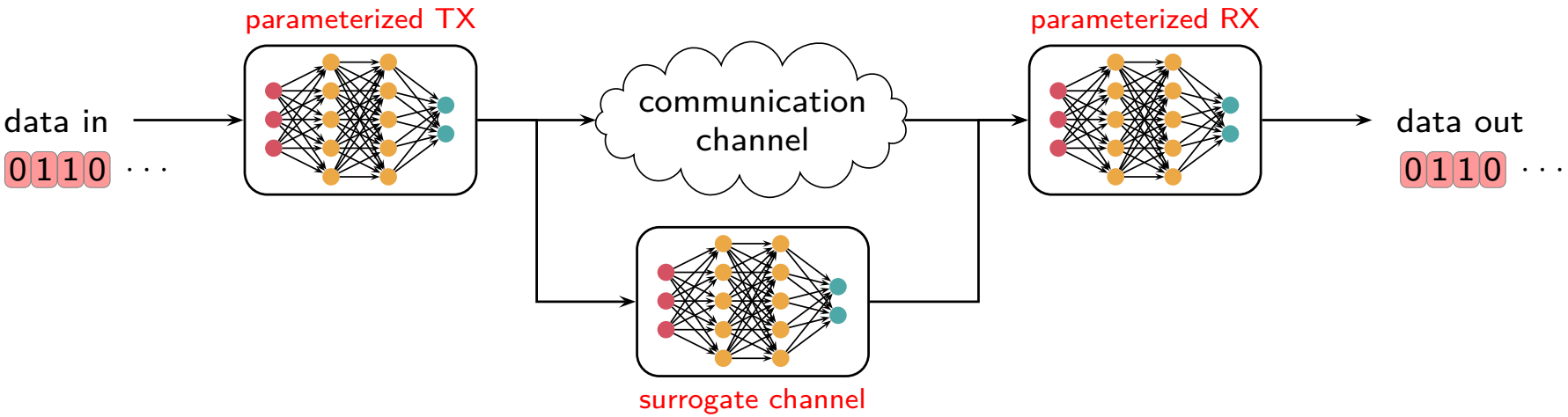

Using (deep) neural networks for $\mathcal{T}_\theta, \mathcal{R}_\theta, \mathcal{C}_\theta$? Possible, but . . .

- How to choose the network architecture (#layers, activation function)?
- How to limit the number of parameters (complexity)?
- How to interpret the solutions? Any insight gained?
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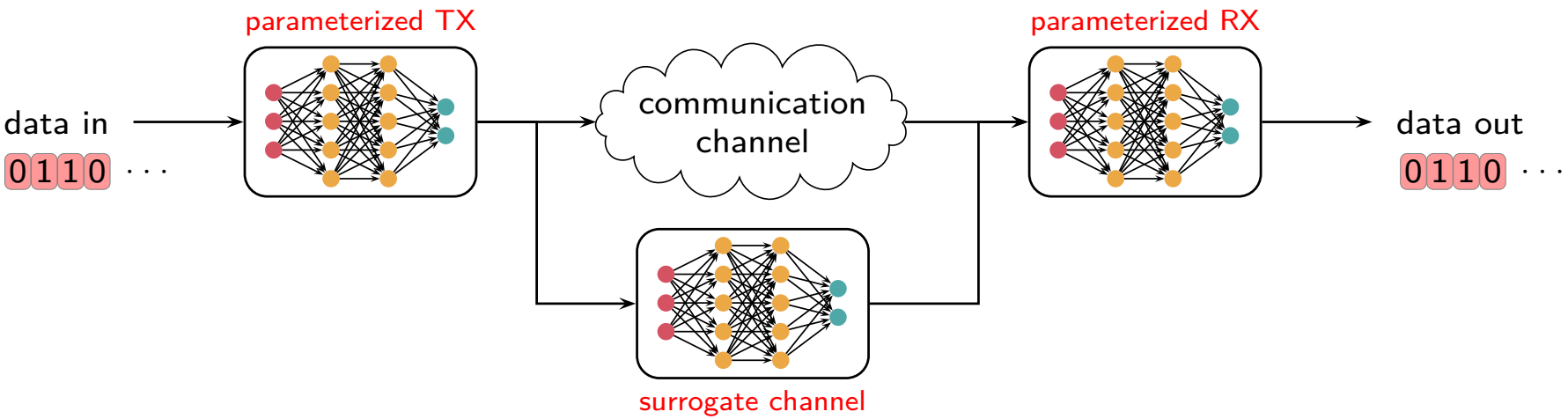

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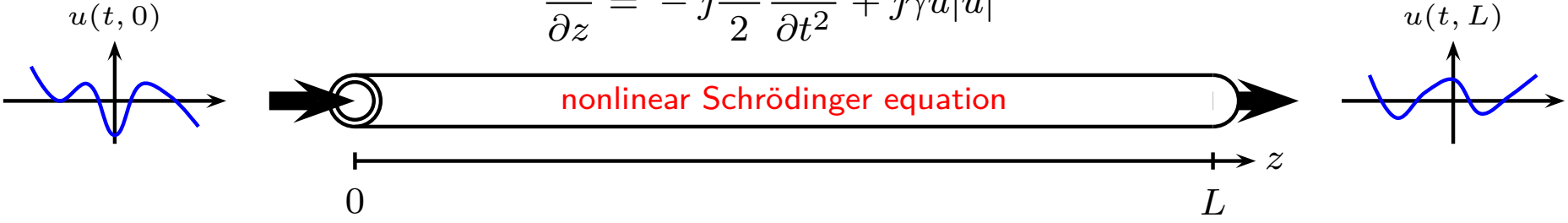
Our contribution: designing "neural-network-like" machine-learning models by exploiting the underlying physics of the propagation.

Outline

Nonlinear Fiber Channel Modeling

$$\frac{\partial u}{\partial z} = -\jmath\frac{\beta_2}{2}\frac{\partial^2 u}{\partial t^2} + \jmath\gamma u|u|^2$$



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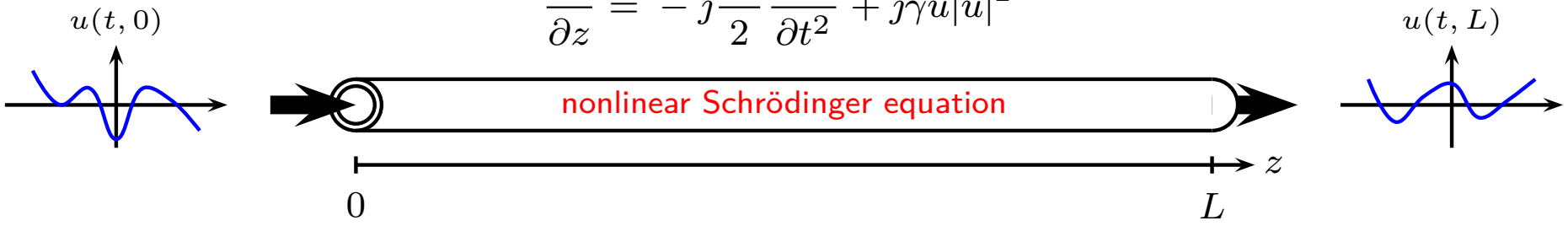

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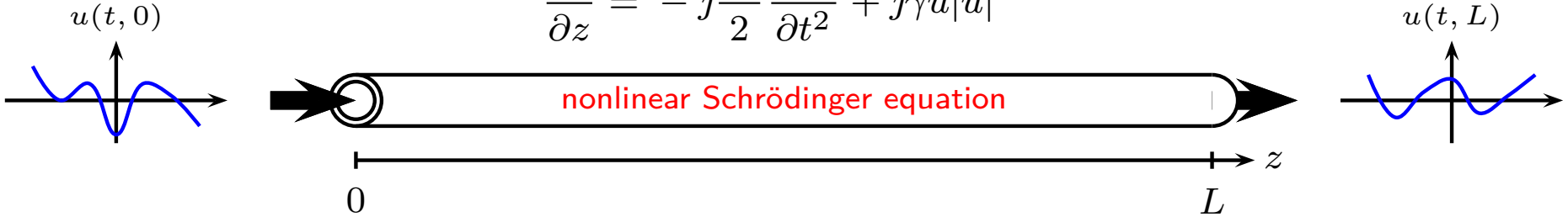

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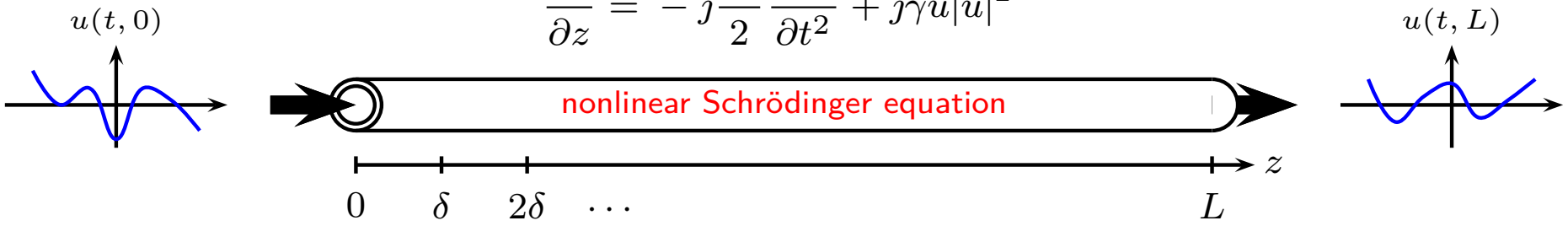

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- Split-step method with M steps ($\delta = L/M$):

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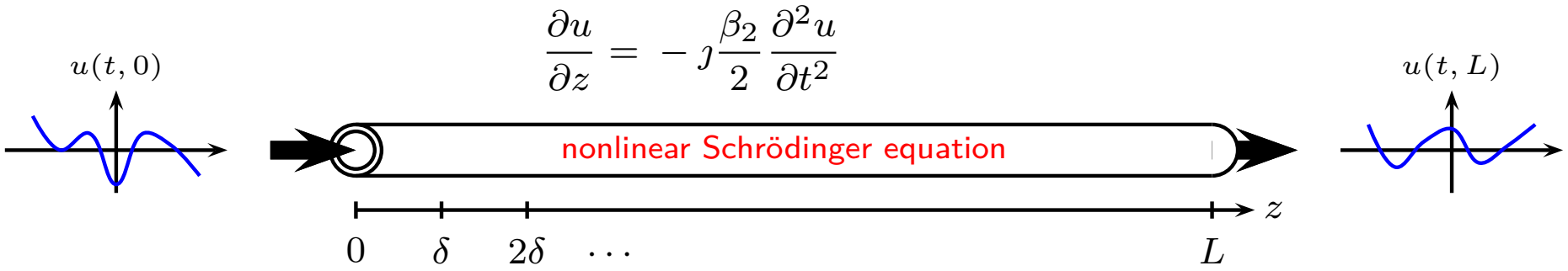

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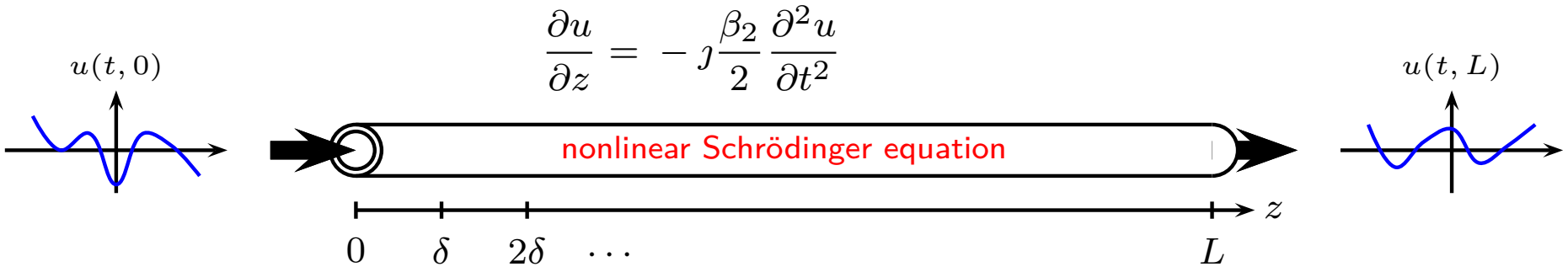

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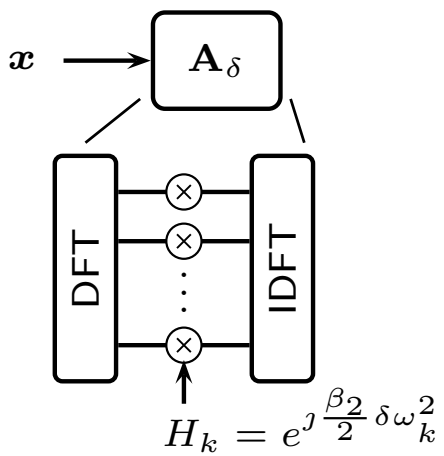

dispersion (all-pass filter)

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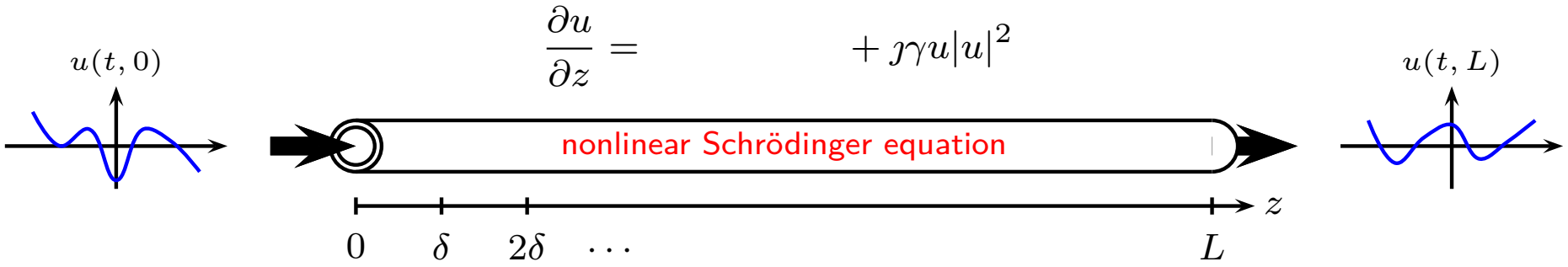

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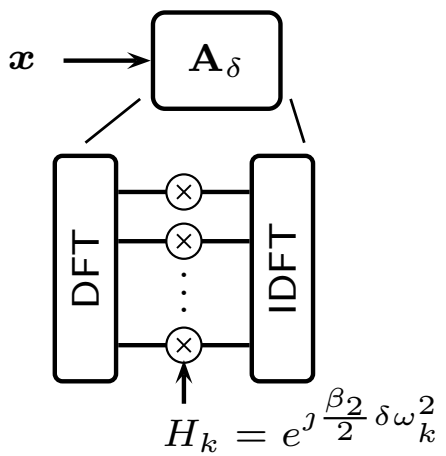

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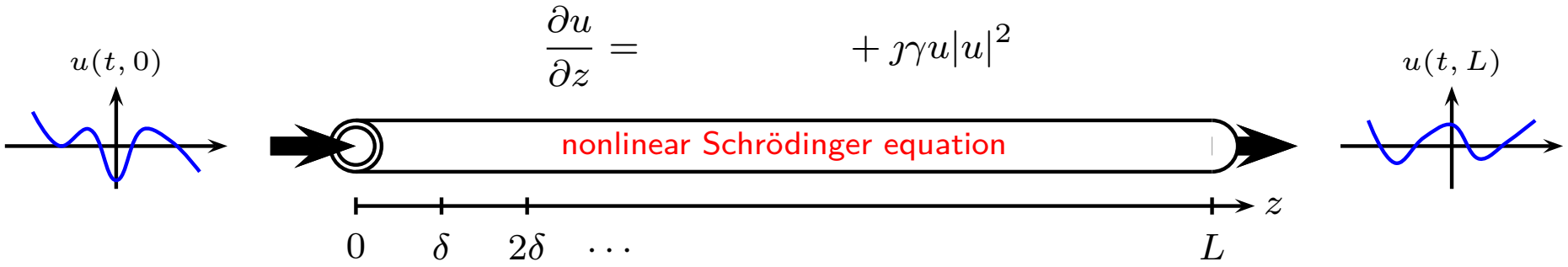

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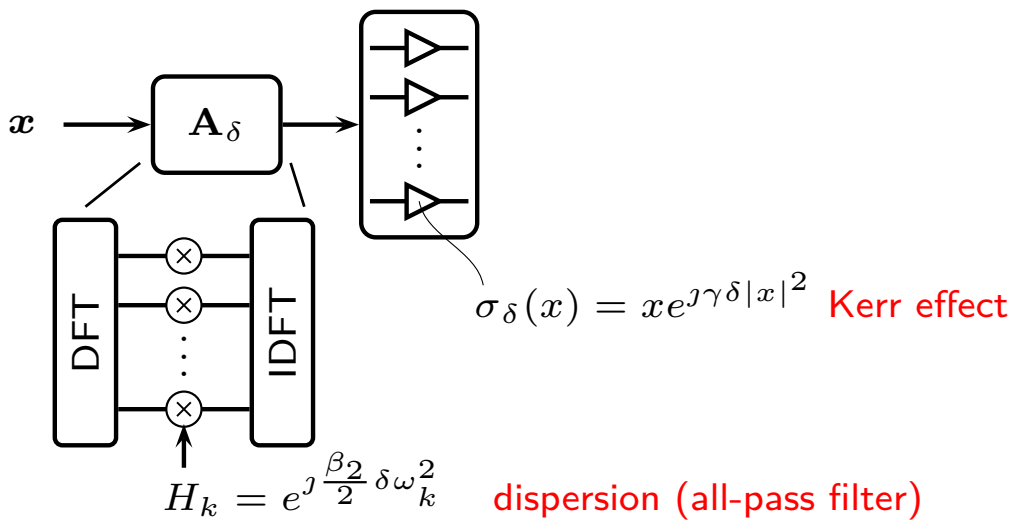

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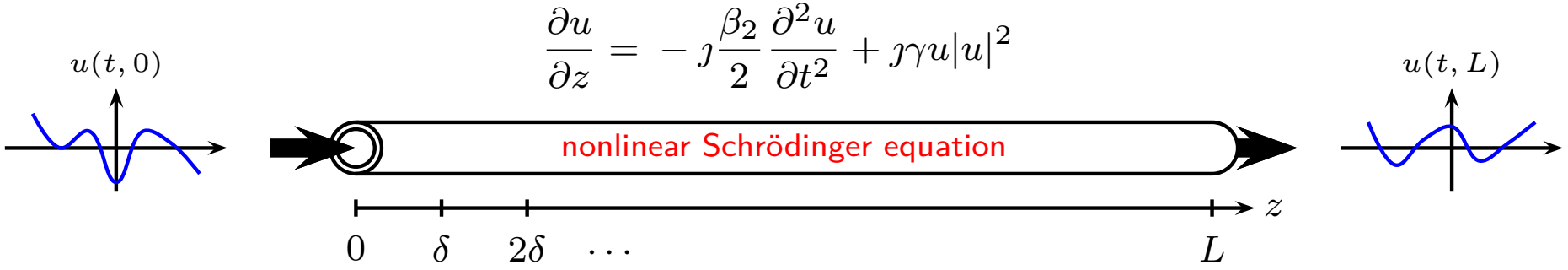

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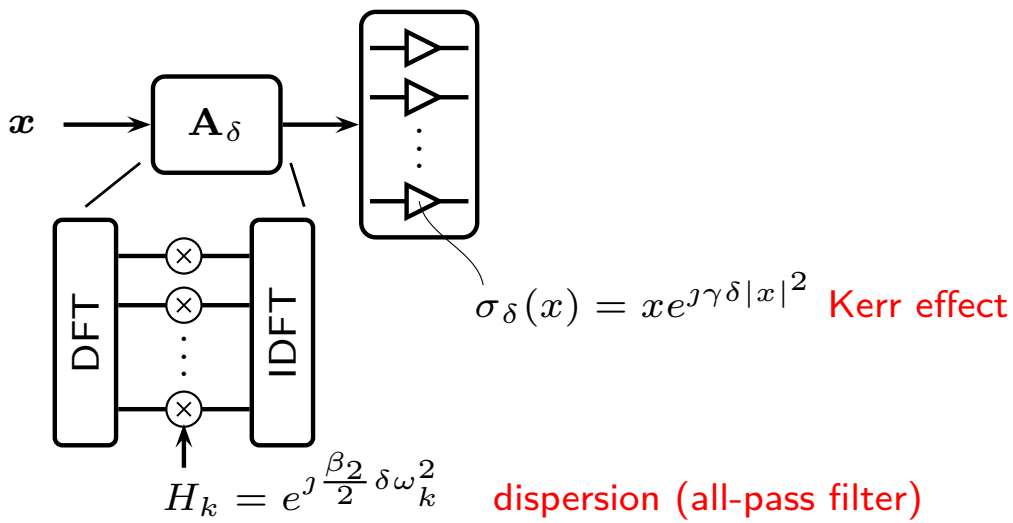

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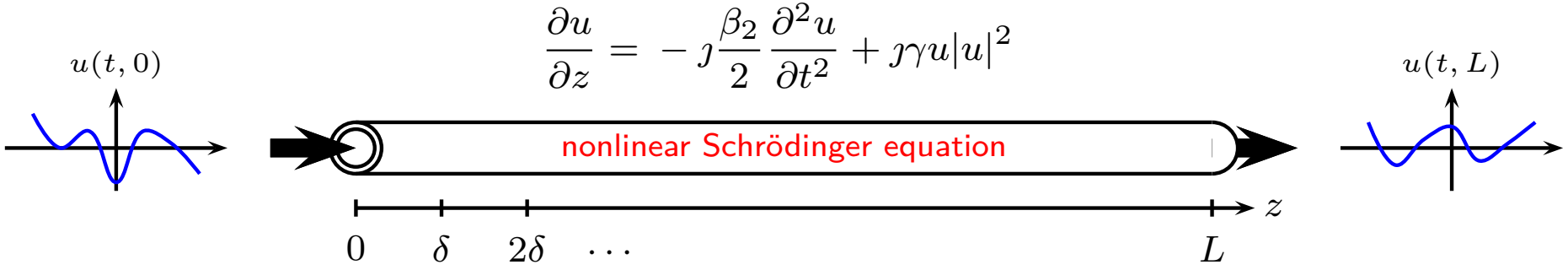

