Revisiting Multi-Step Nonlinearity Compensation with Machine Learning

Christian Häger⁽¹⁾

 $\begin{array}{l} \mbox{Joint work with: Henry D. Pfister^{(2)}, Rick M. Bütler^{(3)}, \\ \mbox{Gabriele Liga}^{(3)}, Alex Alvarado^{(3)}, Christoffer Fougstedt^{(4)}, \\ \mbox{Lars Svensson}^{(4)}, \mbox{and Per Larsson-Edefors}^{(4)} \end{array}$

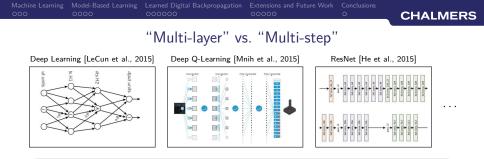
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September 2, 2018

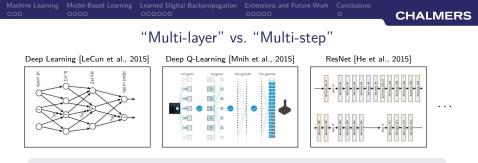


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Multi-layer neural networks: impressive performance, countless applications



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Conventional wisdom: Steps are inefficient \implies reduce as much as possible

- "with only four steps for the entire link" [Du and Lowery, 2010]
- "up to 80% reduction in required [...] steps" [Rafique et al., 2011]
- "it reduces 85% back-propagation stages [...]" [Yan et al., 2011]
- "considerably reduces the number of spans needed " [Napoli et al., 2014]
- "single-step digital backpropagation" [Secondini et al., 2016]



"Multi-layer" vs. "Multi-step"



1. show that multi-layer neural networks and the split-step method have the same functional form: both alternate linear and pointwise nonlinear steps



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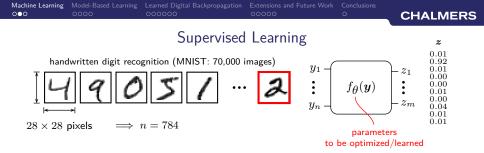
Outline

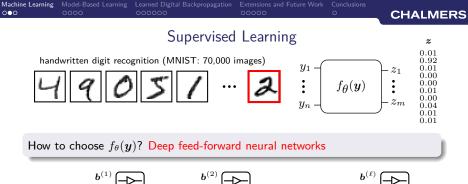
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- 2. Model-Based Machine Learning for Fiber-Optic Communications
- 3. Learned Digital Backpropagation
- 4. Extensions and Future Work
- 5. Conclusions

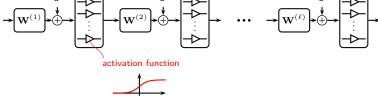
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$$(Marked Learning) (Marked Le$$





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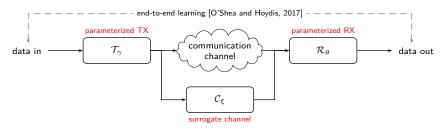
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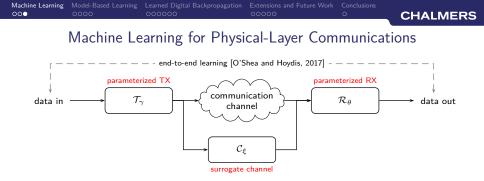
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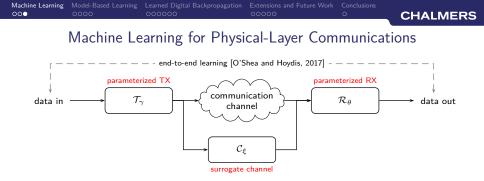
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[O'Shea et al., 2018], Approximating the void: Learning stochastic channel models from observation with variational GANs, (arXiv) [Ye et al., 2018], Channel agnostic end-to-end learning based communication systems with conditional GAN, (arXiv)



Using neural networks for $\mathcal{T}_{\gamma}, \mathcal{R}_{\theta}, \mathcal{C}_{\xi}$

- How to choose network architecture (#layers, activation function)?
- How to initialize parameters?
- How to interpret solutions? Any insight gained?
- . . .