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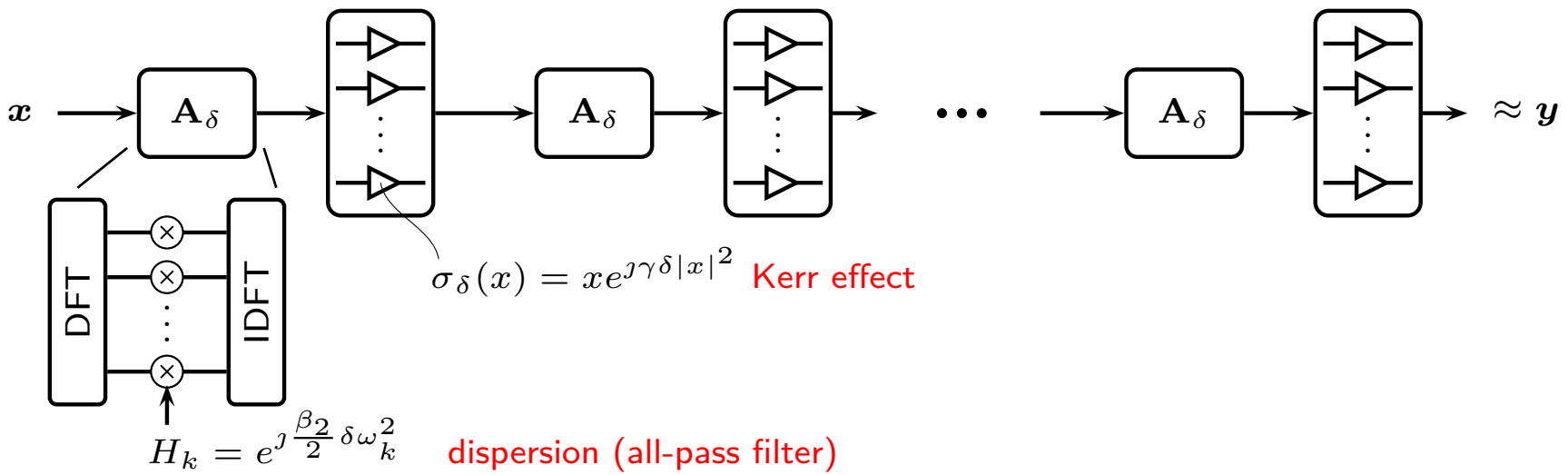
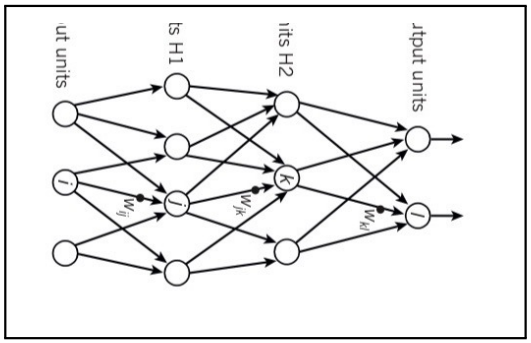
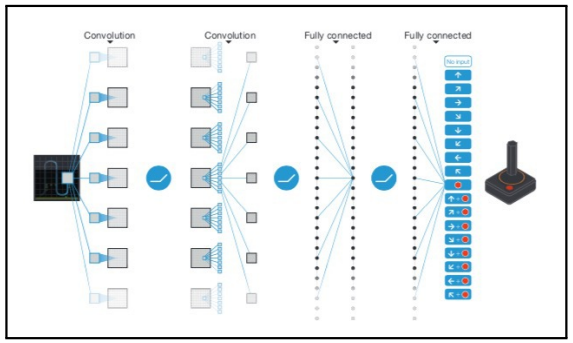


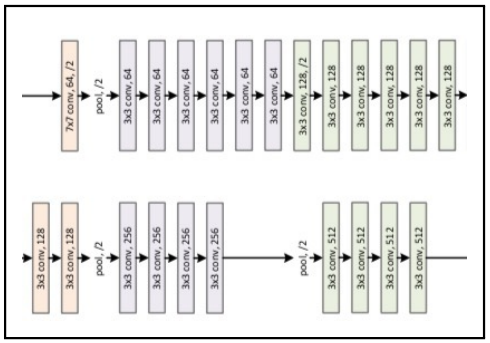
Deep Learning [LeCun et al., 2015]



Deep Q-Learning [Mnih et al., 2015]



ResNet [He et al., 2015]



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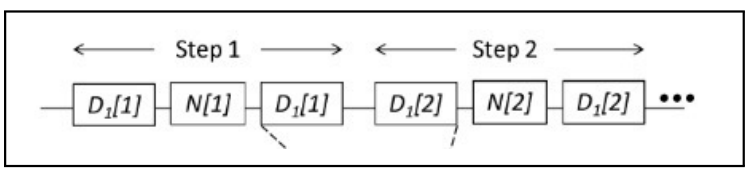
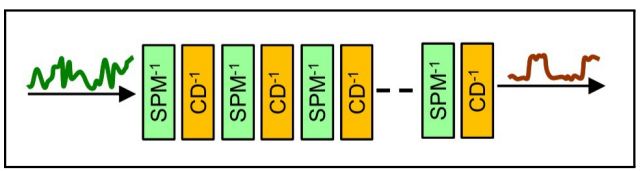
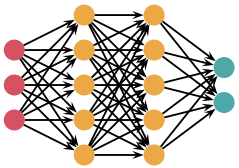


[Du and Lowery, 2010]

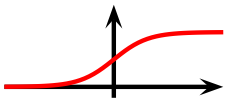


[Nakashima et al., 2017]

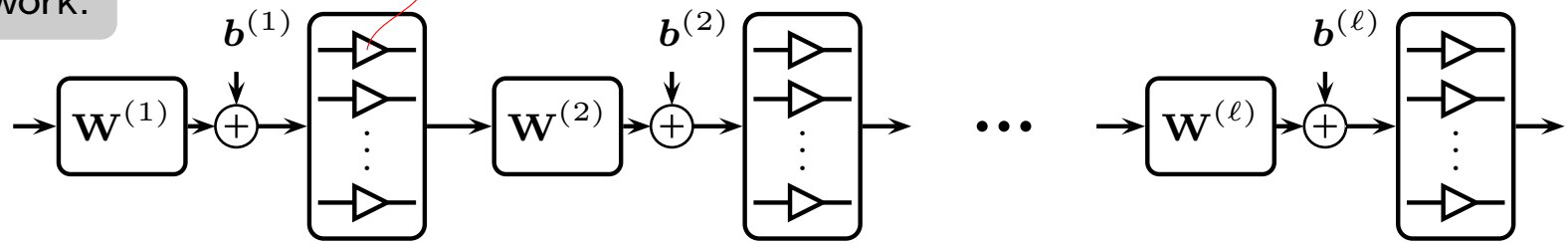
The Main Idea



multi-layer neural network:



activation function



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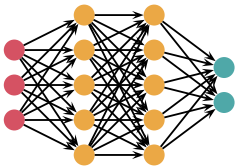

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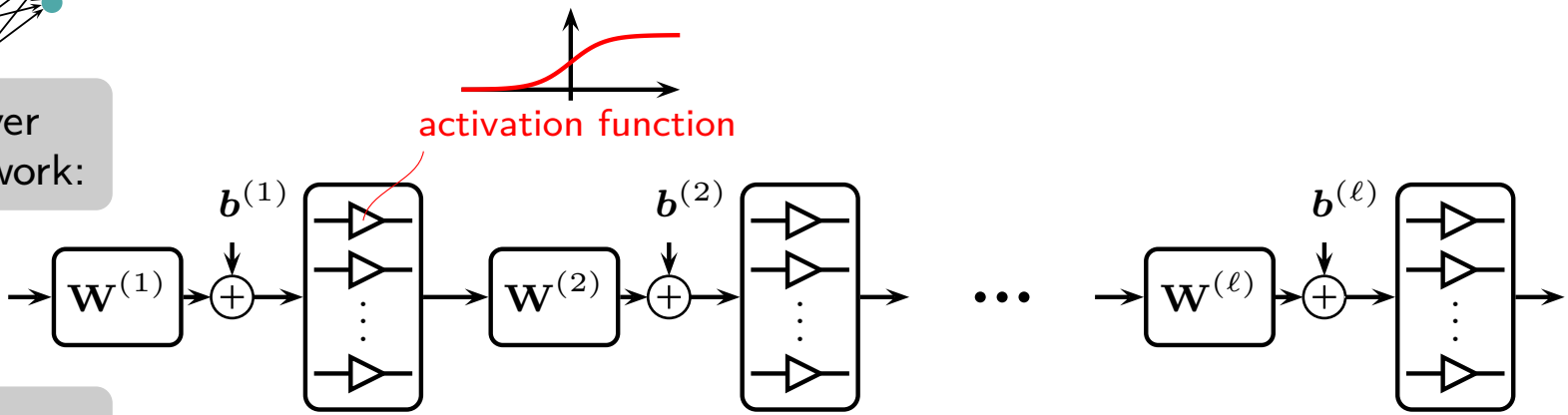
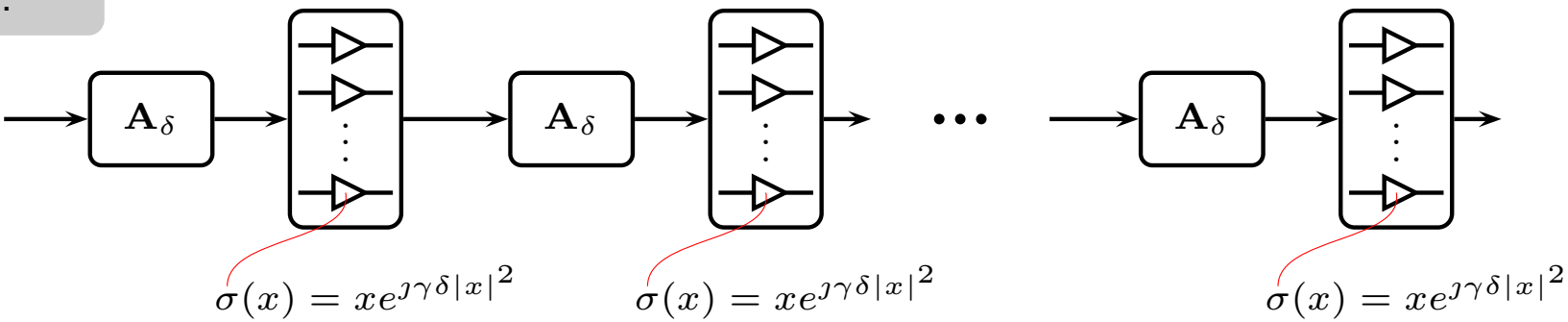
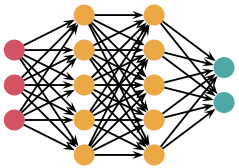


split-step method:



- This almost looks like a deep neural net!

The Main Idea



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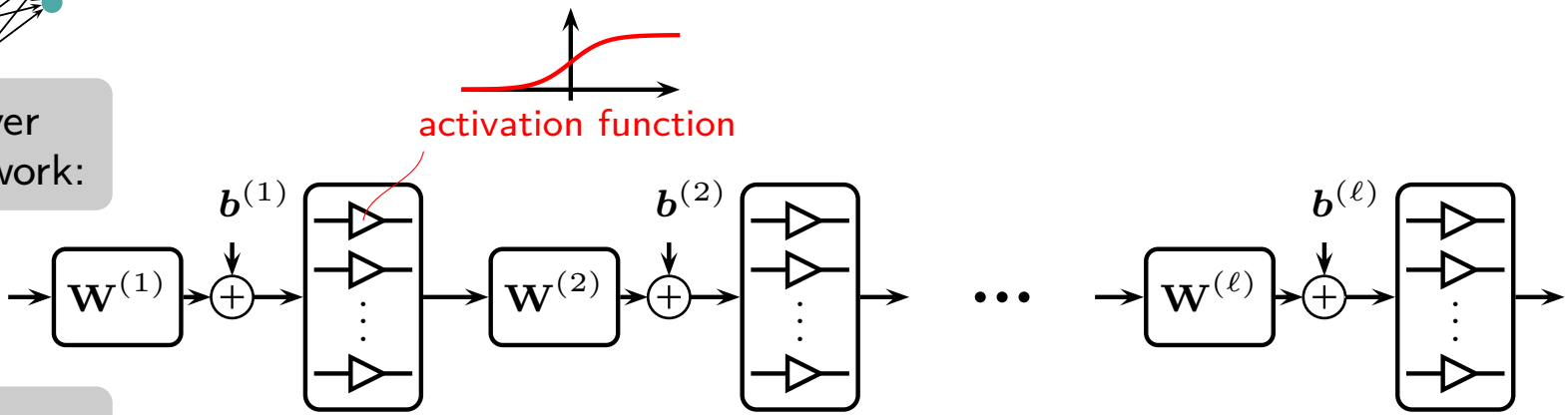

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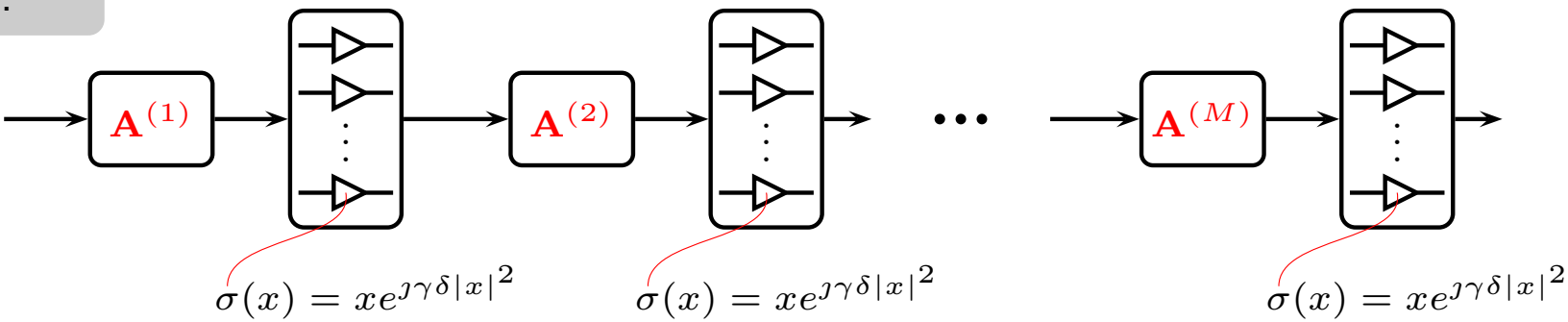

- This almost looks like a deep neural net!
- Parameterize all linear steps: f_θ with $\theta = \{\mathbf{A}^{(1)}, \ldots, \mathbf{A}^{(M)}\}$

[Häger & Pfister, 2018], Nonlinear Interference Mitigation via Deep Neural Networks, *(OFC)*
[Häger & Pfister, 2021], Physics-Based Deep Learning for Fiber-Optic Communication Systems, *IEEE J. Sel. Areas Commun.*

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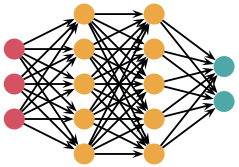

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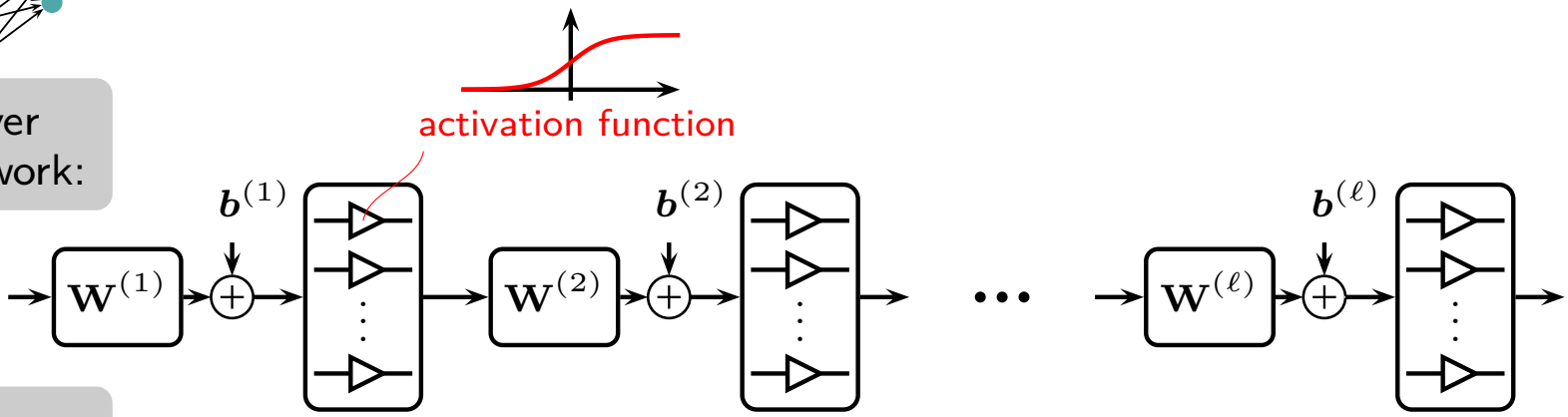

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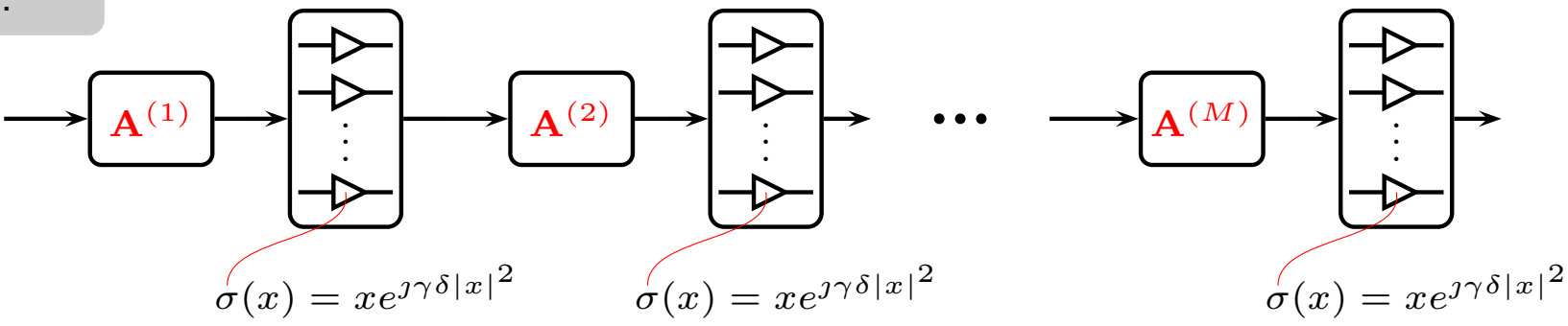

- This almost looks like a deep neural net!
- Parameterize all linear steps: f_θ with $\theta = \{\mathbf{A}^{(1)}, \ldots, \mathbf{A}^{(M)}\}$
- Special cases: step-size optimization, nonlinear operator "placement", ...

[Häger & Pfister, 2018], Nonlinear Interference Mitigation via Deep Neural Networks, *(OFC)*
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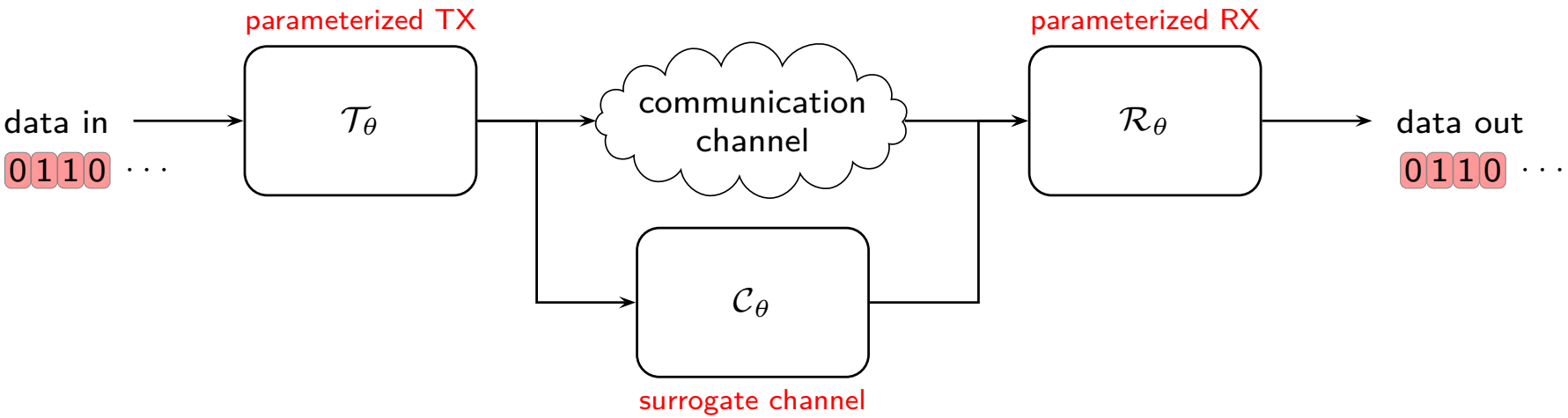
Possible Applications



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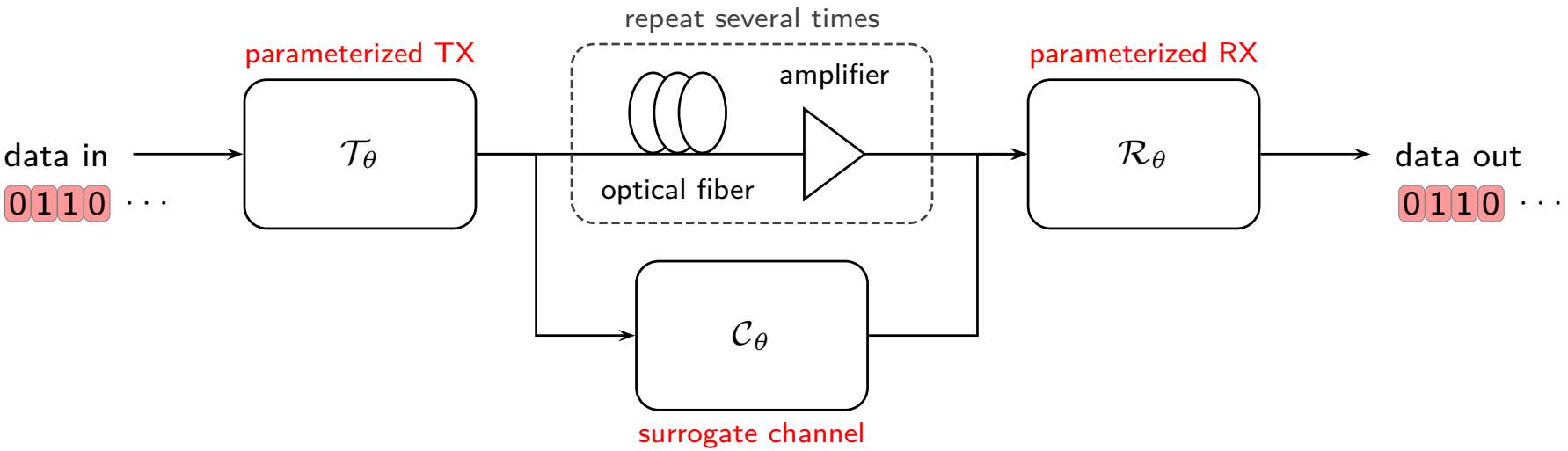

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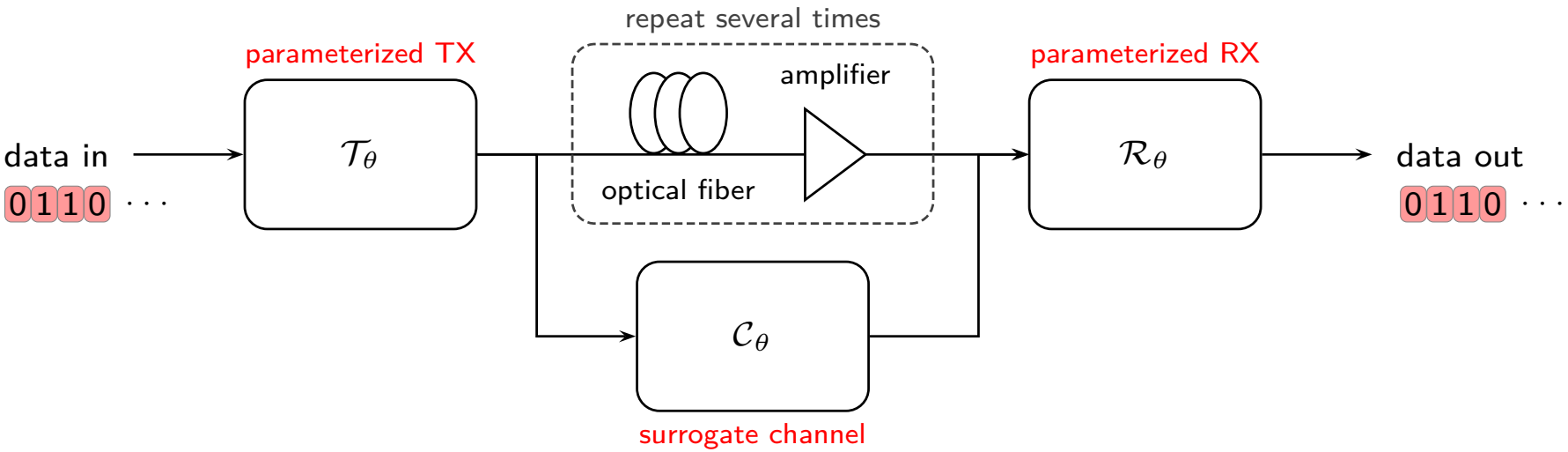
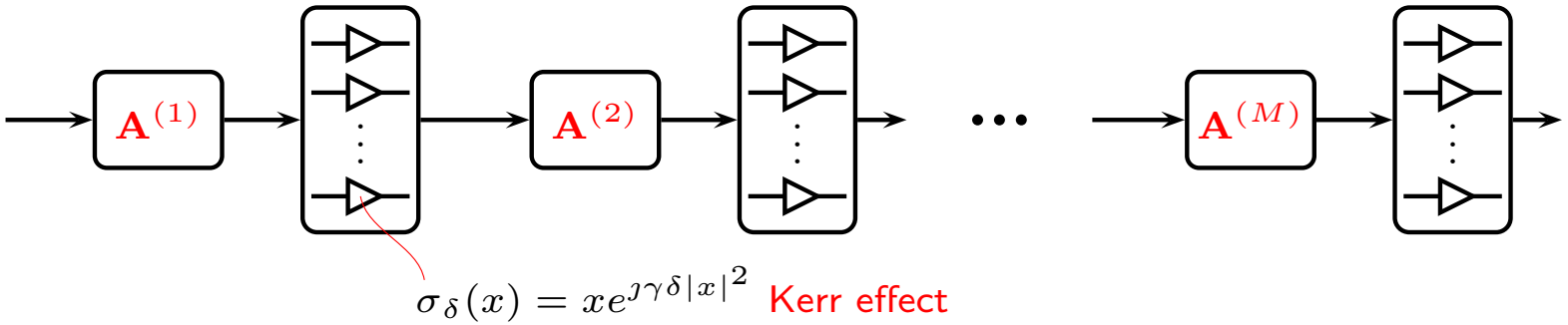


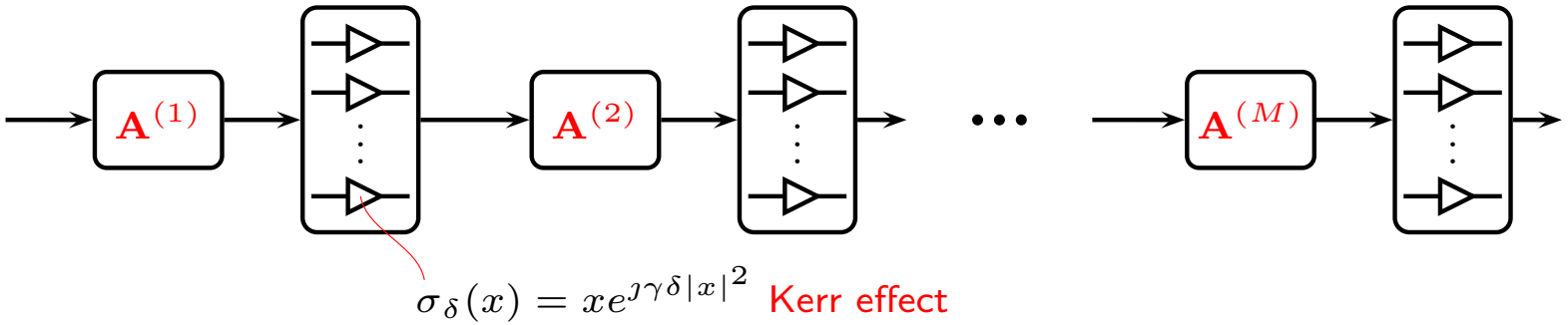
- Channel $\mathcal{C}_\theta$: fine-tune model based on experimental data, reduce simulation time [Leibrich and Rosenkranz, 2003], [Li et al., 2005]
- Receiver $\mathcal{R}_\theta$: nonlinear equalization (focus in this talk)
- Transmitter $\mathcal{T}_\theta$: digital pre-distortion [Essiambre and Winzer, 2005], [Roberts et al., 2006], "split" nonlinearity compensation [Lavery et al., 2016]

Potential Benefits



- How to choose the network architecture (#layers, activation function)?
- How to limit the number of parameters (complexity)?
- How to interpret the solutions? Any insight gained?

Potential Benefits



- How to choose the network architecture (#layers, activation function)? ✓
  - Activation function is fixed; number of layers = number of steps
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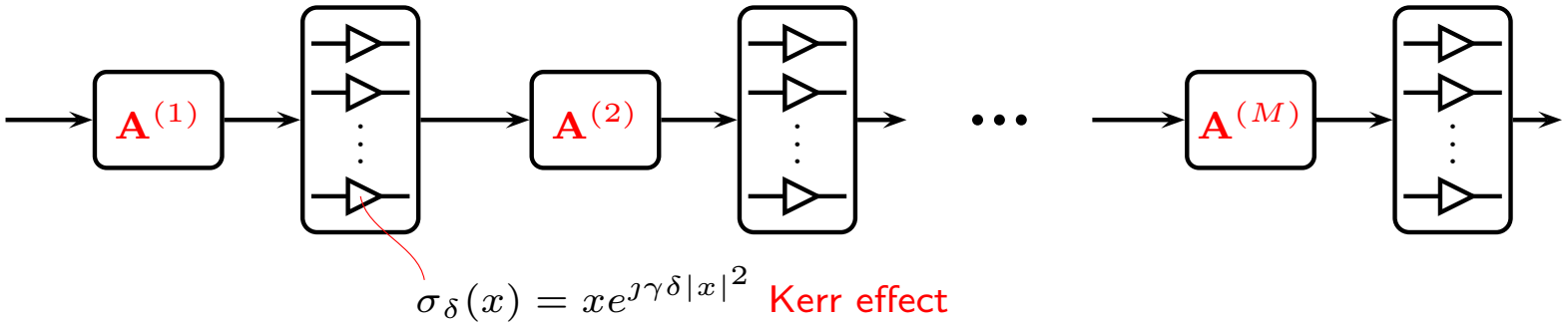

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  - Filter symmetry can be enforced in the linear steps
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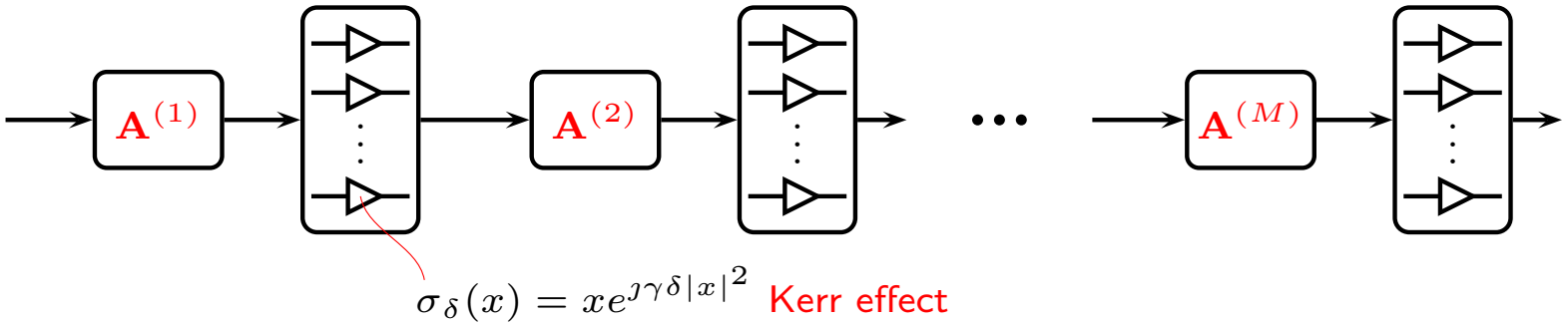

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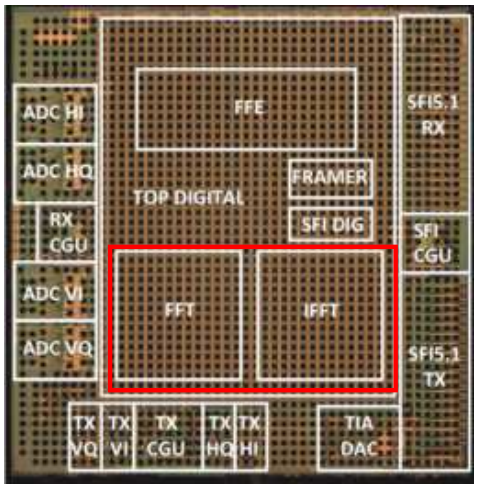
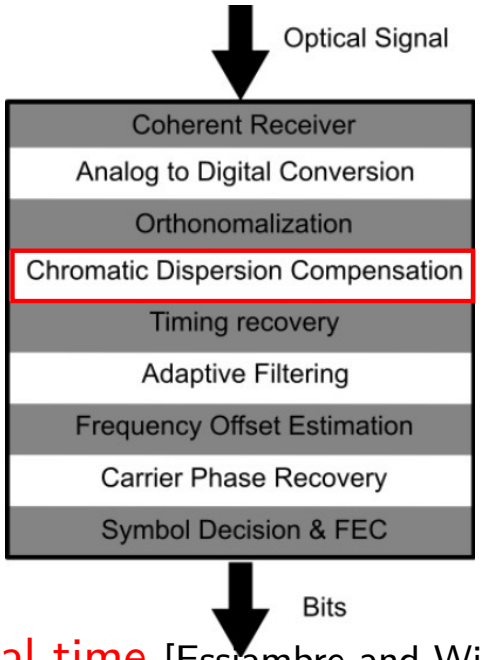
  - Model compression (e.g., parameter pruning, quantization)
- How to interpret the solutions? Any insight gained? ✓
  - Learned parameter configurations are interpretable
  - Satisfactory explanations for benefits over previous handcrafted solutions

Outline

Real-Time Digital Backpropagation

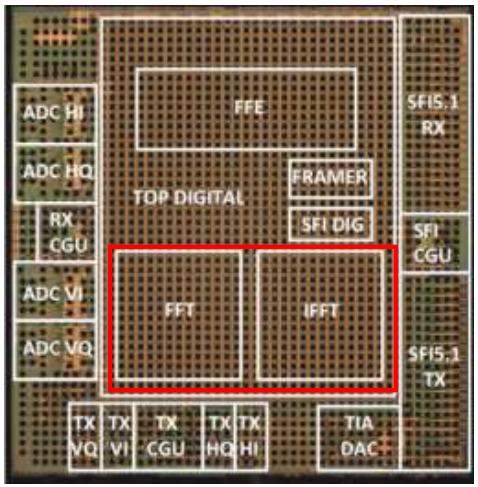


[Crivelli et al., 2014]



- Invert a PDE in real time [Essiambre and Winzer, 2005], [Roberts et al., 2006], [Li et al., 2008], [Ip and Kahn, 2008]: widely considered to be impractical
- Complexity increases with the number of steps $M \implies$ reduce M as much as possible (see, e.g., [Du and Lowery, 2010], [Rafique et al., 2011], [Li et al., 2011], [Yan et al., 2011], [Napoli et al., 2014], [Secondini et al., 2016], . . .)

Real-Time Digital Backpropagation



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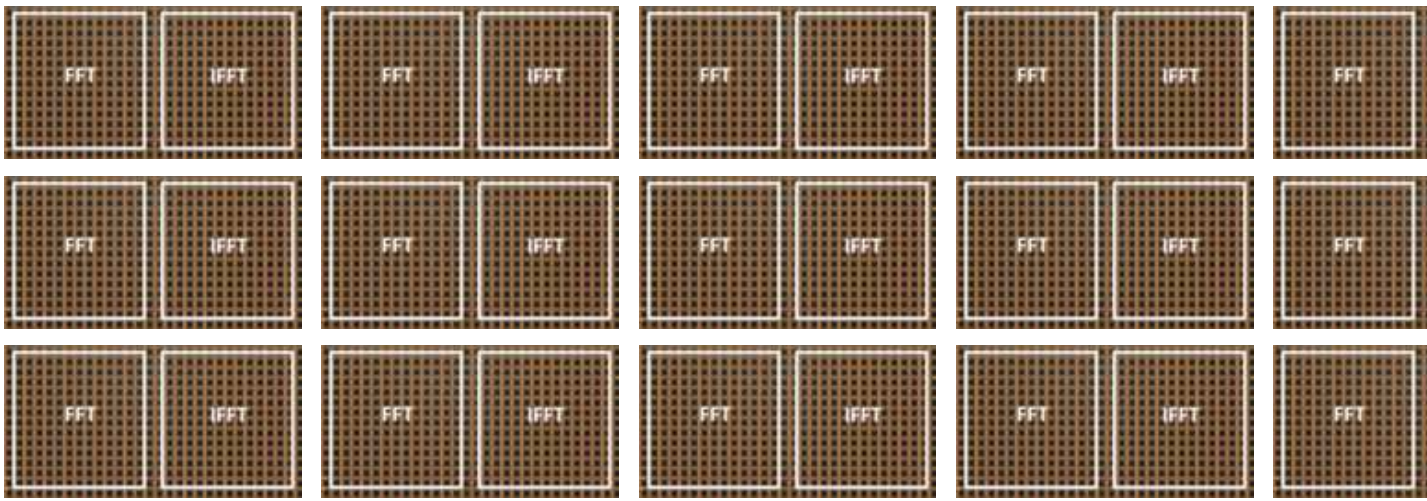

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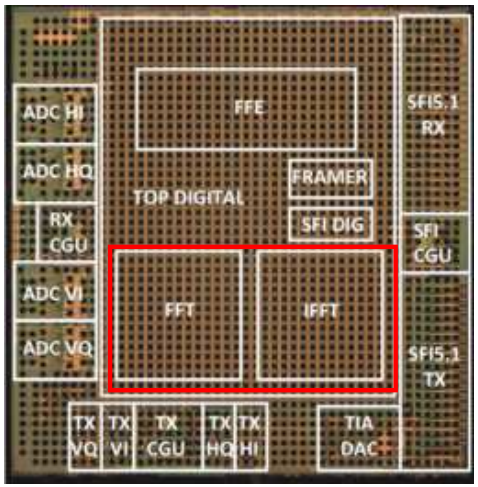

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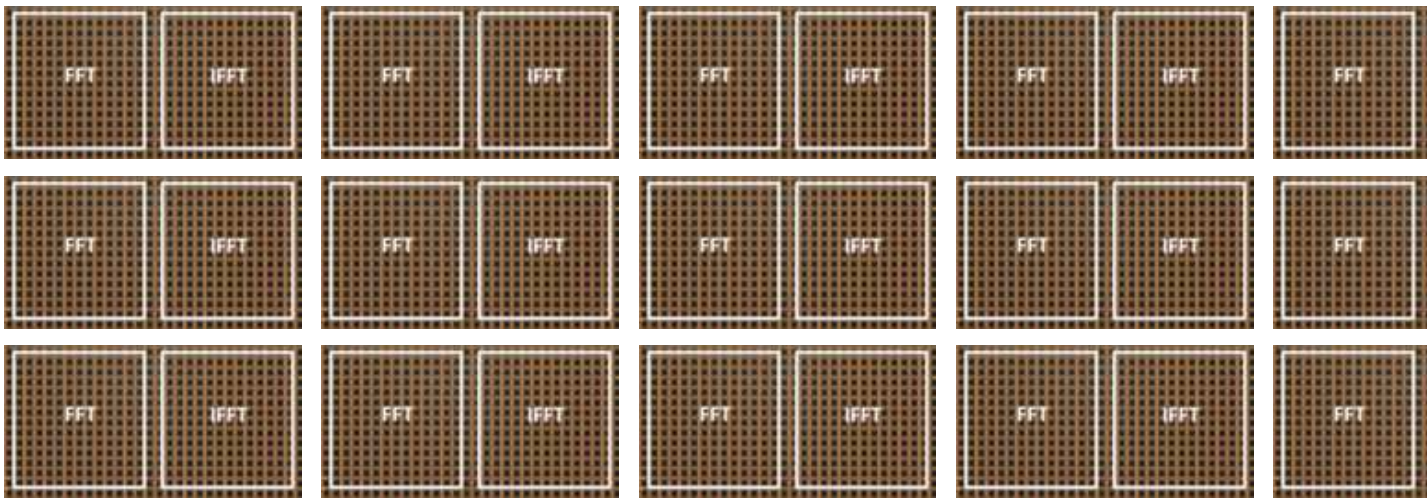

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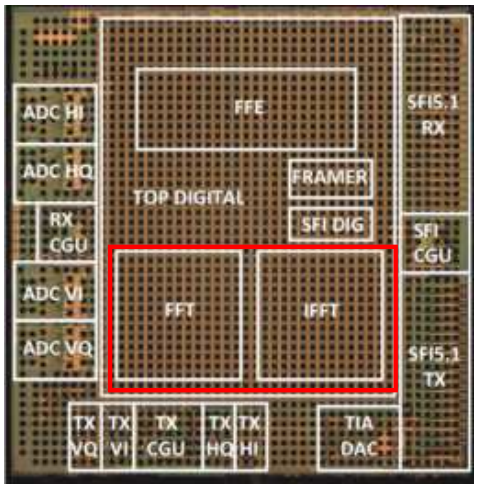

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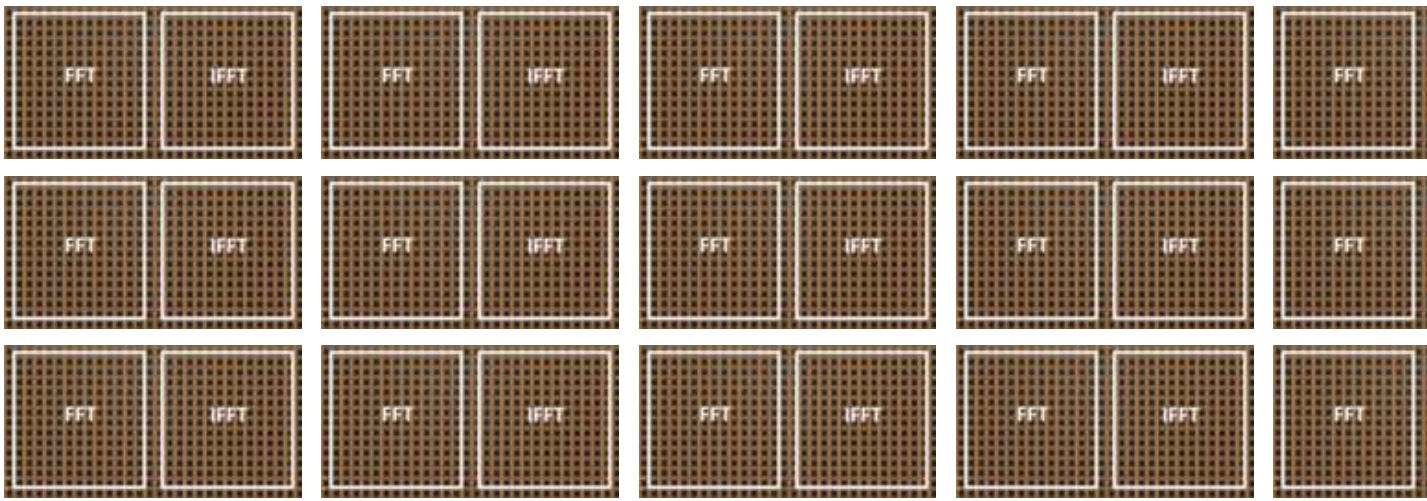

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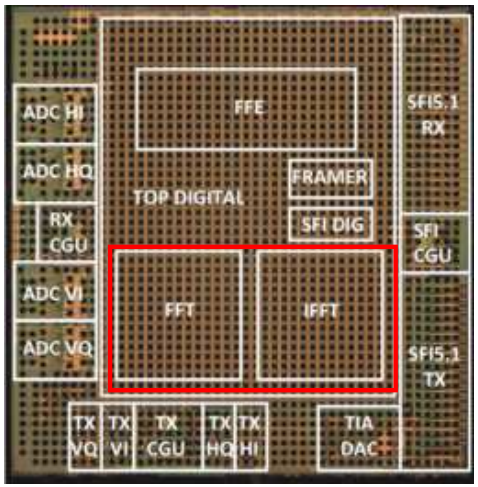

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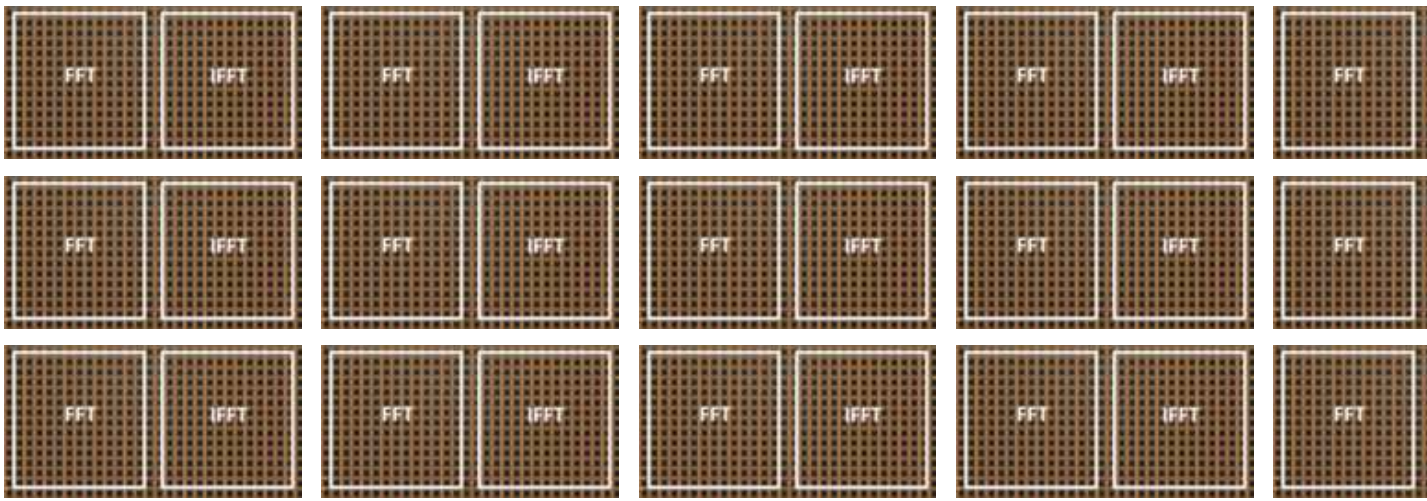