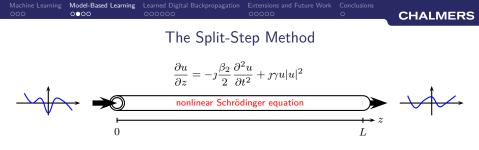
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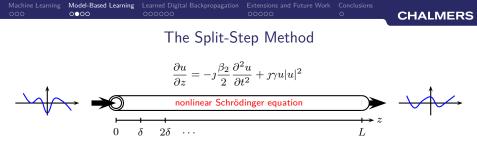
Model-based alternatives: sparse signal recovery [Gregor and Lecun, 2010], [Borgerding and Schniter, 2016], channel coding [Nachmani et al., 2016], ...

Outline

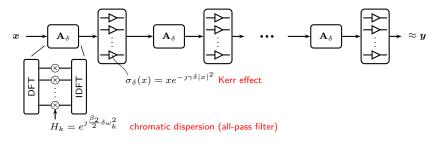
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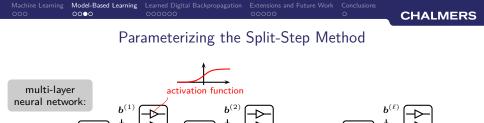


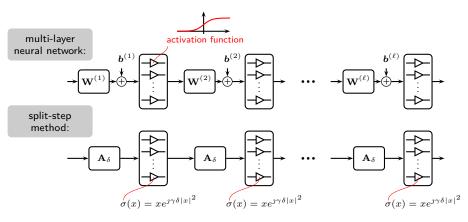
• Deterministic channel model: partial differential equation

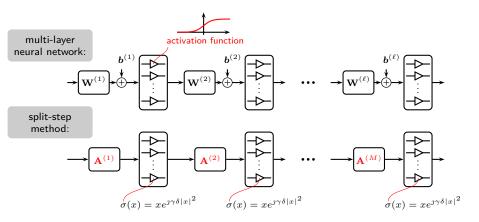


- Deterministic channel model: partial differential equation
- Split-step method with M steps ($\delta = L/M$):

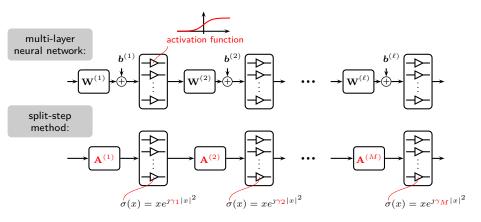






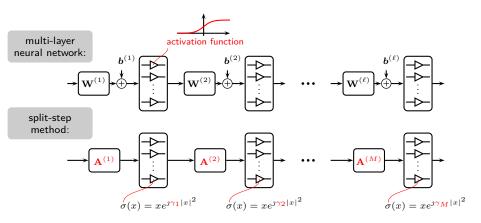


[[]Häger & Pfister, 2018], Nonlinear Interference Mitigation via Deep Neural Networks, (OFC) [Häger & Pfister, 2018], Deep Learning of the Nonlinear Schrödinger Equation in Fiber-Optic Communications, (ISIT)



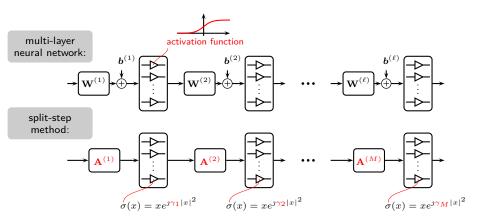
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Parameterizing the Split-Step Method



• Parameterized model f_{θ} with $\theta = \{\mathbf{A}^{(1)}, \dots, \mathbf{A}^{(M)}, \gamma_1, \dots, \gamma_M\}$