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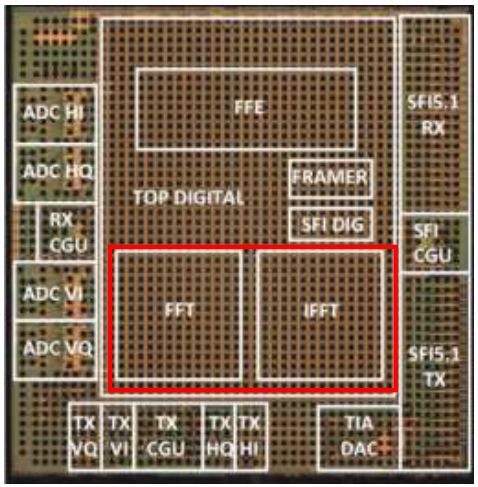

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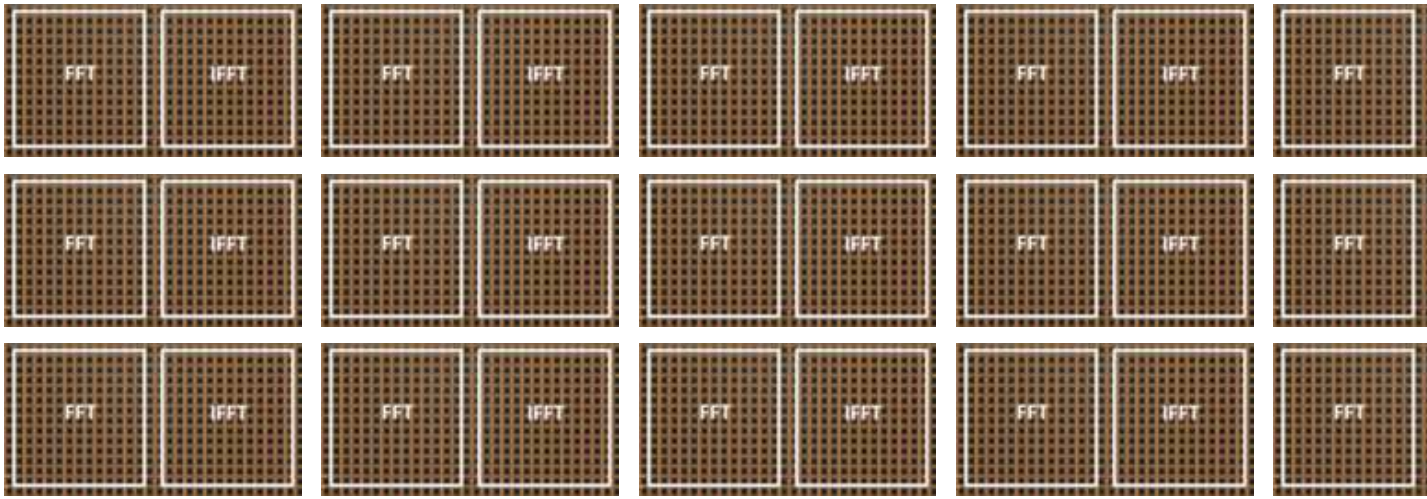

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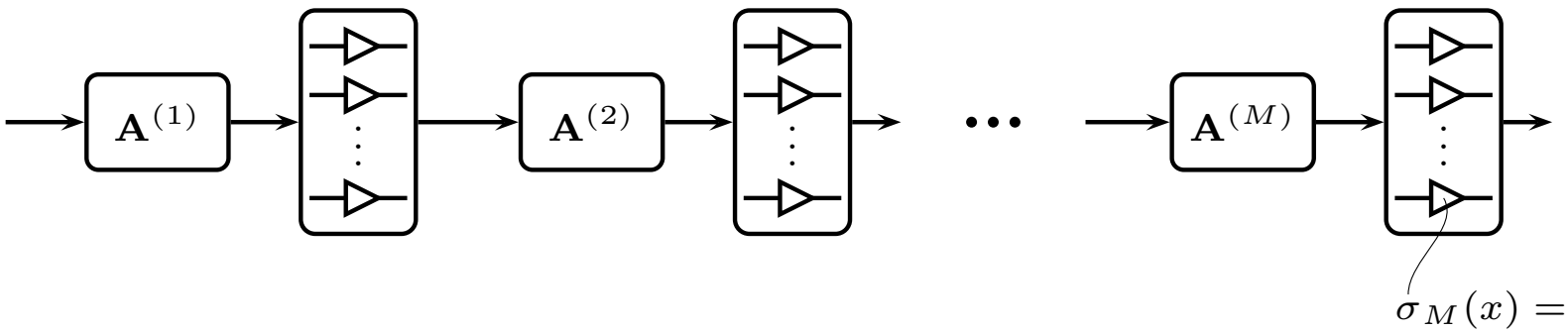
Our approach: many steps but model compression

Joint optimization, pruning, and quantization of all linear steps

Learned Digital Backpropagation

TensorFlow implementation of the computation graph $f_\theta(\boldsymbol{y})$:



Learned Digital Backpropagation

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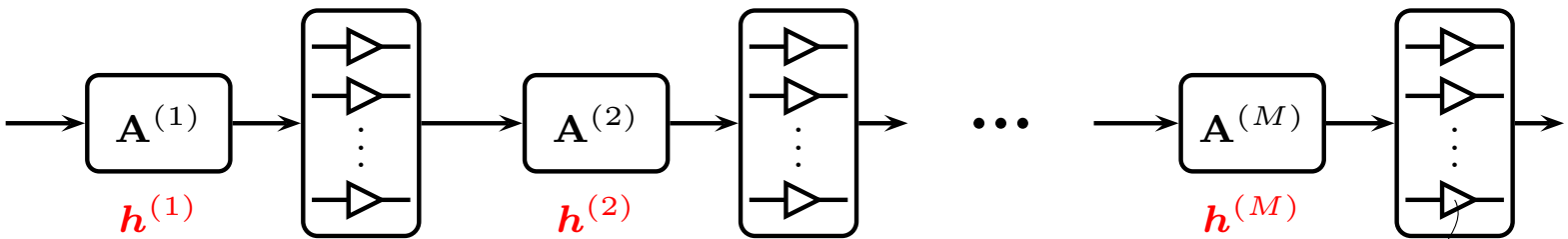

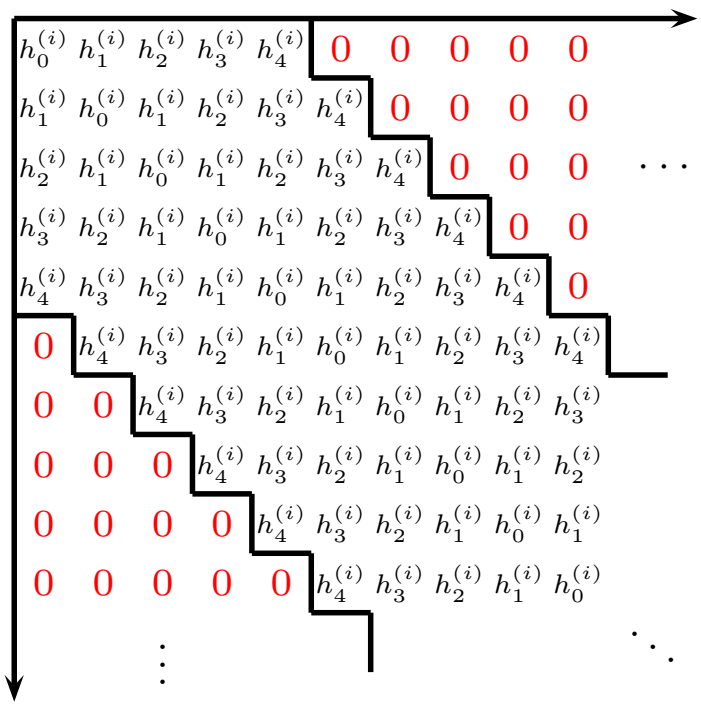

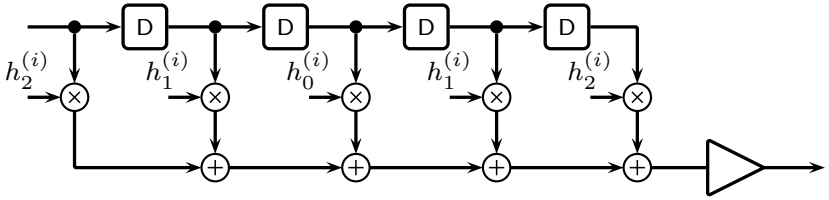

finite impulse response (FIR) filter
complex & symmetric coefficients

Learned Digital Backpropagation

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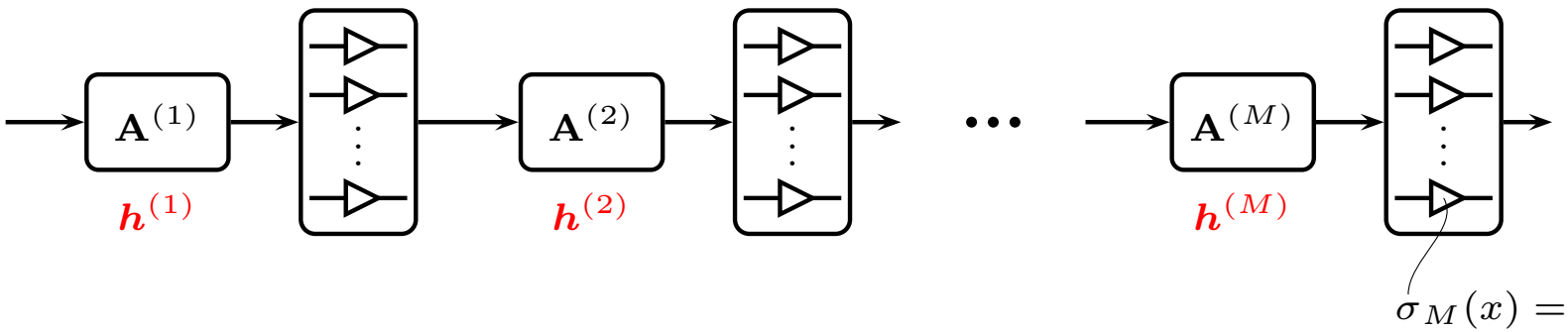

Deep learning of all FIR filter coefficients $\theta = \{\boldsymbol{h}^{(1)}, \ldots, \boldsymbol{h}^{(M)}\}$:

$$\min_\theta \sum_{i=1}^{N} \text{Loss}(f_\theta(\boldsymbol{y}^{(i)}), \boldsymbol{x}^{(i)}) \triangleq g(\theta) \qquad \text{using} \quad \theta_{k+1} = \theta_k - \lambda \nabla_\theta g(\theta_k)$$

mean squared error — Adam optimizer, fixed learning rate

Learned Digital Backpropagation

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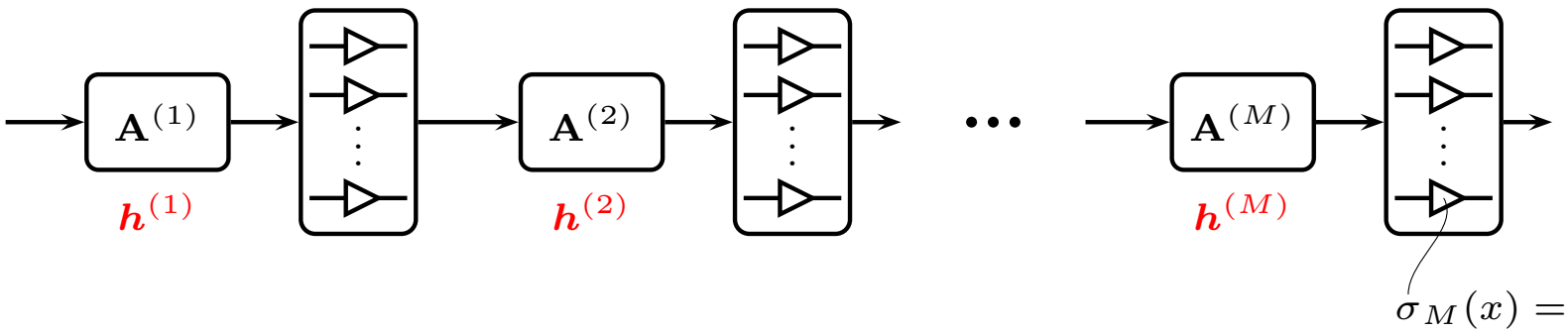

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mean squared error — Adam optimizer, fixed learning rate

Iteratively prune (set to 0) outermost filter taps during gradient descent

Iterative Filter Tap Pruning

$$\theta = \begin{cases} \boldsymbol{h}^{(1)} \\ \boldsymbol{h}^{(2)} \\ \vdots \\ \boldsymbol{h}^{(M)} \end{cases}$$

Iterative Filter Tap Pruning

starting length $2K' + 1$

$$\theta = \left\{ \begin{array}{lll} \boldsymbol{h}^{(1)} = (\; h_{K'}^{(1)} \; \cdots \; h_{K}^{(1)} \; \cdots \; h_{1}^{(1)} \; h_{0}^{(1)} \; h_{1}^{(1)} \; \cdots \; h_{K}^{(1)} \; \cdots \; h_{K'}^{(1)} \;) & \text{step } 1 \\ \boldsymbol{h}^{(2)} = (\; h_{K'}^{(2)} \; \cdots \; h_{K}^{(2)} \; \cdots \; h_{1}^{(2)} \; h_{0}^{(2)} \; h_{1}^{(2)} \; \cdots \; h_{K}^{(2)} \; \cdots \; h_{K'}^{(2)} \;) & \text{step } 2 \\ \vdots \quad \vdots \quad \vdots & \\ \boldsymbol{h}^{(M)} = (\; h_{K'}^{(M)} \; \cdots \; h_{K}^{(M)} \; \cdots \; h_{1}^{(M)} \; h_{0}^{(M)} \; h_{1}^{(M)} \; \cdots \; h_{K}^{(M)} \; \cdots \; h_{K'}^{(M)} \;) & \text{step } M \end{array} \right.$$

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- Initially: constrained least-squares coefficients (LS-CO) [Sheikh et al., 2016]

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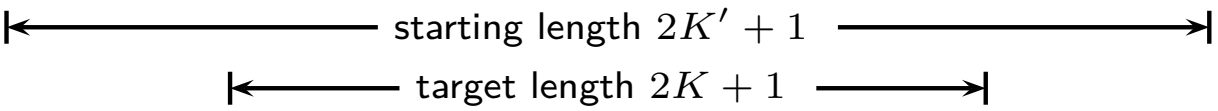

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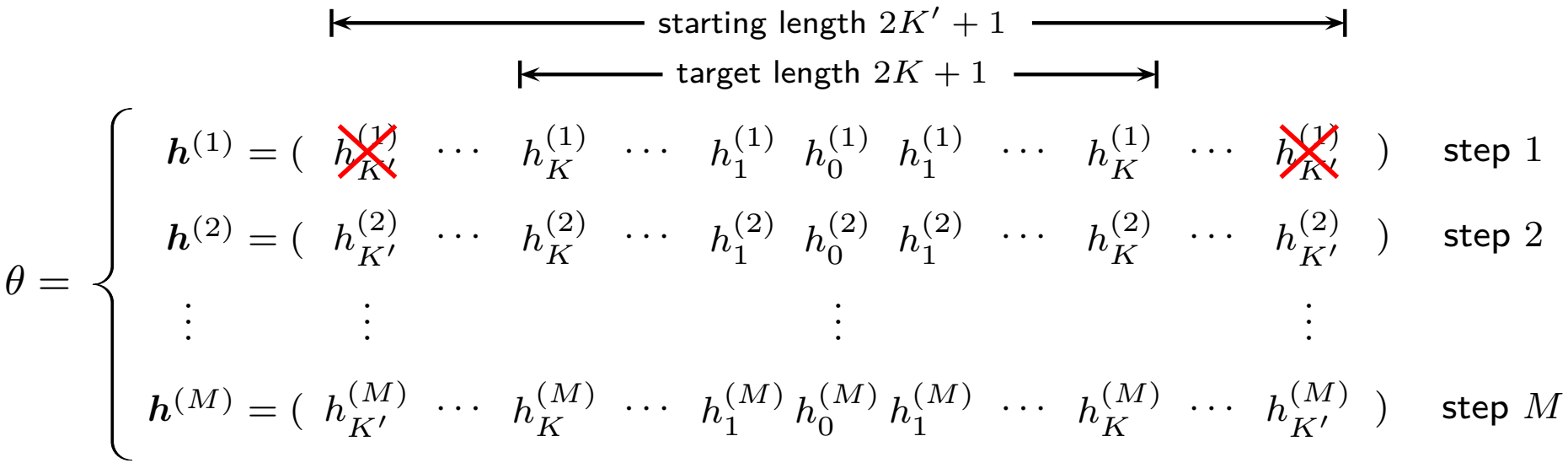

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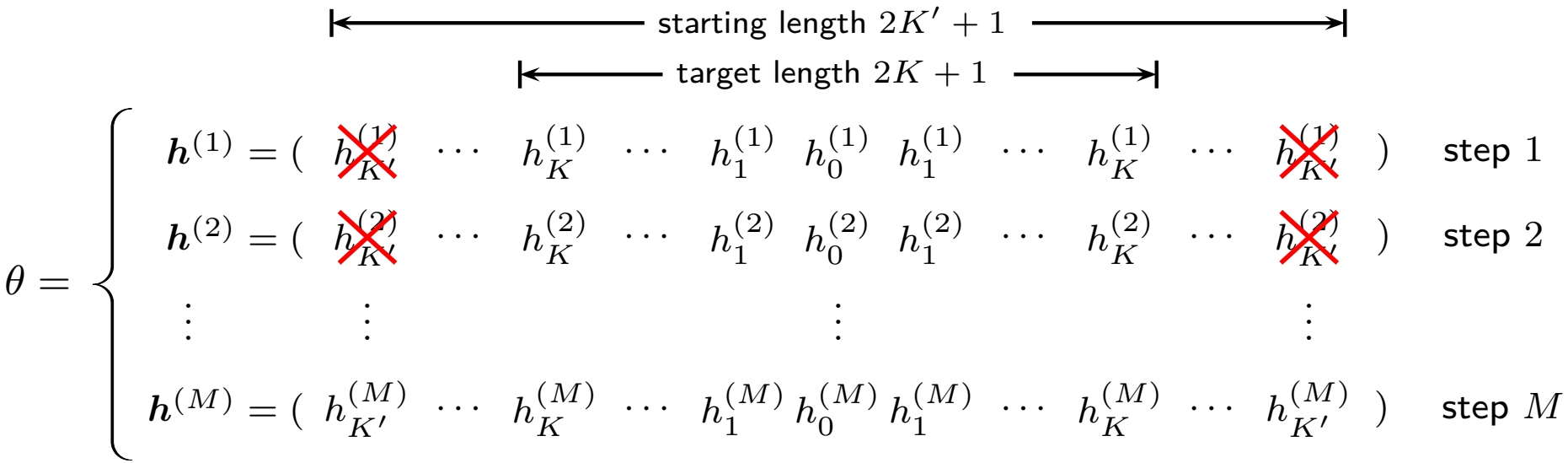

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Iterative Filter Tap Pruning

starting length $2K' + 1$

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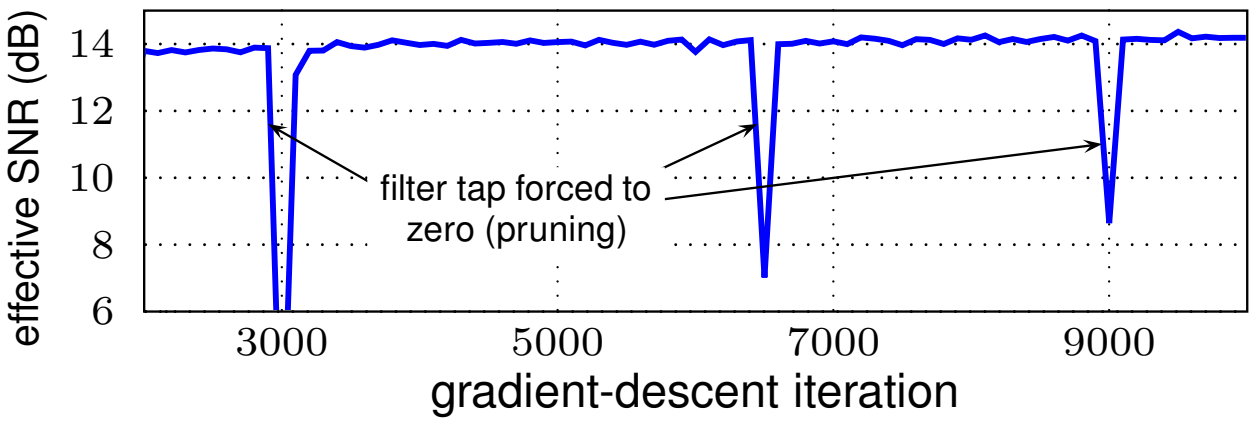
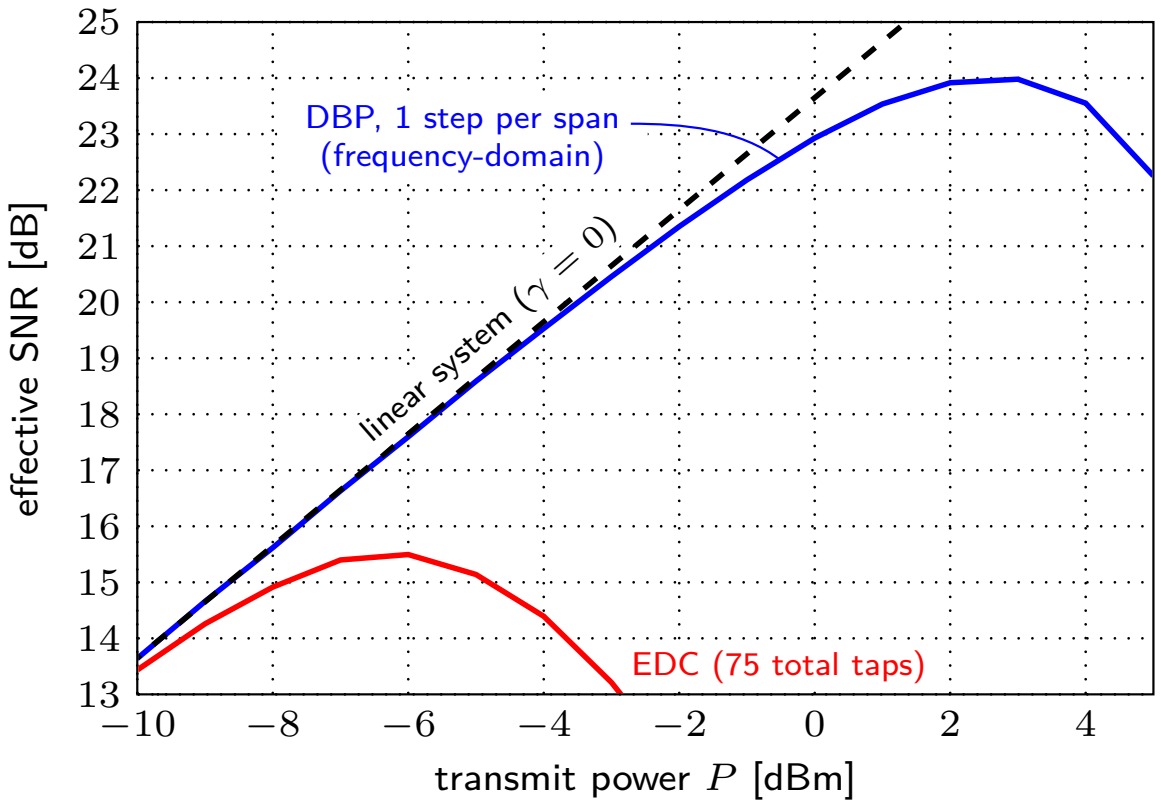
- Typical learning curve:



Revisiting Ip and Kahn (2008)



Parameters similar to [Ip and Kahn, 2008]:

- 25×80 km SSFM
- Gaussian modulation
- RRC pulses (0.1 roll-off)
- 10.7 Gbaud
- 2 samples/symbol processing
- single channel, single pol.

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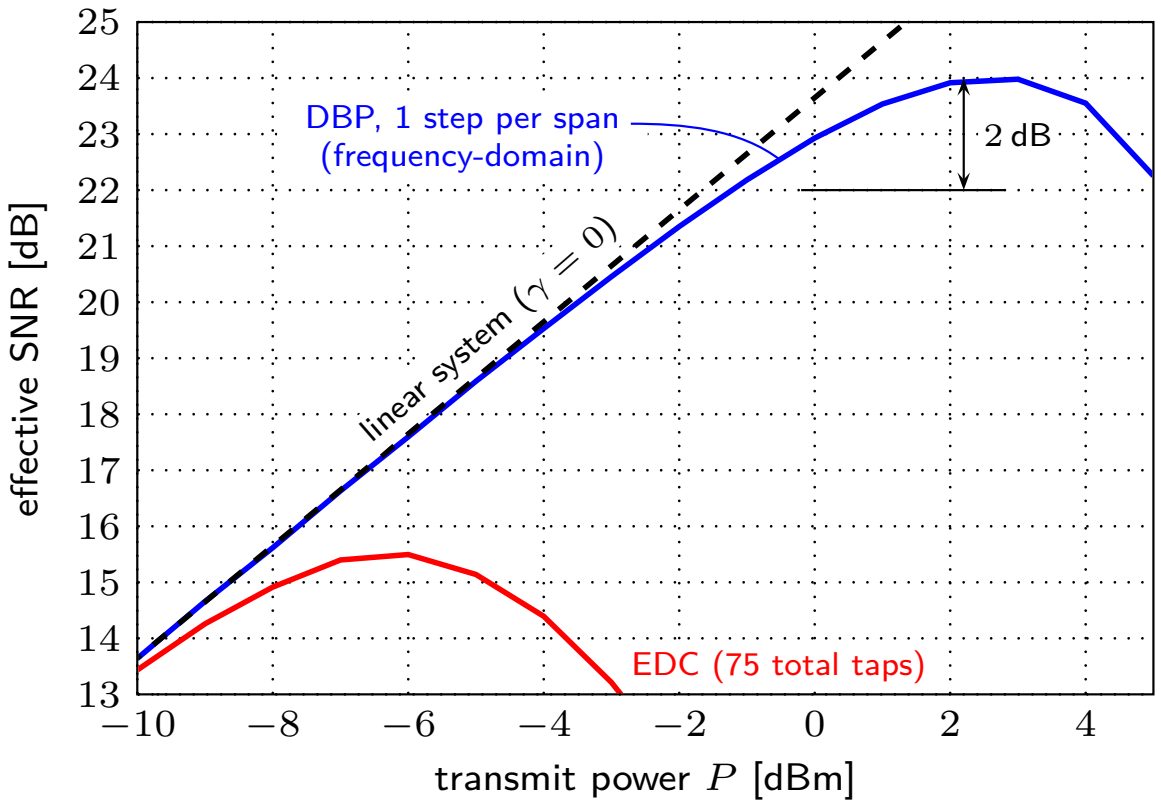

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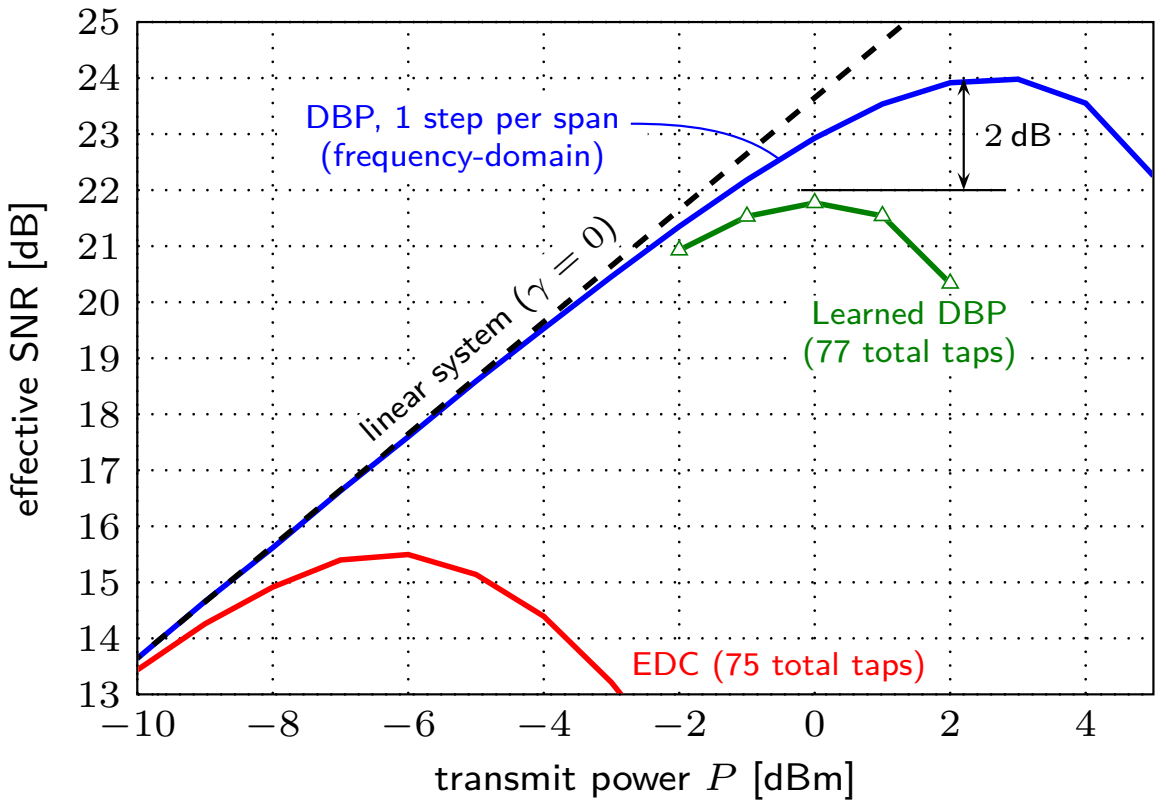

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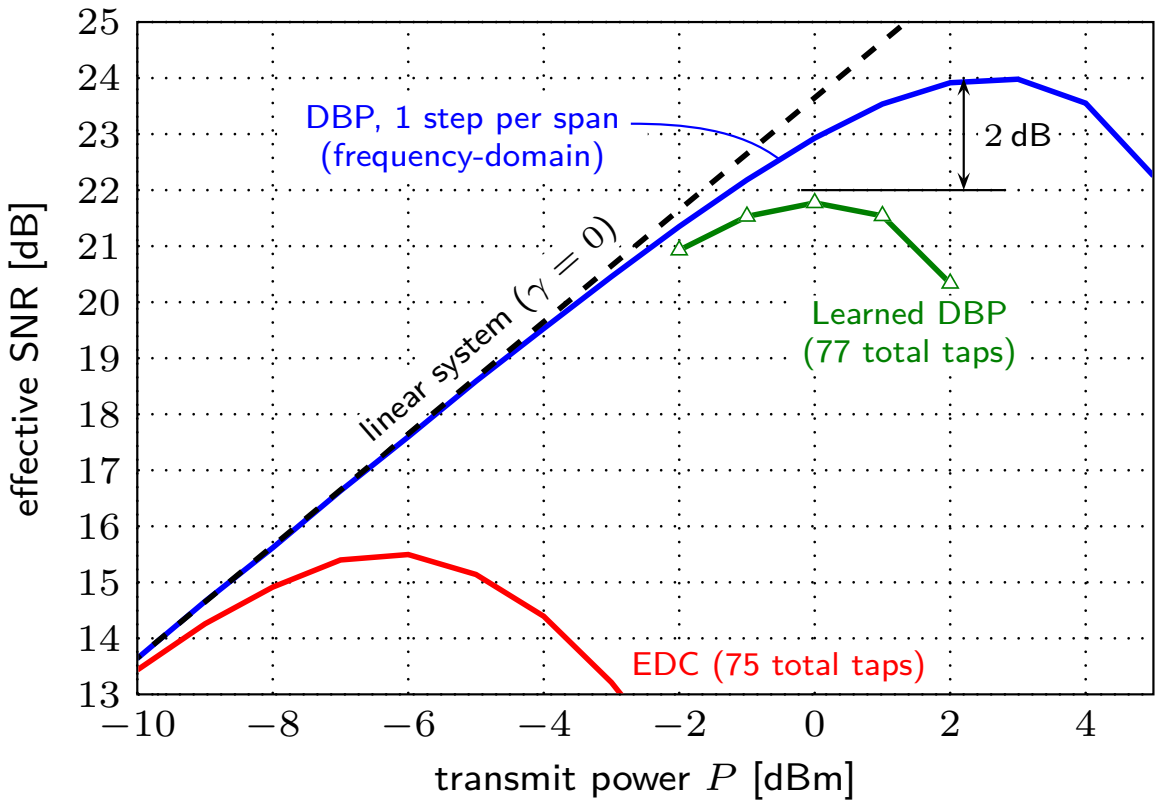

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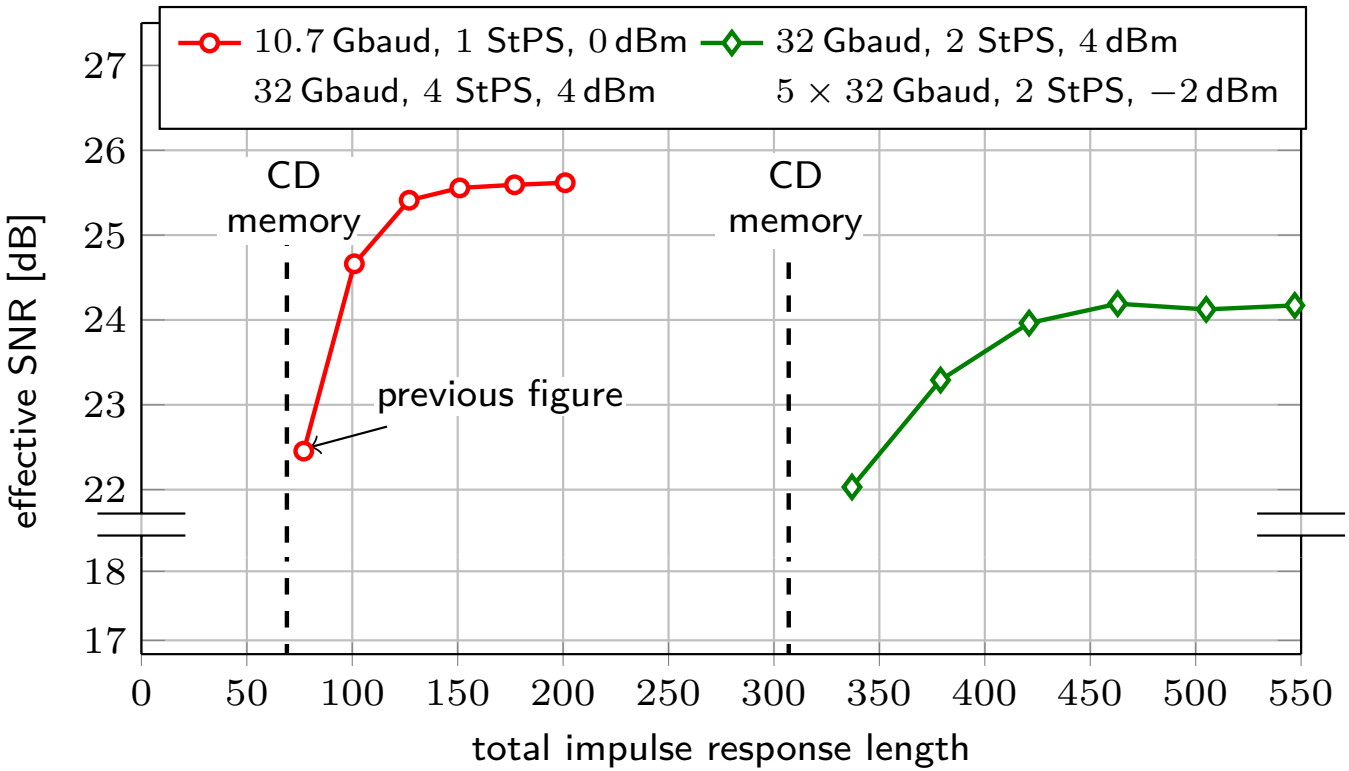
- Learned approach uses only 77 total taps: alternate 5 and 3 taps/step and use different filter coefficients in all steps [Häger and Pfister, 2018a]
- Can outperform "ideal DBP" in the nonlinear regime [Häger and Pfister, 2018b]

Performance–Complexity Trade-off



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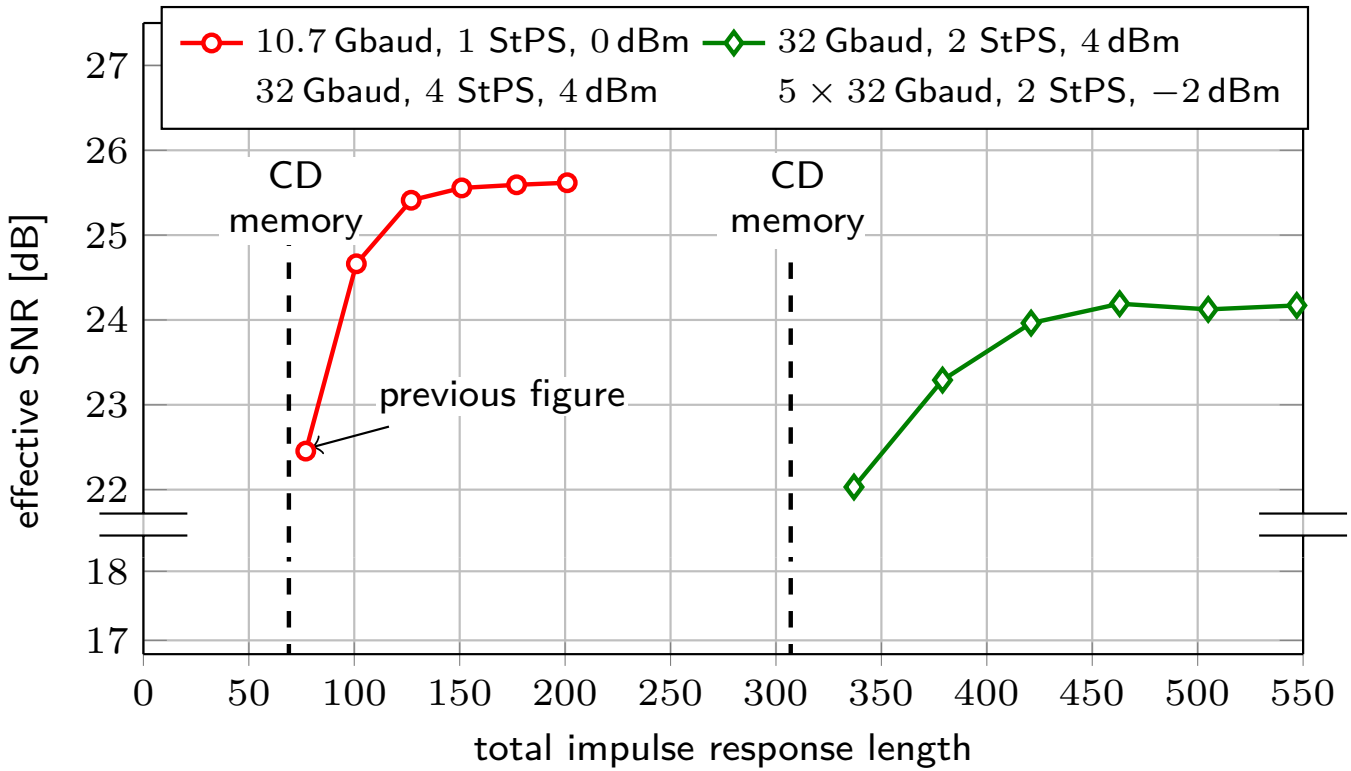

Conventional wisdom: Steps are inefficient $\Longrightarrow$ reduce as much as possible

Complexity $\overset{?}{\approx}$ Number of Steps

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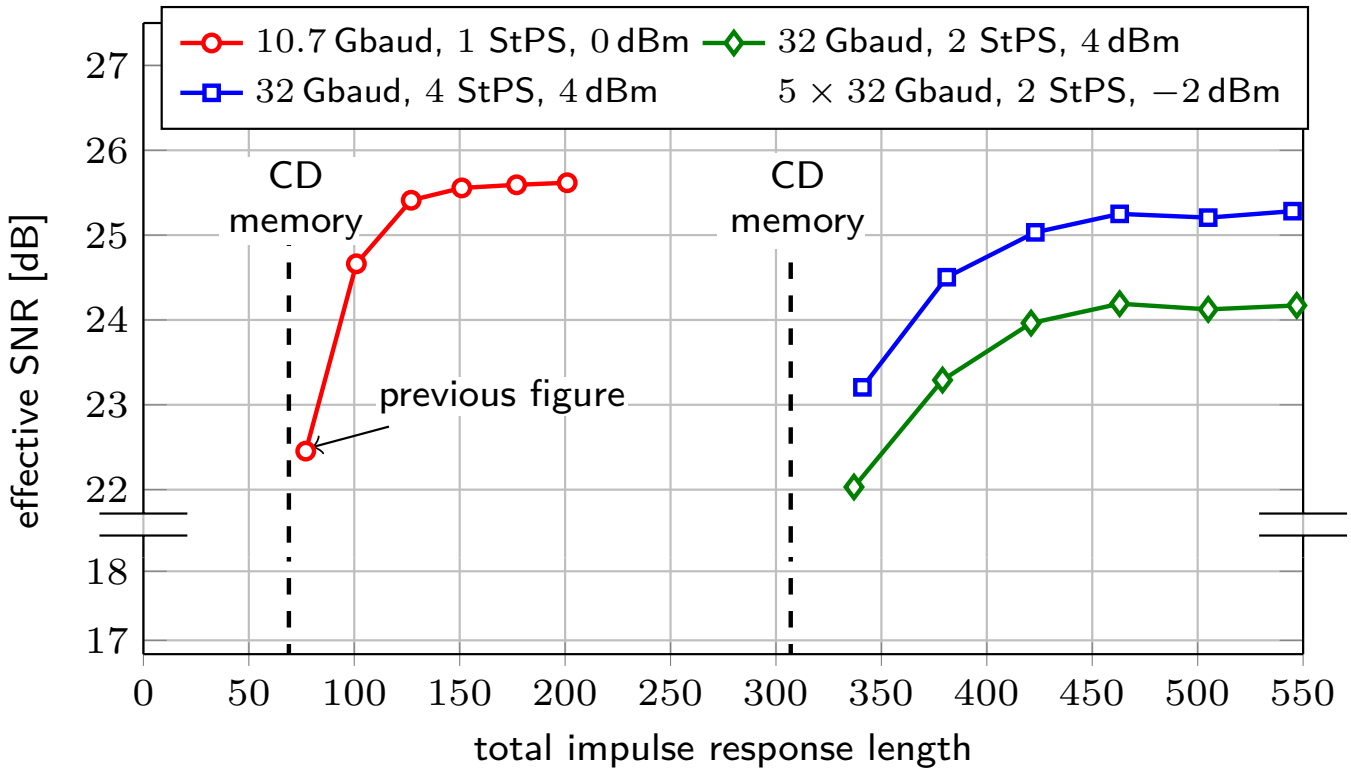

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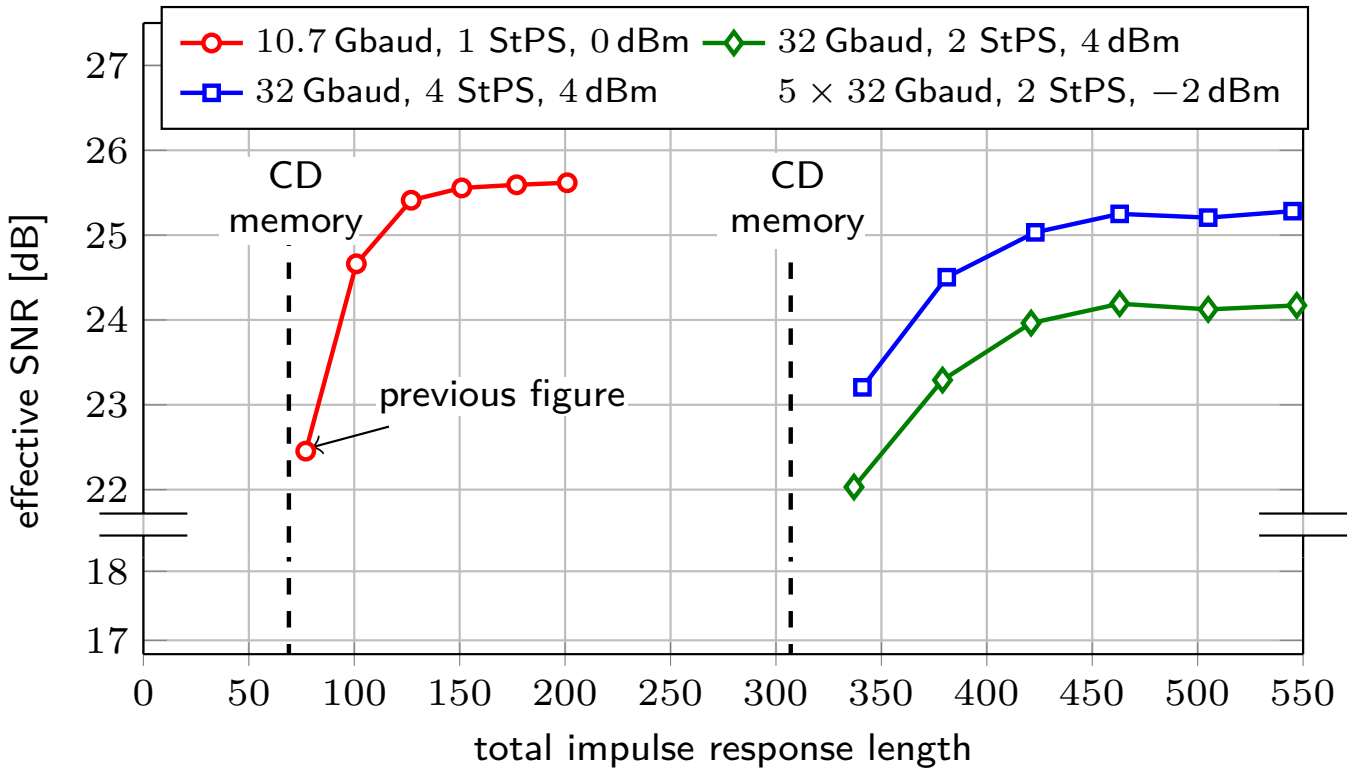