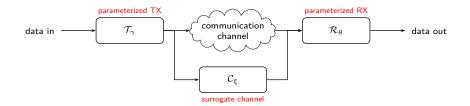
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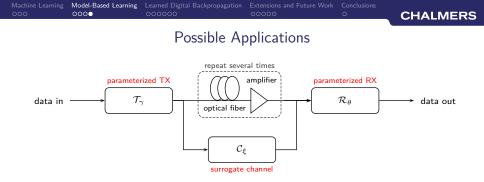


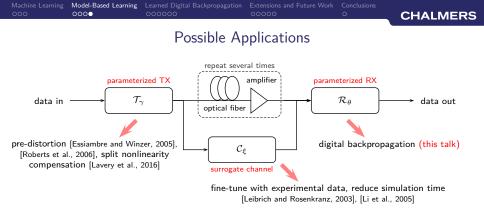
- Parameterized model f_{θ} with $\theta = \{\mathbf{A}^{(1)}, \dots, \mathbf{A}^{(M)}, \gamma_1, \dots, \gamma_M\}$
- Includes as special cases: step-size optimization, "placement" of nonlinear operator, higher-order dispersion, matched filtering ...

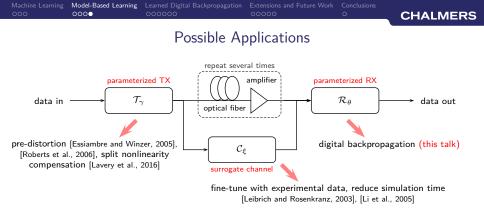


Possible Applications









Model-based learning approaches

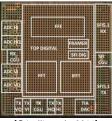
- How to choose network architecture (#layers, activation function)? \checkmark
- How to initialize parameters? ✓
- How to interpret solutions? Any insight gained? \checkmark

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Real-Time Digital Backpropagation



[Crivelli et al., 2014]

 Invert a partial differential equation in real time ([Paré et al., 1996], [Essiambre and Winzer, 2005], [Roberts et al., 2006], [Li et al., 2008], [Ip and Kahn, 2008])



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Our approach

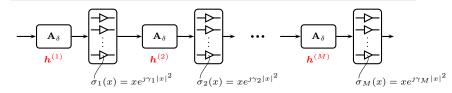
Joint optimization, pruning, and quantization of all chromatic-dispersion filters leads to efficient digital backpropagation, even with many steps.

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Learned Digital Backpropagation

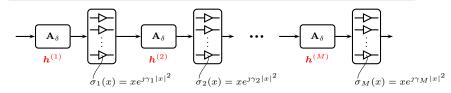
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TensorFlow implementation of the computation graph $f_{\theta}(\boldsymbol{y})$:



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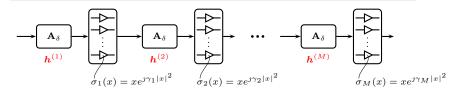
Deep learning of parameters
$$heta = \{oldsymbol{h}^{(1)}, \dots, oldsymbol{h}^{(M)}\}$$
:

$$\min_{\boldsymbol{\theta}} \sum_{i=1}^{N} \mathsf{Loss}(f_{\boldsymbol{\theta}}(\boldsymbol{y}^{(i)}), \boldsymbol{x}^{(i)}) \triangleq g(\boldsymbol{\theta})$$
mean squared error

 $\begin{array}{ll} \text{using} \quad \theta_{k+1} = \theta_k - \lambda \nabla_\theta g(\theta_k) \\ \text{Adam optimizer, fixed learning rate} \end{array}$

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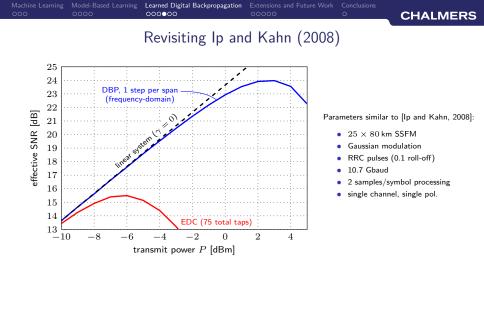