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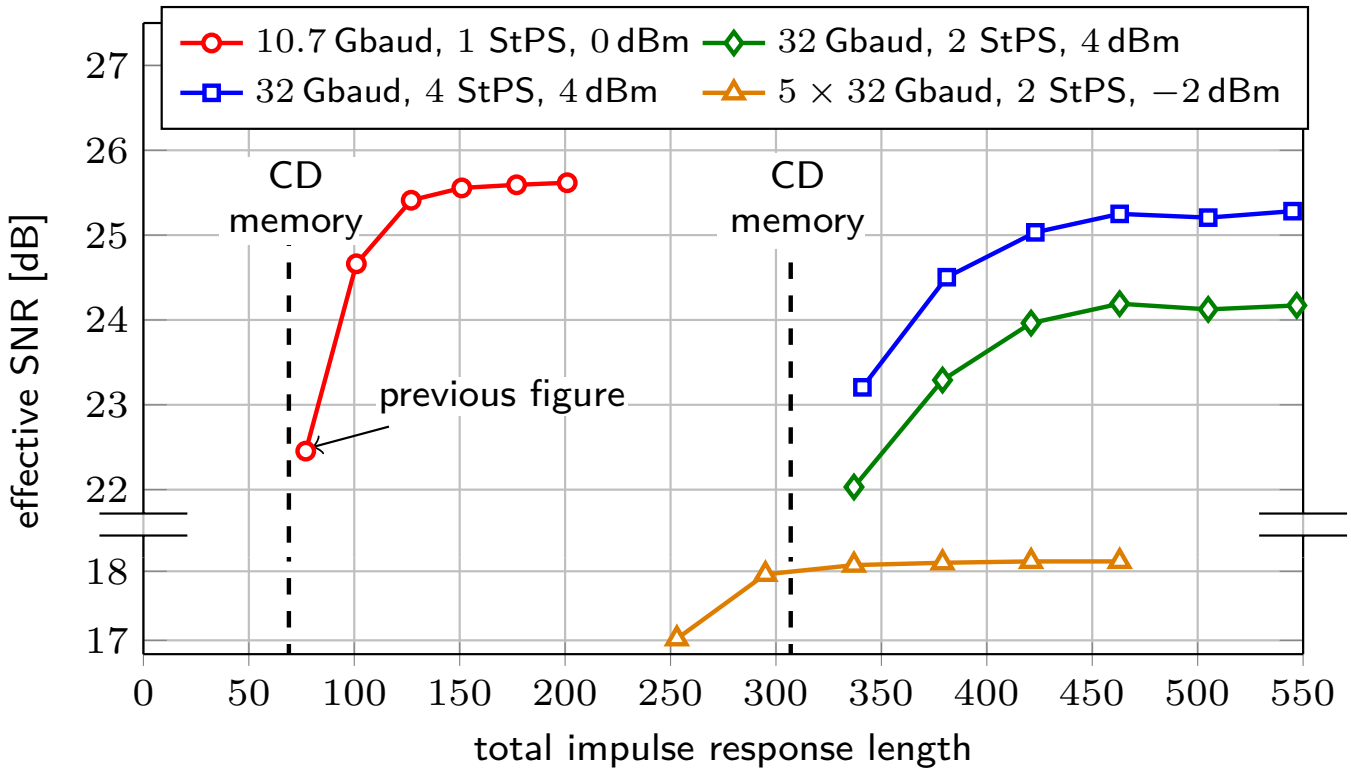

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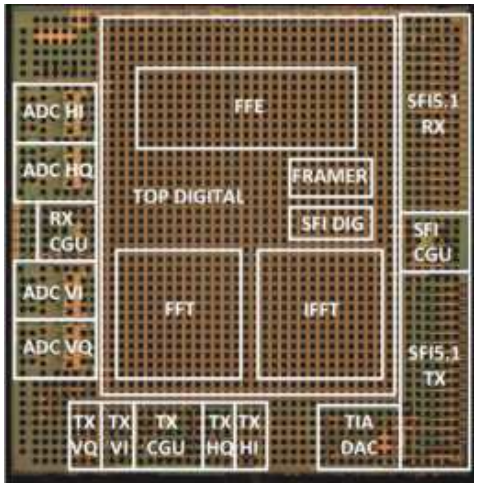
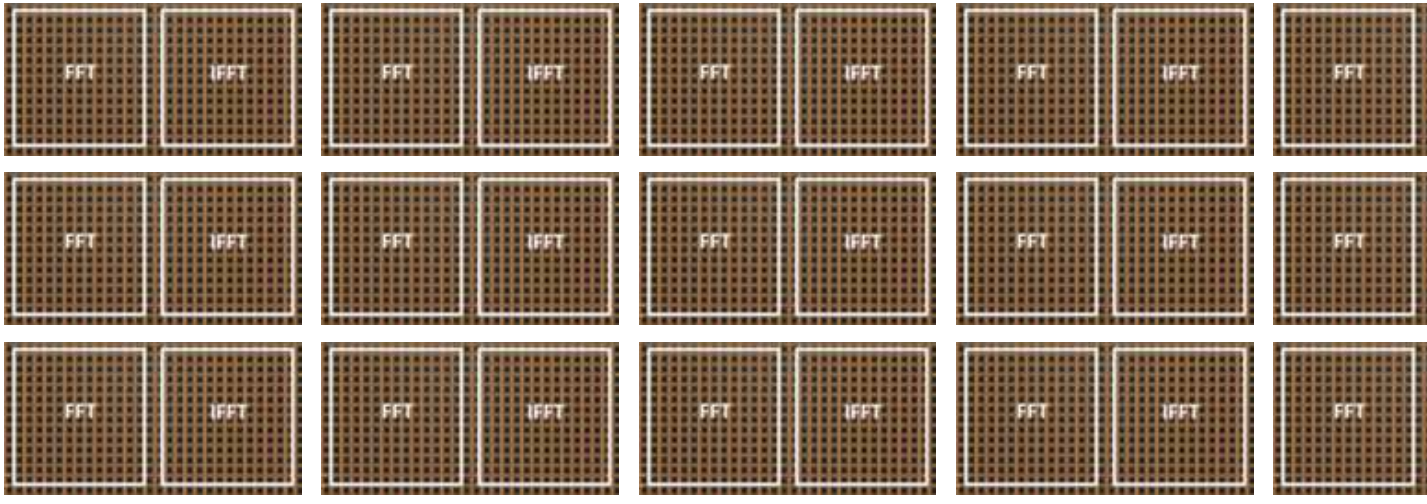
Real-Time ASIC Implementation



[Crivelli et al., 2014]



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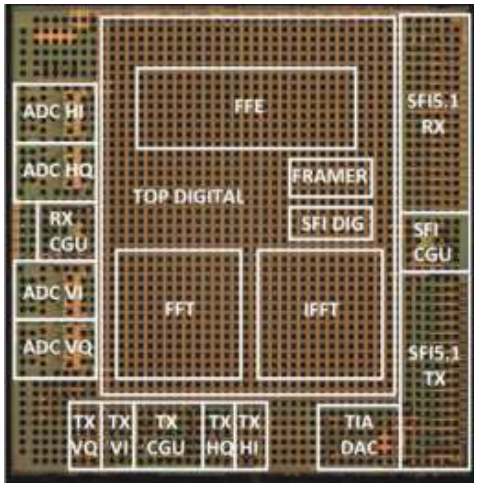

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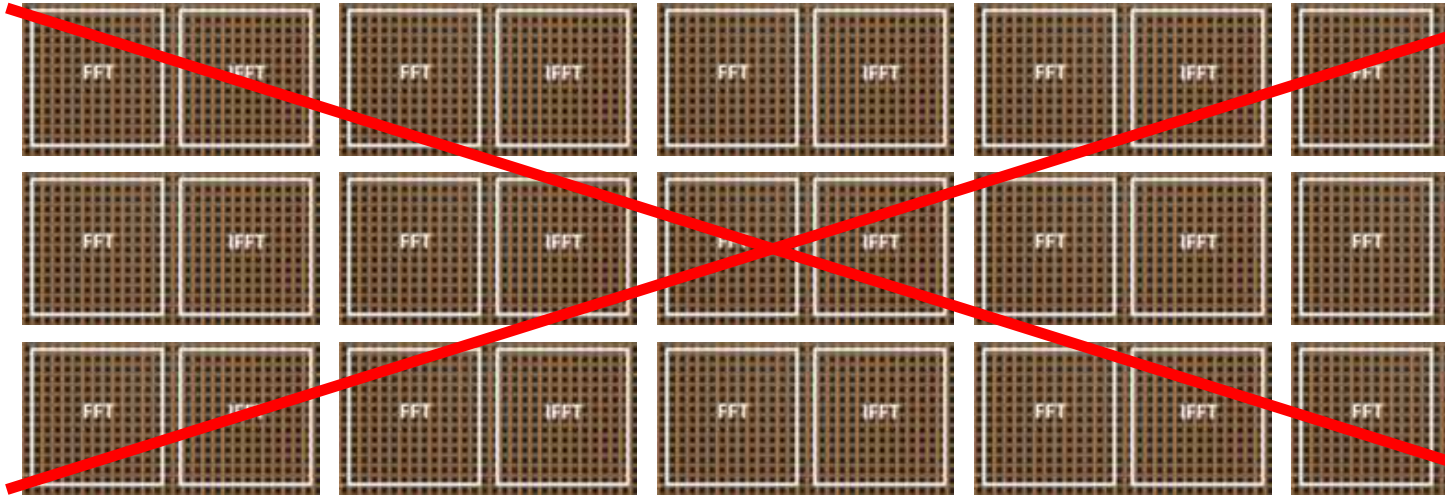

[Fougstedt et al., 2017], Time-domain digital back propagation: Algorithm and finite-precision implementation aspects, (*OFC*)
[Fougstedt et al., 2018], ASIC implementation of time-domain digital back propagation for coherent receivers, *(PTL)*
[Sherborne et al., 2018], On the impact of fixed point hardware for optical fiber nonlinearity compensation algorithms, (*JLT*)

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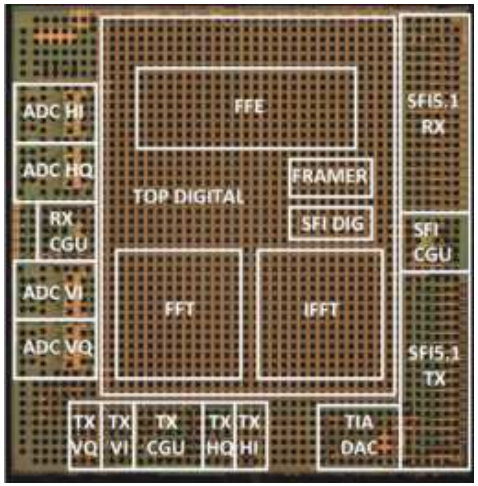

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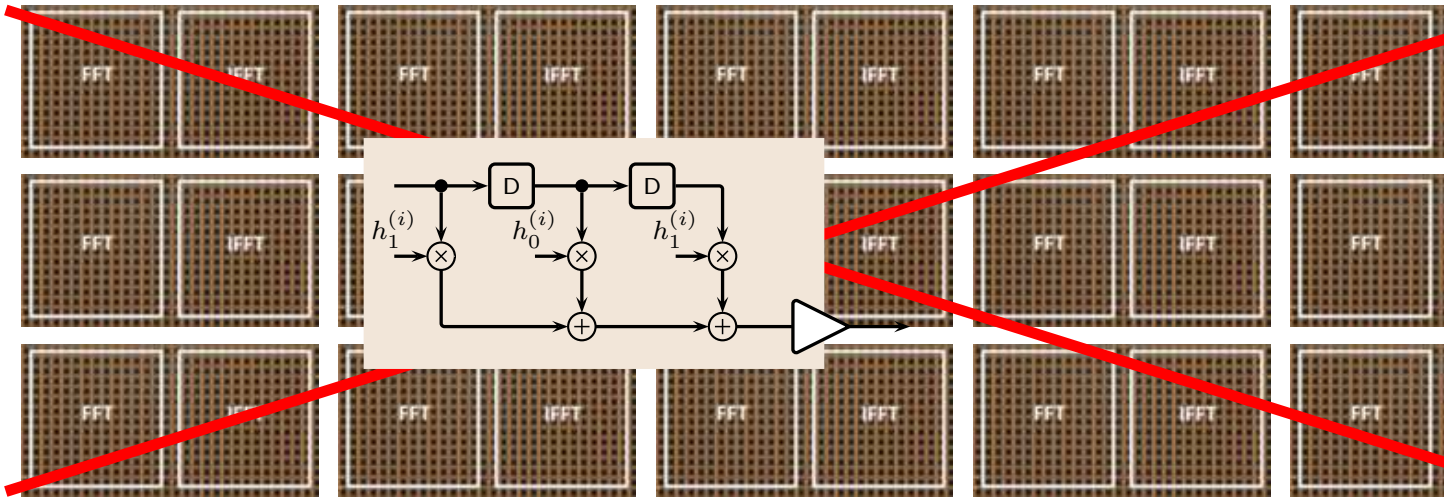

- Our linear steps are very short symmetric FIR filters (as few as 3 taps)

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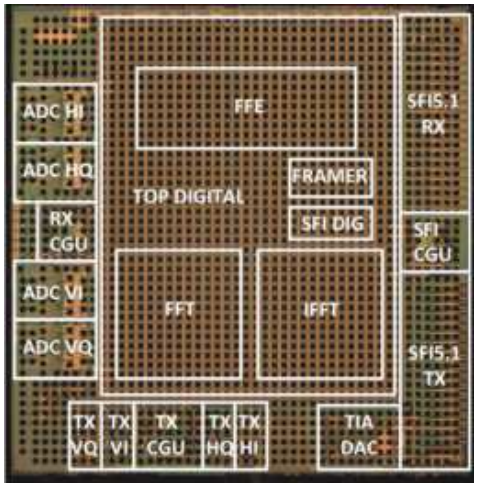

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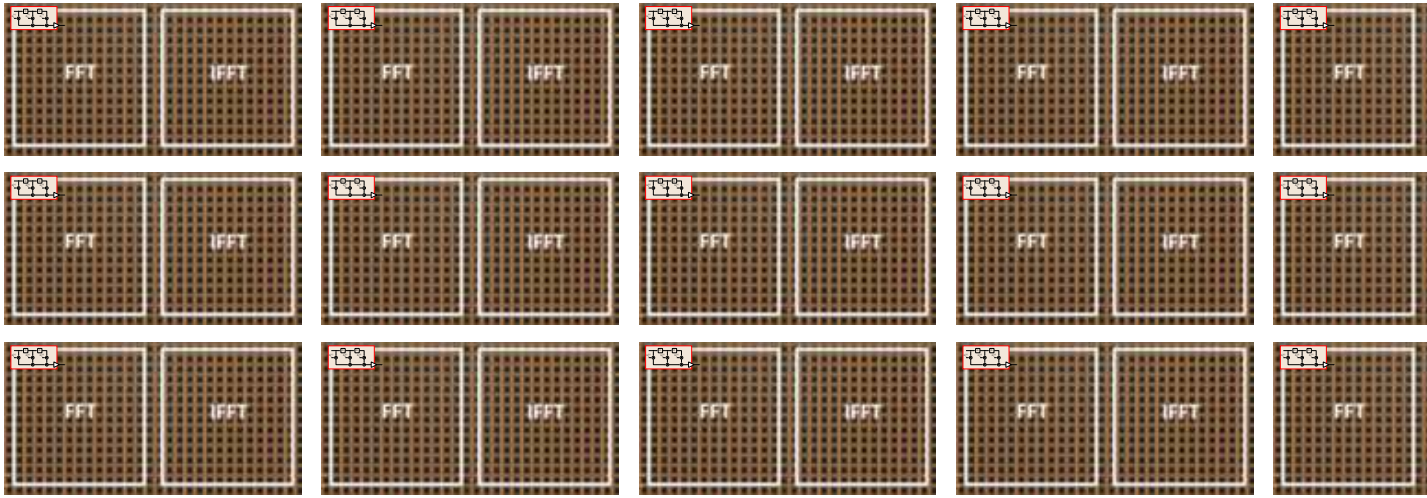

- Our linear steps are very short symmetric FIR filters (as few as 3 taps)
- 28-nm ASIC at 416.7 MHz clock speed (40 GHz signal)
  - Only 5-6 bit filter coefficients via learned quantization
  - Hardware-friendly nonlinear steps (Taylor expansion)
  - All FIR filters are fully reconfigurable

[Fougstedt et al., 2018], ASIC implementation of time-domain digital backpropagation with deep-learned chromatic dispersion filters, *(ECOC)*

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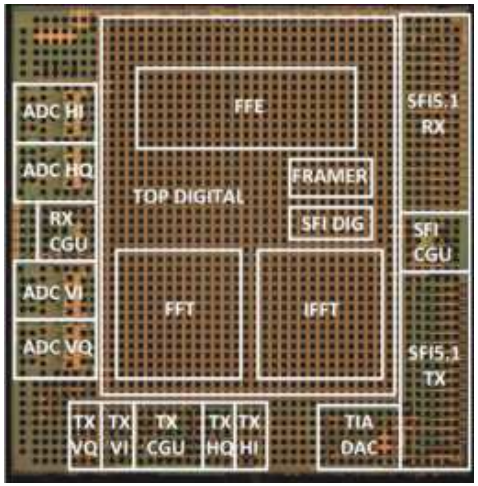

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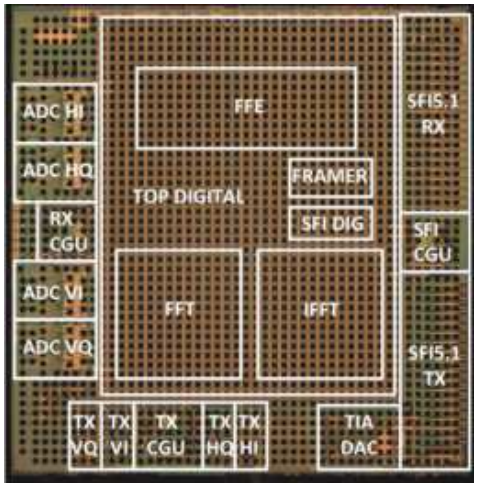

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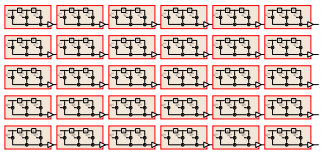

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  - All FIR filters are fully reconfigurable
- $< 2\times$ power compared to EDC [Crivelli et al., 2014, Pillai et al., 2014]

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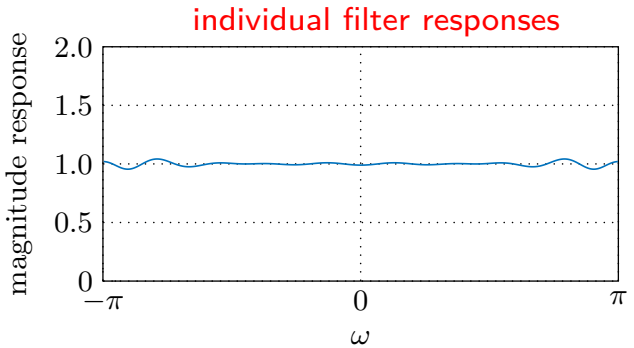
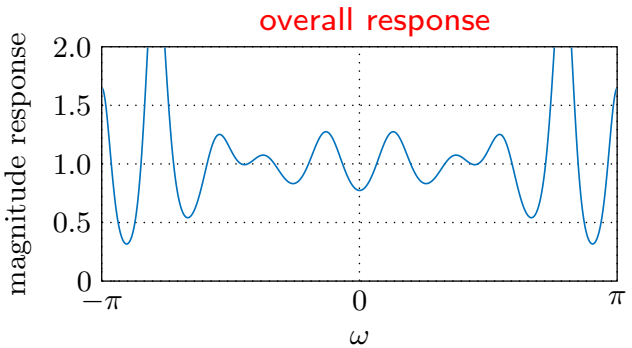
Why Does The Learning Approach Work?

Previous work: design a single filter or filter pair and use it repeatedly.

$\Longrightarrow$ Good overall response only possible with very long filters.





From [Ip and Kahn, 2009]:

- "We also note that [...] 70 taps, is much larger than expected"
- "This is due to amplitude ringing in the frequency domain"
- "Since backpropagation requires multiple iterations of the linear filter, amplitude distortion due to ringing accumulates (Goldfarb & Li, 2009)"

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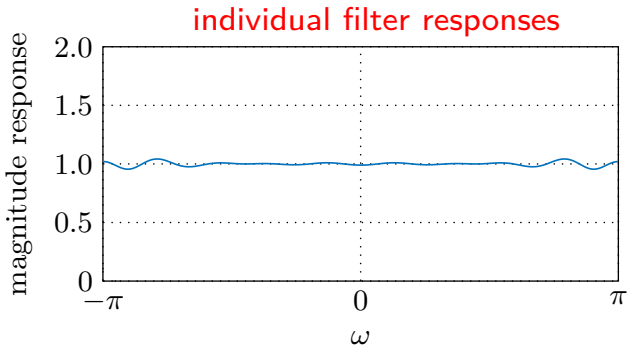

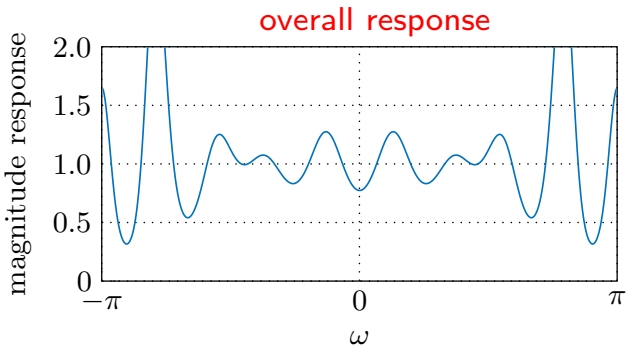

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The learning approach uncovered that there is no such requirement!

[Lian, Häger, Pfister, 2018], What can machine learning teach us about communications? (*ITW*)

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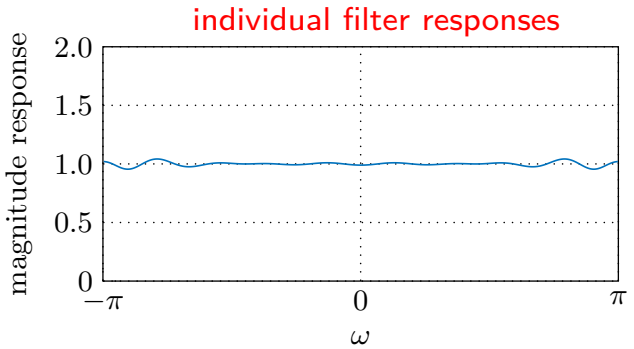
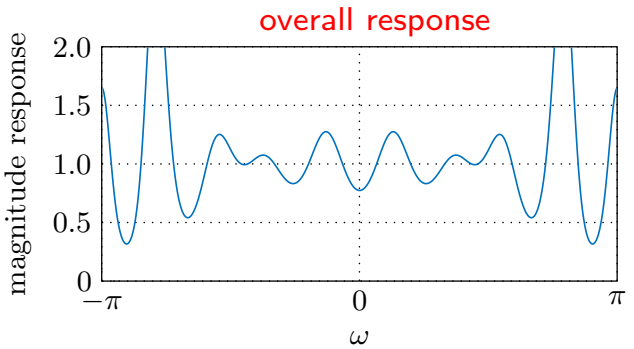
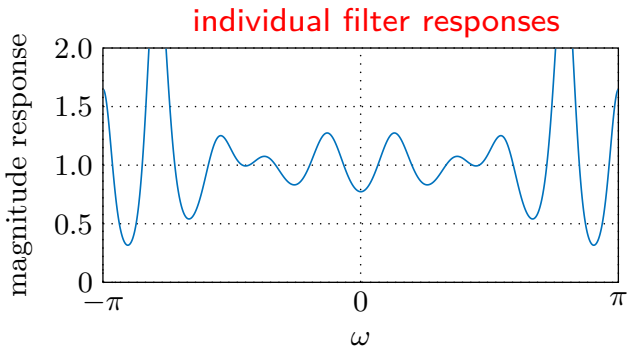
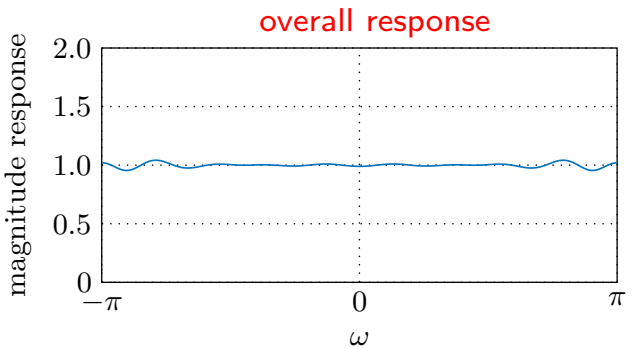




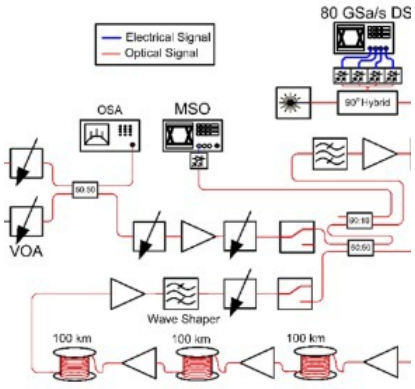
Sacrifice individual filter accuracy, but different response per step.

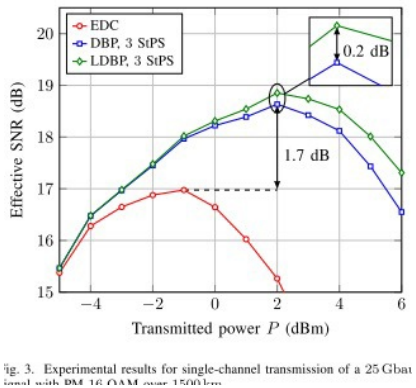
$\Longrightarrow$ Good overall response even with very short filters by joint optimization.

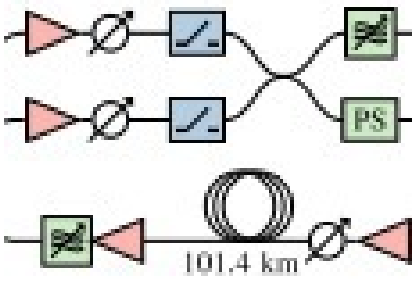


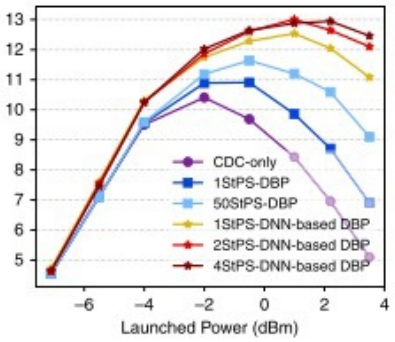


Experimental Investigations









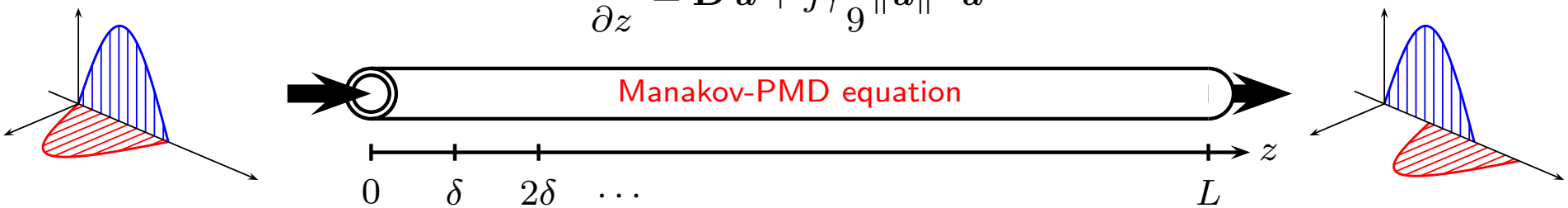
Training with real-world data sets including presence of various hardware impairments (phase noise, timing error, frequency offset, etc.)

- [Oliari et al., 2020], Revisiting Efficient Multi-step Nonlinearity Compensation with Machine Learning: An Experimental Demonstration, *(J. Lightw. Technol.)*
- [Sillekens et al., 2020], Experimental Demonstration of Learned Time-domain Digital Back-propagation, *(Proc. IEEE Workshop on Signal Processing Systems)*
- [Fan et al., 2020], Advancing Theoretical Understanding and Practical Performance of Signal Processing for Nonlinear Optical Communications through Machine Learning, *(Nat. Commun.)*
- [Bitachon et al., 2020], Deep learning based Digital Back Propagation Demonstrating SNR gain at Low Complexity in a 1200 km Transmission Link, *(Opt. Express)*

Outline

Evolution of Polarization-Multiplexed Signals

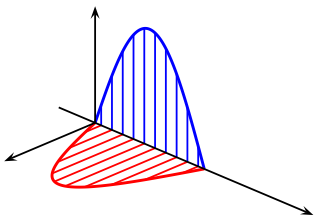
$$\frac{\partial \boldsymbol{u}}{\partial z} = \hat{\mathbf{D}}\,\boldsymbol{u} + \jmath\gamma\frac{8}{9}\|\boldsymbol{u}\|^2\,\boldsymbol{u}$$

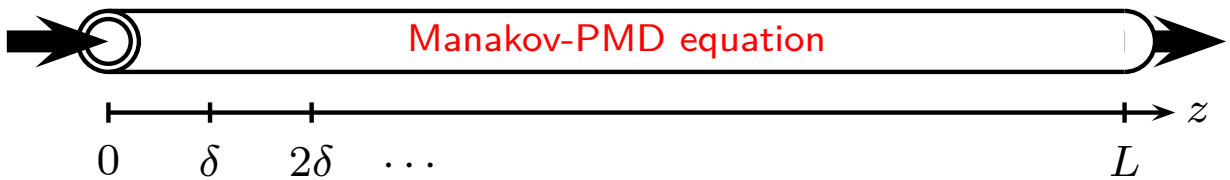


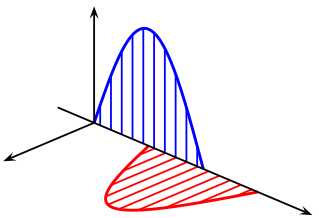
- Jones vector $\boldsymbol{u} \triangleq (u_1(t,z), u_2(t,z))^\top$ with complex baseband signals
- linear operator $\hat{\mathbf{D}}$: attentuation, chromatic & polarization mode dispersion

Evolution of Polarization-Multiplexed Signals

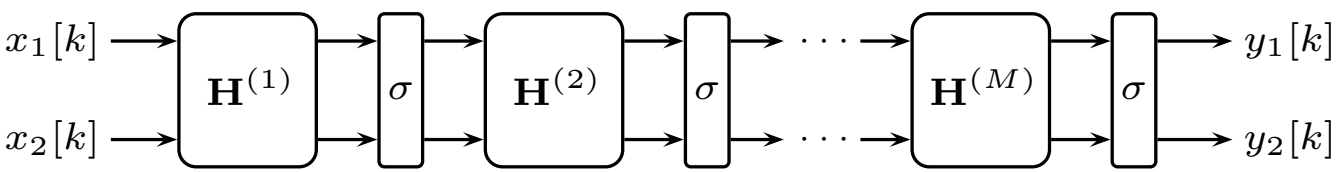
$$\frac{\partial \boldsymbol{u}}{\partial z} = \hat{\mathbf{D}}\,\boldsymbol{u} + \jmath\gamma\frac{8}{9}\|\boldsymbol{u}\|^2\,\boldsymbol{u}$$



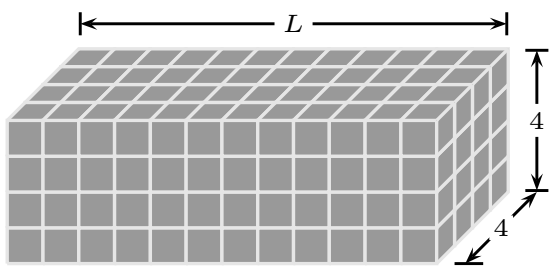




- Jones vector $\boldsymbol{u} \triangleq (u_1(t,z), u_2(t,z))^\top$ with complex baseband signals
- linear operator $\hat{\mathbf{D}}$: attentuation, chromatic & polarization mode dispersion
- Split-step method: alternate linear and nonlinear steps $\sigma(\boldsymbol{x}) = \boldsymbol{x}e^{\jmath\gamma\frac{8}{9}\delta\|\boldsymbol{x}\|^2}$

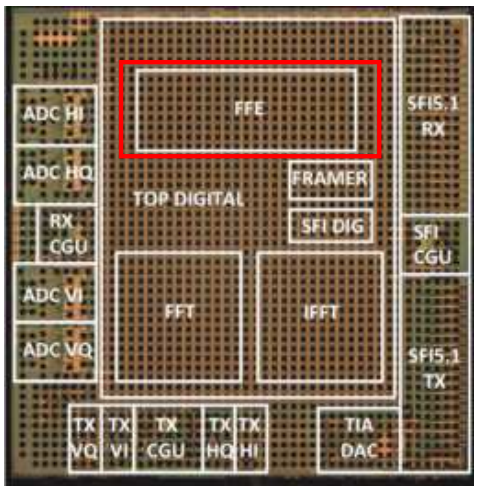


$e^{\delta\hat{\mathbf{D}}} \approx$

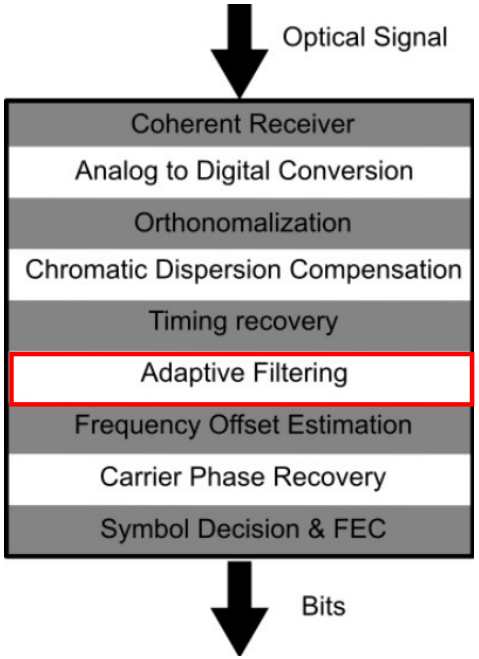


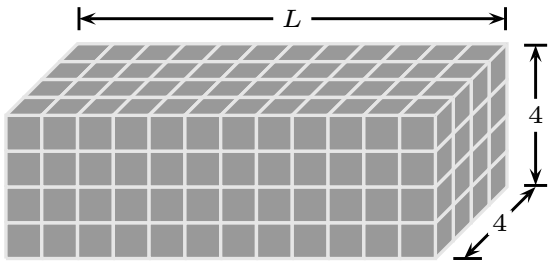
2×2 complex or
4×4 real MIMO filters $\Longrightarrow$ complexity!

Real-Time Compensation of Polarization Impairments



[Crivelli et al., 2014]





- time-varying effects (e.g., drifts) & apriori unknown realizations
- $\Longrightarrow$ adaptive filtering (via stochastic gradient descent) required

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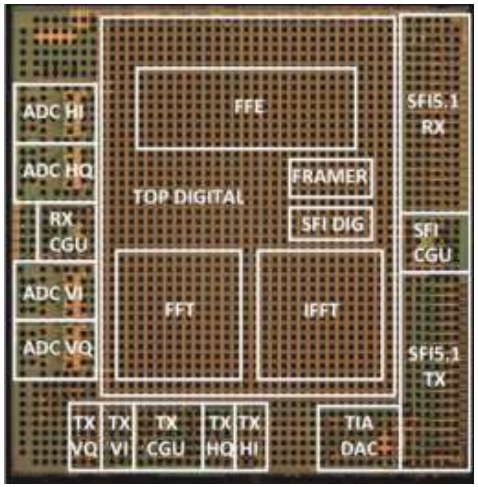

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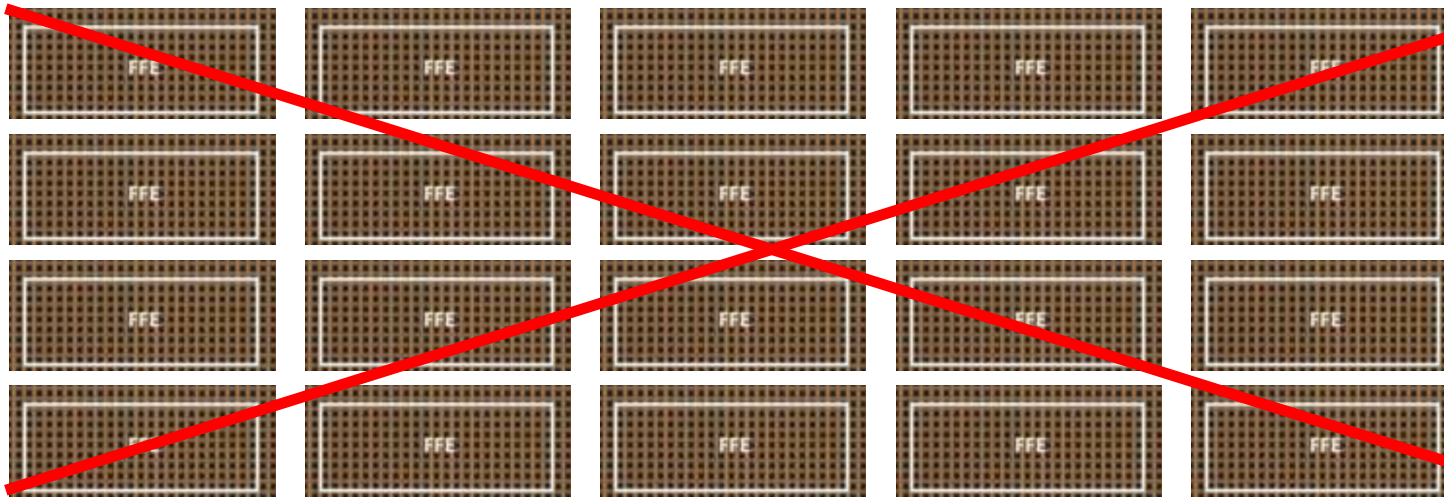

- time-varying effects (e.g., drifts) & apriori unknown realizations
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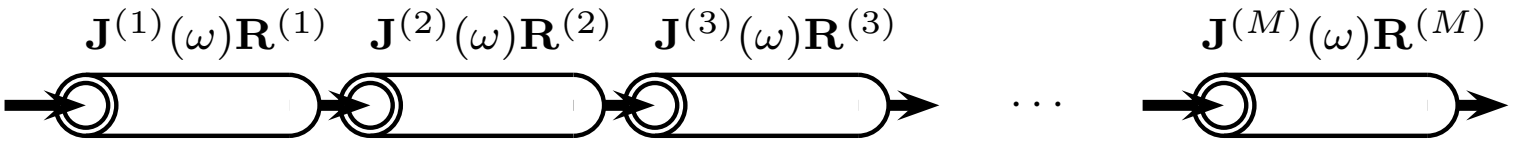
- Using (and updating) full MIMO filters in each step is not feasible.
- We propose a hardware-efficient machine-learning model based on the propagation characteristics

Modeling of Polarization Mode Dispersion (PMD)



The overall PMD is modeled via M sections, where each section introduces

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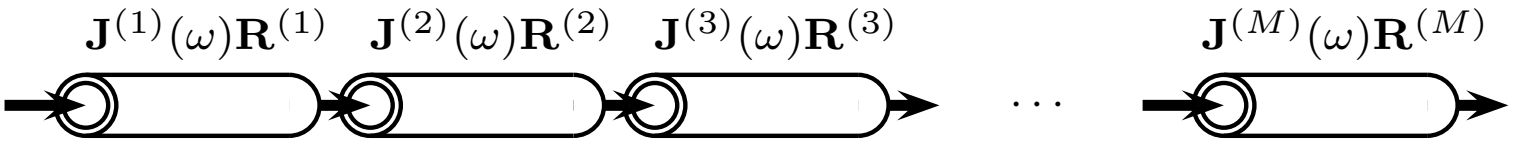

The overall PMD is modeled via M sections, where each section introduces

1. a differential group delay (DGD) $\tau^{(k)}$, described by

$$\mathbf{J}^{(k)}(\omega) = \begin{pmatrix} \exp\left(-\jmath\omega\frac{\tau^{(k)}}{2}\right) & 0 \\ 0 & \exp\left(\jmath\omega\frac{\tau^{(k)}}{2}\right) \end{pmatrix}$$

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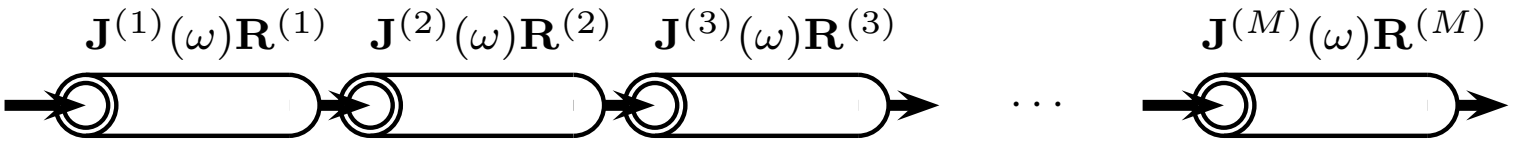

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2. a rotation of the polarization state, described by $\mathbf{R}^{(k)} \in \mathrm{SU}(2)$, where

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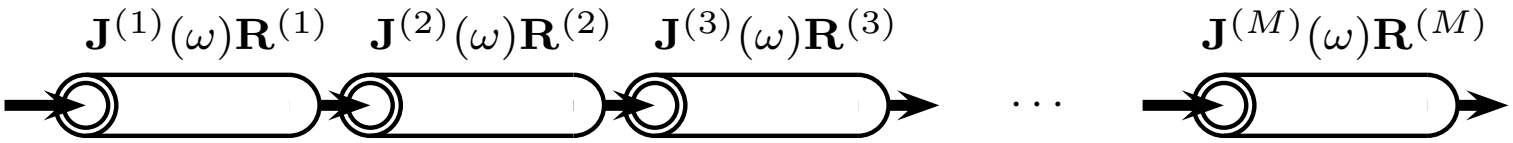

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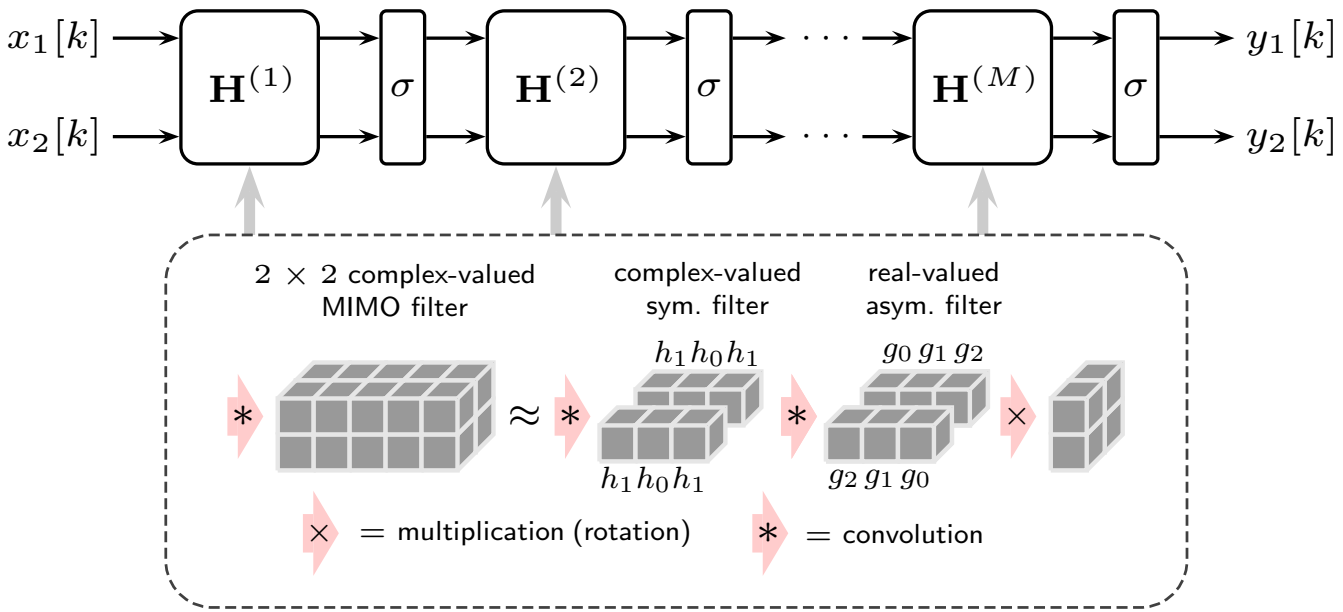
(i) We integrate these operations in each step/layer