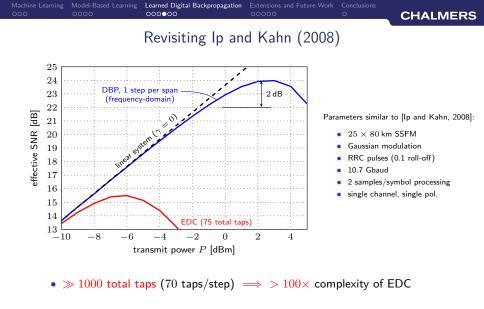
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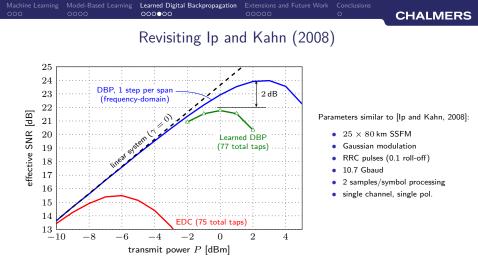
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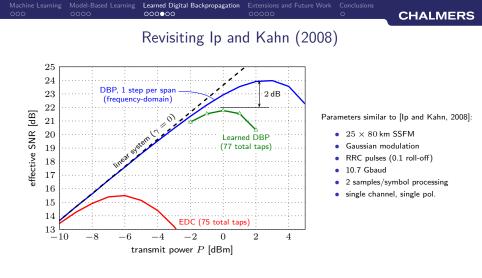
Iteratively prune (set to 0) outermost filter taps during gradient descent







- $\gg 1000$ total taps (70 taps/step) $\implies > 100 \times$ complexity of EDC
- Learned approach uses only 77 total taps: alternate 5 and 3 taps/step and use different filter coefficients in all steps [Häger and Pfister, 2018a]

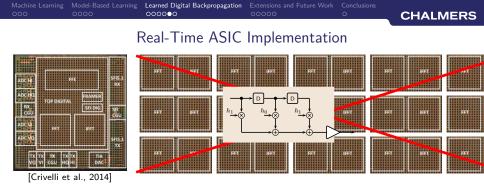


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- Learned approach uses only 77 total taps: alternate 5 and 3 taps/step and use different filter coefficients in all steps [Häger and Pfister, 2018a]
- Can outperform "ideal DBP" in the nonlinear regime [Häger and Pfister, 2018b]





[[]Fougstedt et al., 2017], Time-domain digital back propagation: Algorithm and finite-precision implementation aspects, (OFC) [Fougstedt et al., 2018], ASIC implementation of time-domain digital back propagation for coherent receivers, (PTL) [Sherborne et al., 2018], On the impact of fixed point hardware for optical fiber nonlinearity compensation algorithms, (JLT)



• Our linear steps are very short symmetric FIR filters (as few as 3 taps)



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- 28-nm ASIC at 416.7 MHz clock speed (40 GHz signal)
 - Only 5-6 bit filter coefficients via learned quantization
 - Hardware-friendly nonlinear steps (Taylor expansion)
 - All FIR filters are fully reconfigurable

[[]Fougstedt et al., 2018], ASIC implementation of time-domain digital backpropagation with deep-learned chromatic dispersion filters, (ECOC)

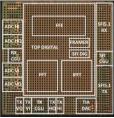


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Real-Time ASIC Implementation



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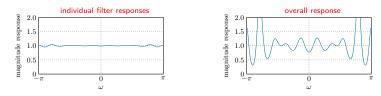
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 - Only 5-6 bit filter coefficients via learned quantization
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 - All FIR filters are fully reconfigurable
- $< 2 \times$ power compared to EDC [Crivelli et al., 2014, Pillai et al., 2014]

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Why Does The Learning Approach Work?