(ii) We use real-valued (asymmetric) FIR filters to approximate DGD

The Final Machine-Learning Model



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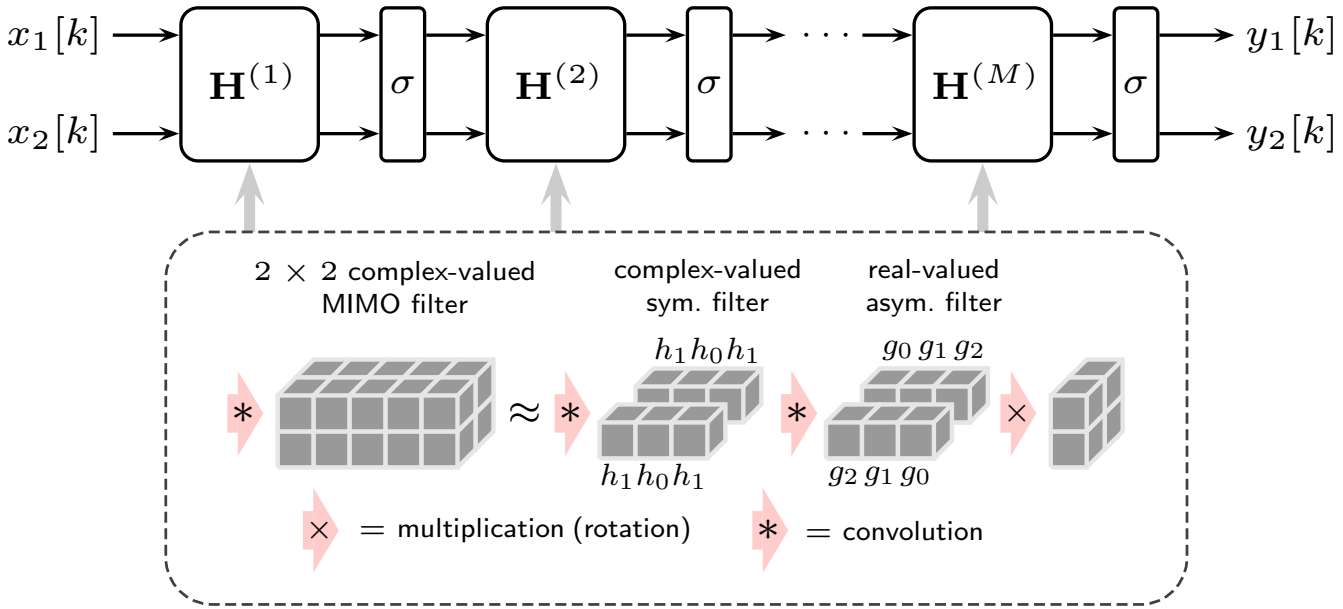

Each linear step consists of 3 trainable components

1. complex-valued (symmetric) filters that mainly account for dispersion
2. real-valued (asymmetric) filters for DGD
3. memoryless "rotation" matrices $\begin{pmatrix} a & -b^* \\ b & a^* \end{pmatrix}$, where $a, b \in \mathbb{C}$ (4 real parameters)

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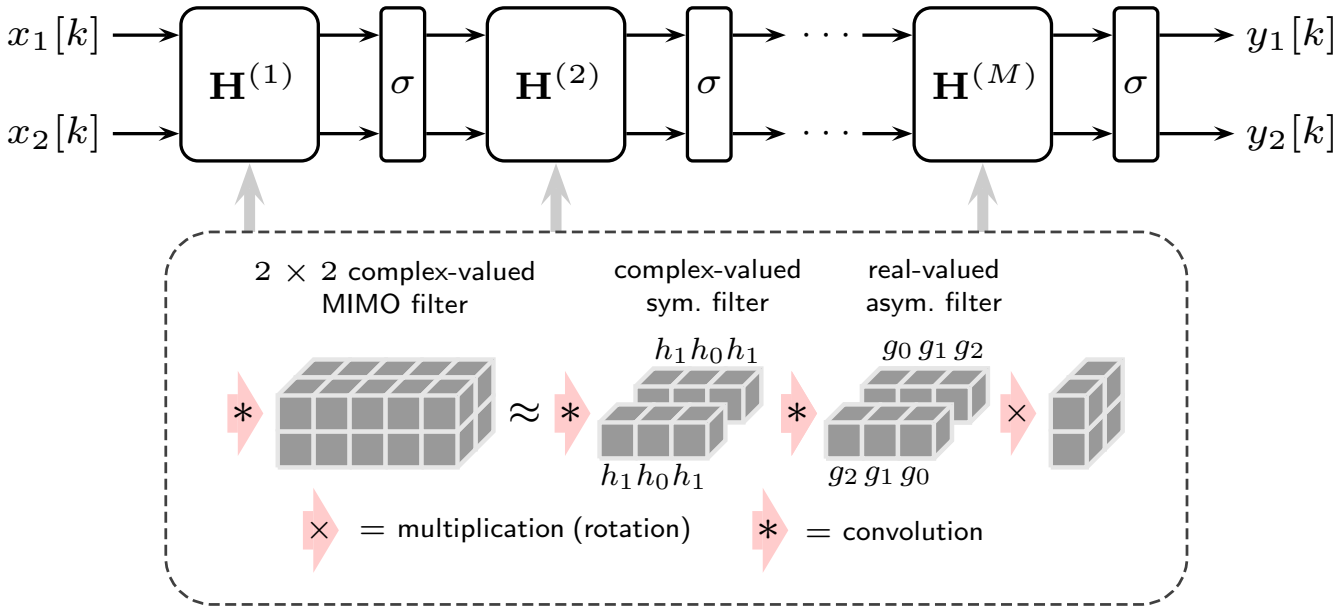
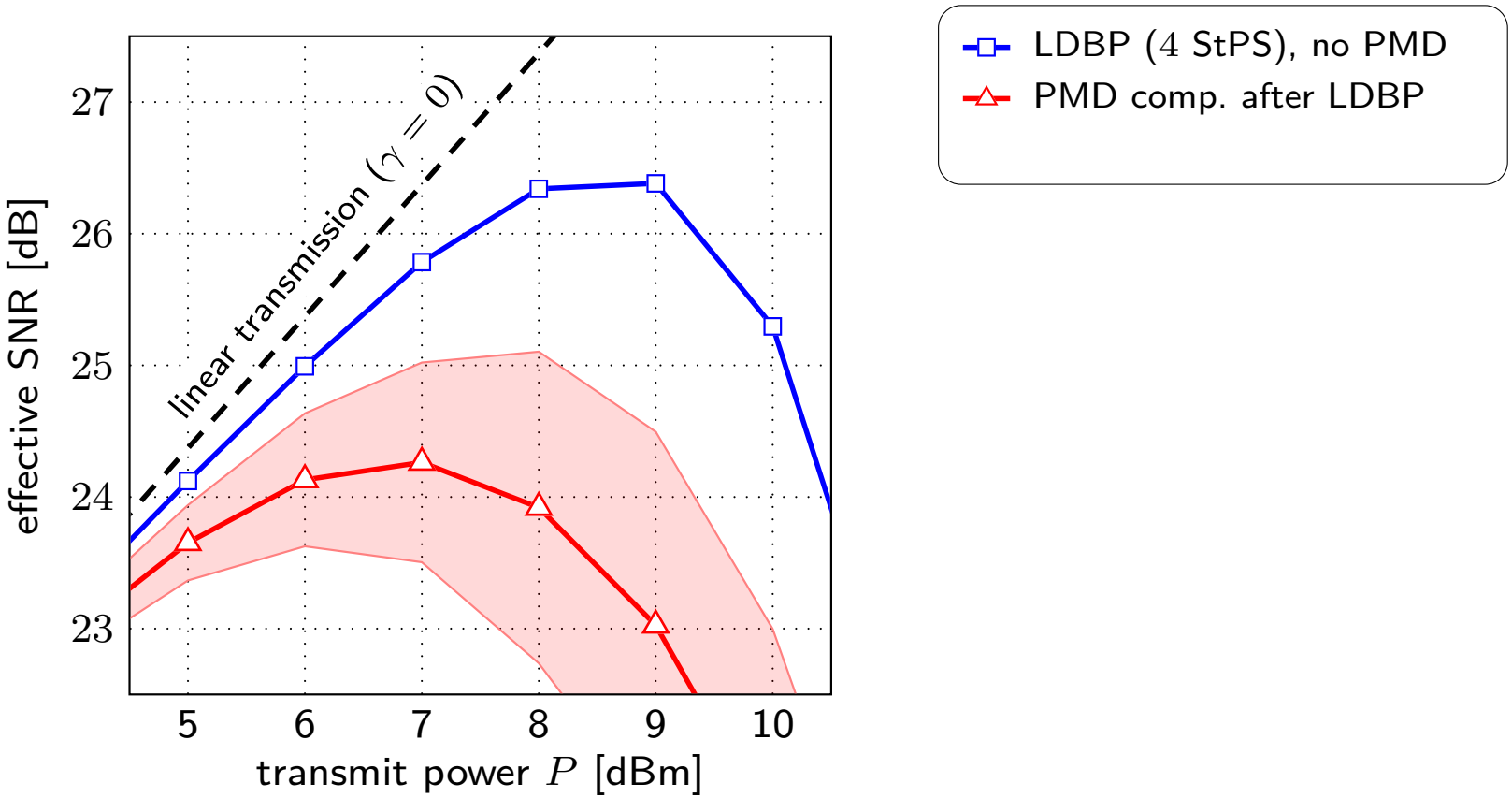


Compared to prior work, our learning-based approach . . .

- assumes no knowledge about PMD realizations or accumulated PMD
- is FIR-filter based! Avoids frequency-domain (FFT-based) filtering

[Goroshko et al., 2016], Overcoming performance limitations of digital back propagation due to polarization mode dispersion, *(CTON)*
[Czegledi et al., 2017], Digital backpropagation accounting for polarization-mode dispersion, *(Opt. Express)*
[Liga et al., 2018], A PMD-adaptive DBP receiver based on SNR optimization, *(OFC)*

Results (32 Gbaud, 10×100 km, 0.2 ps/$\sqrt{\text{km}}$ PMD)



- Similar parameters & simulation setup compared to [Czegledi et al., 2016], results averaged over 40 PMD realizations

[Bütler et al., 2021], Model-based Machine Learning for Joint Digital Backpropagation and PMD Compensation, *(J. Lightw. Technol.)*, see arXiv:2010.12313

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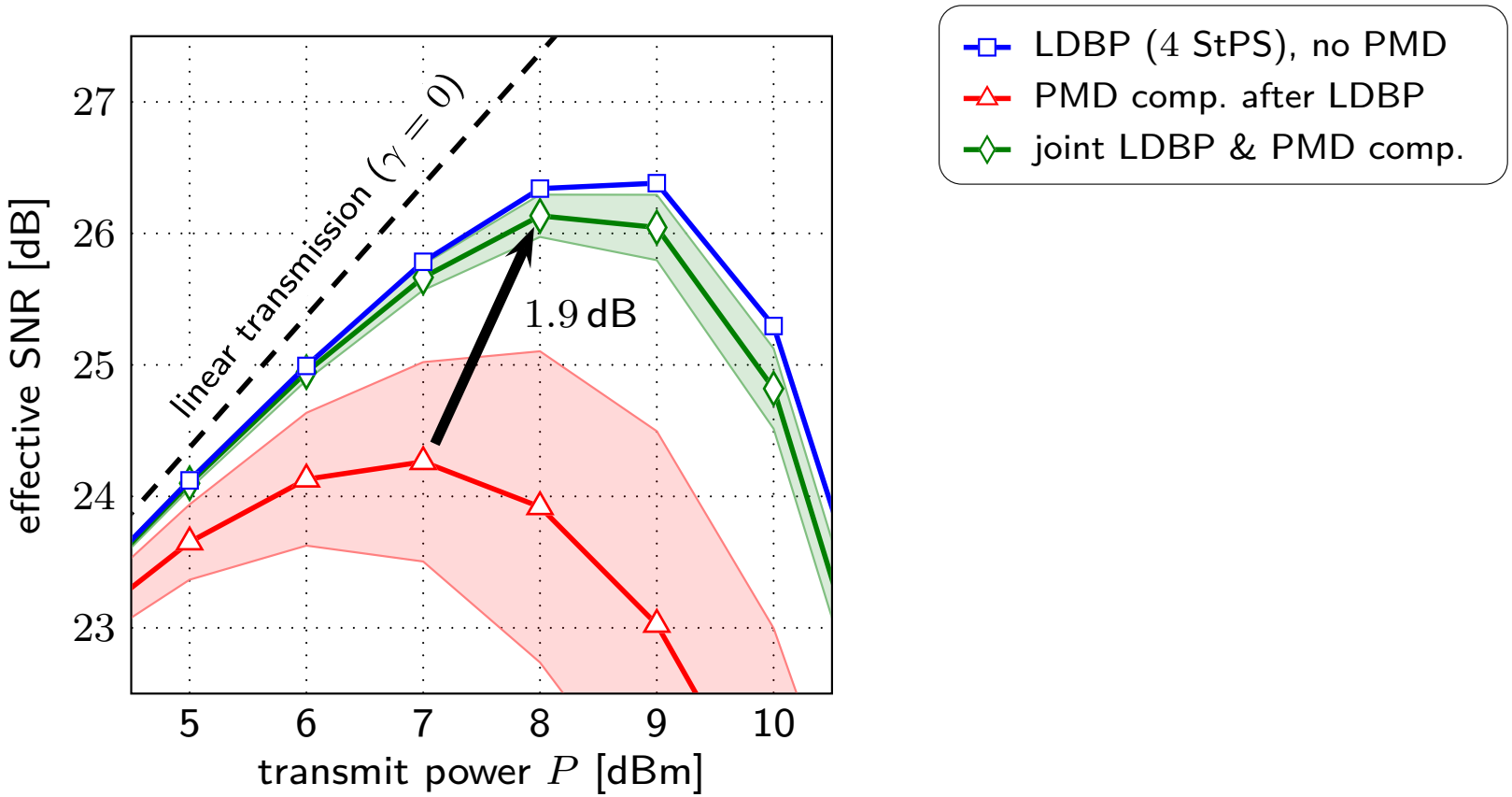

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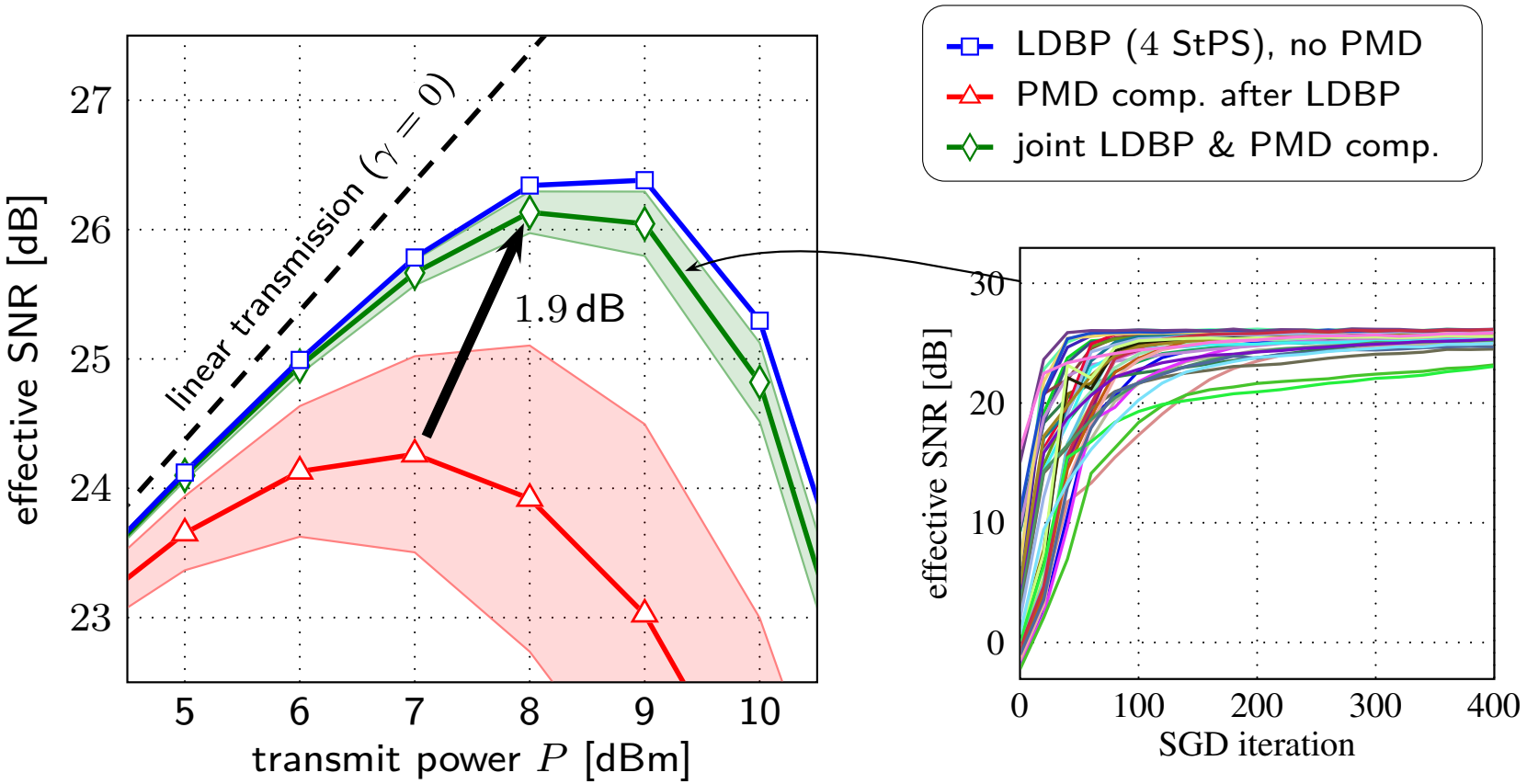

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- Reliable convergence "from scratch" + only 9 real parameters per step

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Outline

The Bigger Picture

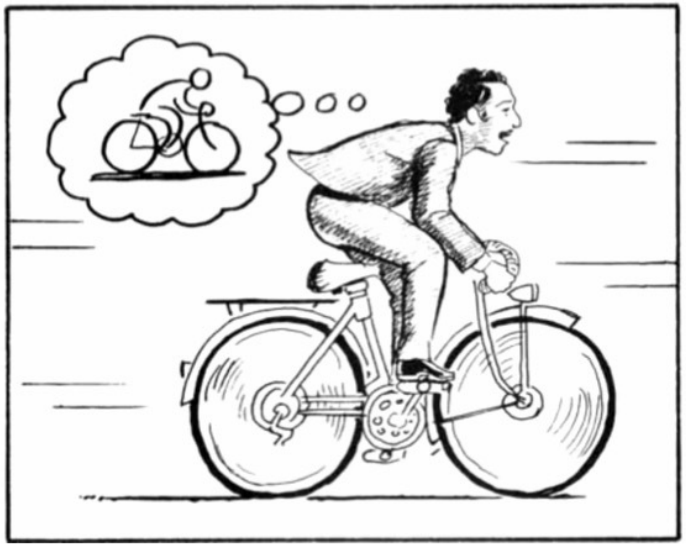
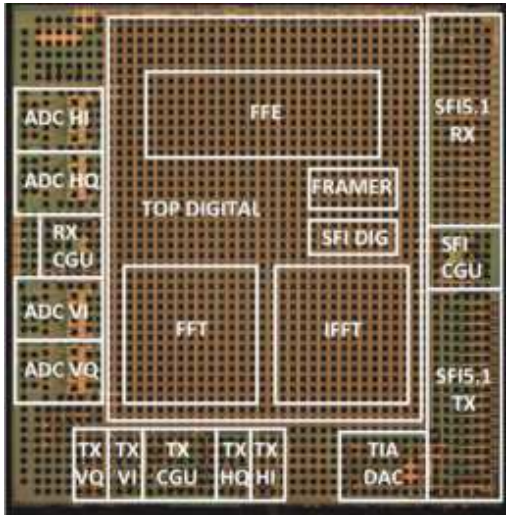


Figure 1. A World Model, from Scott McCloud's *Understanding Comics.* (McCloud, 1993; E, 2012)



[Crivelli et al., 2014]

- Optical receivers build models of their "environment"

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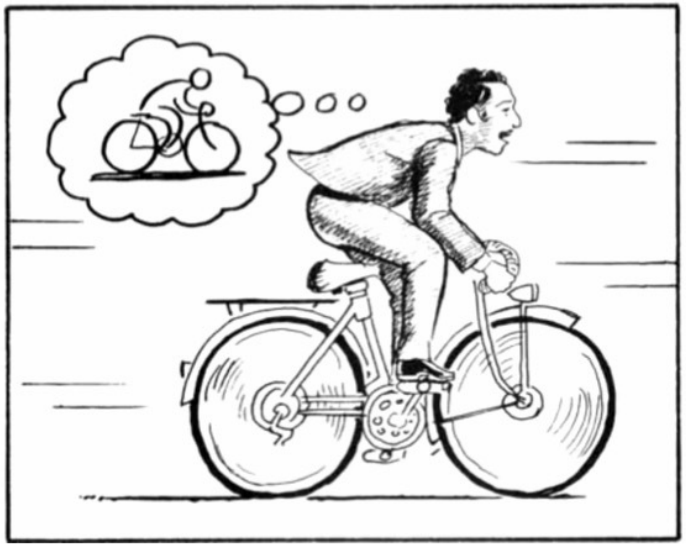

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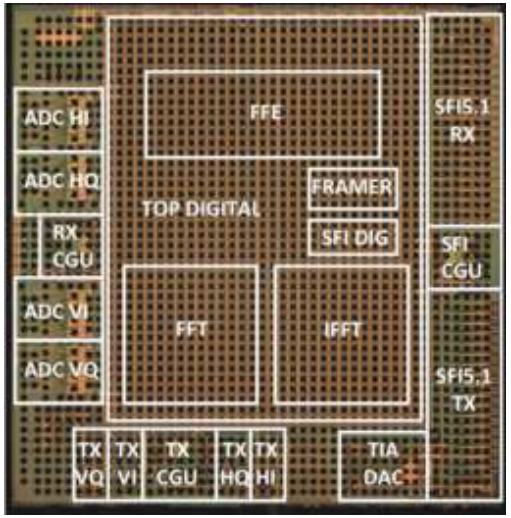

[Crivelli et al., 2014]

- Optical receivers build models of their "environment"
- Currently these models are linear and/or rigid (non-adaptive)
- Interpretable physics-based "multi-layer" models for machine learning can be obtained by exploiting our existing domain knowledge

[Ha & Schmidhuber, 2018], "World Models", arXiv:1803.10122 [cs.LG]

Conclusions

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neural-network-based ML

---

universal function approximators

good designs require
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[Häger & Pfister, 2020], "Physics-Based Deep Learning for Fiber-Optic Communication Systems", in *IEEE J. Sel. Areas Commun.* (to appear), see `https://arxiv.org/abs/2010.14258`

Code: `https://github.com/chaeger/LDBP`

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Thank you!

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