Previous work: design a single filter or filter pair and use it repeatedly. \implies Good overall response only possible with very long filters.

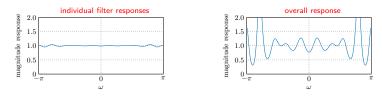


From [Ip and Kahn, 2009]:

- "We also note that [...] 70 taps, is much larger than expected"
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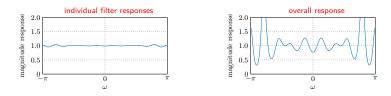
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The learning approach uncovered that there is no such requirement! [Lian, Häger, Pfister, 2018]. What can machine learning teach us about communications? (*ITW*)

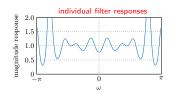
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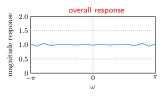
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Sacrifice individual filter accuracy, but different response per step.

 \Rightarrow Good overall response even with very short filters by joint optimization.



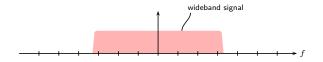


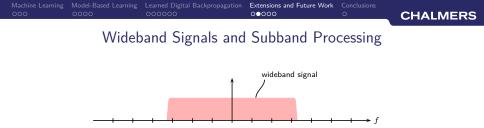
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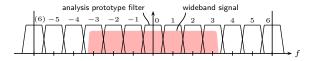
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Wideband Signals and Subband Processing





• Quadratic dependence of overall system memory on the backpropagated bandwidth \implies problematic for time-domain DBP (#taps $\propto f^2$)



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- Subband processing: split received signal into N parallel signals

[Nazarathy and Tolmachev, 2014], Subbanded DSP architectures based on underdecimated filter banks ..., (Signal Proc. Mag.)

[[]Taylor, 2008], Compact digital dispersion compensation algorithms, (OFC)

[[]Ho, 2009], Subband equaliser for chromatic dispersion of optical fibre, (Electronics Lett.)

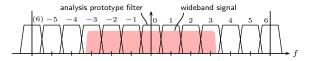
[[]Slim et al., 2013], Delayed single-tap frequency-domain chromatic-dispersion compensation, (PTL)

[[]Mateo et al., 2010], Efficient compensation of inter-channel nonlinear effects via digital backward ..., (Opt. Express)

[[]Ip et al., 2011], Complexity versus performance tradeoff for fiber nonlinearity compensation ... (OFC)

[[]Oyama et al., 2015], Complexity reduction of perturbation-based nonlinear compensator by sub-band processing, (OFC)

Wideband Signals and Subband Processing



- Quadratic dependence of overall system memory on the backpropagated bandwidth \implies problematic for time-domain DBP (#taps $\propto f^2$)
- Subband processing: split received signal into N parallel signals
- MIMO filter accounts for cross-phase modulation (XPM) between subbands [Leibrich and Rosenkranz, 2003]

[Ho, 2009], Subband equaliser for chromatic dispersion of optical fibre, (Electronics Lett.)

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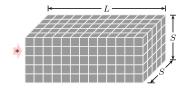
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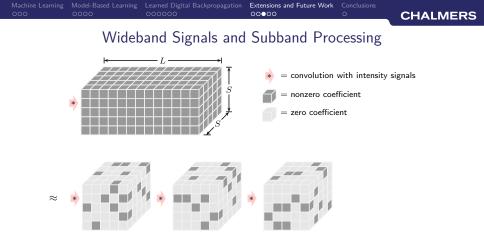
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Wideband Signals and Subband Processing



- = convolution with intensity signals
- nonzero coefficient
 - = zero coefficient

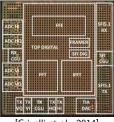


- L₁-norm regularization applied to filter coefficients during gradient descent
- \implies 92% of coefficients are zero with little performance penality

[[]Häger and Pfister, 2018], Wideband time-domain digital backpropagation via subband processing and deep learning, (ECOC)



Polarization Mode Dispersion



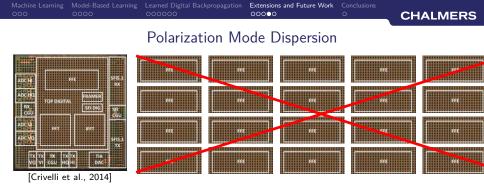
[Crivelli et al., 2014]

- Modeling via PMD sections, $\mathbf{R}^{(i)}\mathbf{J}^{(i)}(\omega)$, in the split-step method:
 - **R**⁽ⁱ⁾: complex unitary rotation matrix with determinant one
 - $\mathbf{J}^{(i)}(\omega)$: first-order PMD matrix with differential group delay (DGD) au_i , i.e.,

$$\mathbf{J}^{(i)}(\omega) = \begin{pmatrix} e^{-\jmath\omega\frac{\tau_i}{2}} & 0\\ 0 & e^{\jmath\omega\frac{\tau_i}{2}} \end{pmatrix}$$

• PMD transfer matrix: $\mathbf{J}(\omega) = \prod_{i=1}^{M} \mathbf{R}^{(i)} \mathbf{J}^{(i)}(\omega)$ for large M

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 - $\mathbf{R}^{(i)}$: complex unitary rotation matrix with determinant one
 - $\mathbf{J}^{(i)}(\omega)$: first-order PMD matrix with differential group delay (DGD) au_i , i.e.,

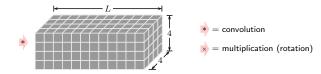
$$\mathbf{J}^{(i)}(\omega) = \begin{pmatrix} e^{-\jmath\omega\frac{\tau_i}{2}} & 0\\ 0 & e^{\jmath\omega\frac{\tau_i}{2}} \end{pmatrix}$$

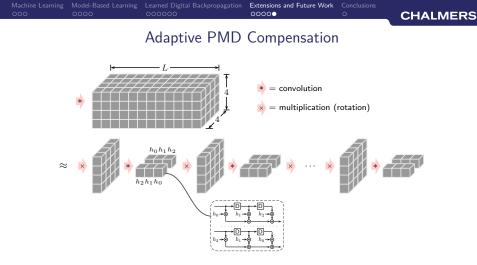
• PMD transfer matrix: $\mathbf{J}(\omega) = \prod_{i=1}^{M} \mathbf{R}^{(i)} \mathbf{J}^{(i)}(\omega)$ for large M

Model-Based Learning	Learned Digital Backpropagation	Extensions and Future Work	
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Adaptive PMD Compensation





 Ongoing work: characterize optimization behavior (saddle points), integrate into digital backpropagation, ...

[Goroshko et al., 2016], Overcoming performance limitations of digital back propagation due to polarization mode dispersion, (CTON) [Czegledi et al., 2017], Digital backpropagation accounting for polarization-mode dispersion, (Opt. Express) [Liga et al., 2018], A PMD-adaptive DBP receiver based on SNR optimization, (OPC)

000 0000 00000 •	Model-Based Learning	Learned Digital Backpropagation	Conclusions
			•



Conclusions



Conclusions

- We have proposed a model-based machine-learning approach for fiber-optic communication systems
- We have revisited efficient multi-step digital backpropagation and shown that deep-learning tools can be used to
 - jointly optimize all linear substeps
 - prune filter taps to get very short filters
 - jointly quantize all filter coefficients
- Multi-step enables factorization into simple, elementary steps

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Conclusions

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Thank you!



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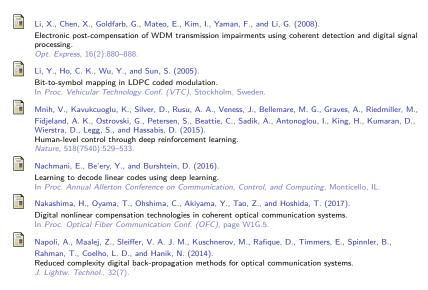
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