Comparison of Terminated and Tailbiting Spatially Coupled LDPC Codes With Optimized Bit Mapping for PM-64-QAM

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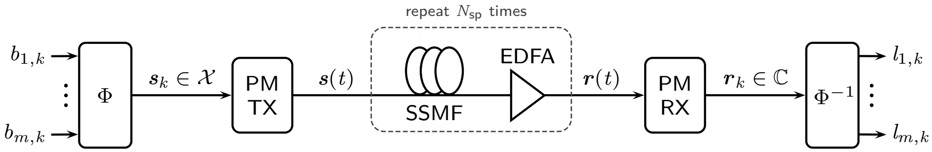
Main Result

Unequal error protection of a nonbinary modulation format can be used to significantly improve performance of tailbiting SC-LDPC codes. Comparable gap to capacity at a lower FEC overhead.

Outline

System Model



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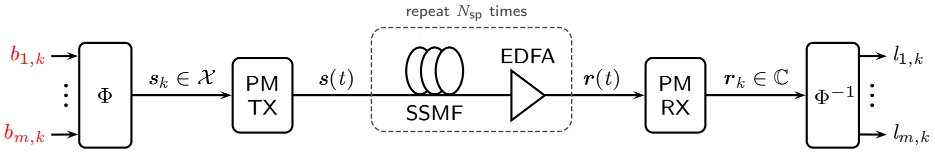

- Coded bits from the FEC encoder

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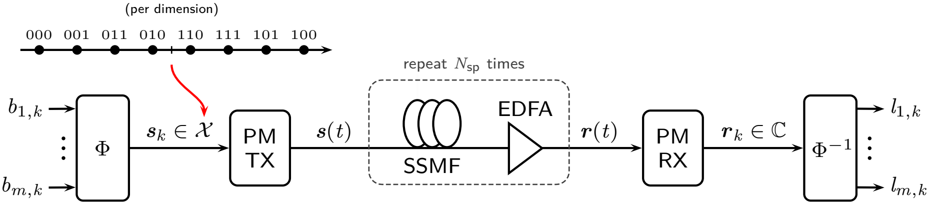

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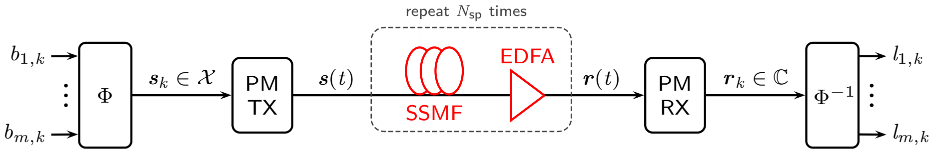

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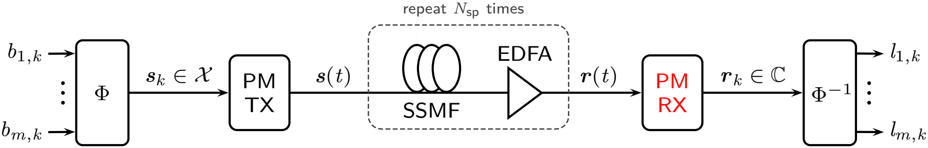

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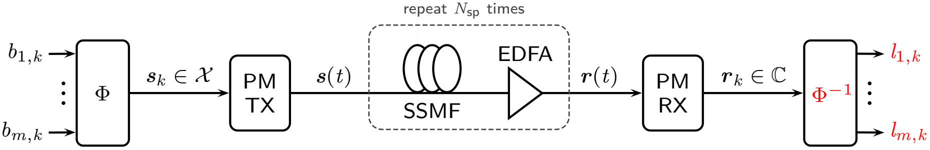

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- Demodulator Φ^{-1}: log-likelihood ratio (LLR) computation

Spatially Coupled LDPC Codes

- Can be represented in terms of a protograph = prototype graph

Example: Terminated Tailbiting

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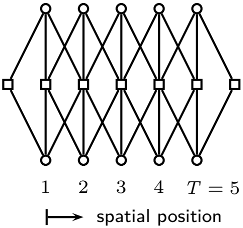
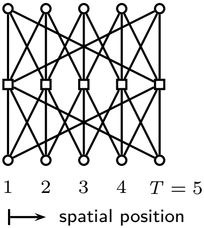
Terminated

Tailbiting

protograph





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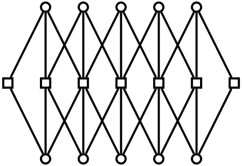

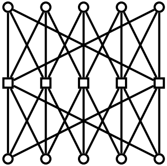

base matrix

$$\begin{pmatrix} 1 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 1 & 1 & 1 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 1 & 1 & 1 & 1 & 1 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 1 & 1 & 1 & 1 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 1 & 1 & 1 & 1 & 1 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 & 1 & 1 & 1 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 1 \end{pmatrix}$$

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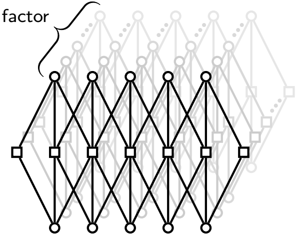

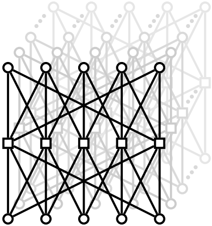

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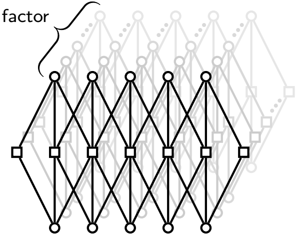

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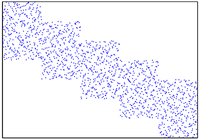
parity-check matrix



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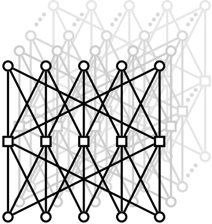

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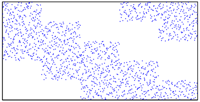

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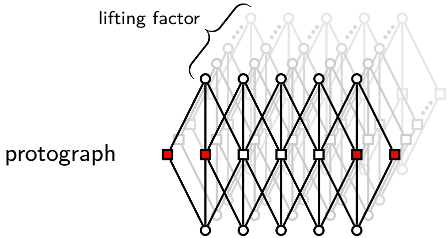

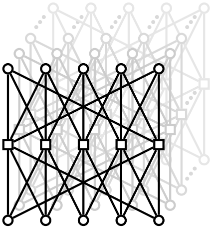

check node degrees — slightly irregular — regular

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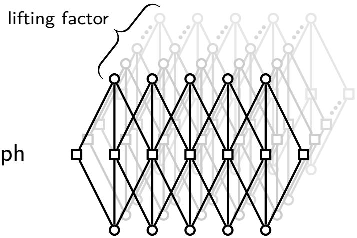
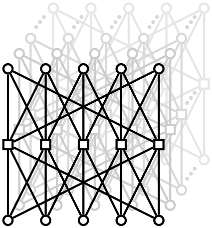
protograph		
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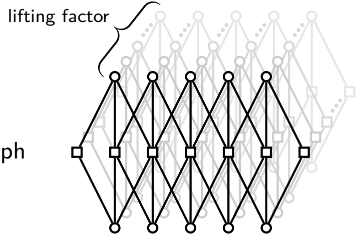
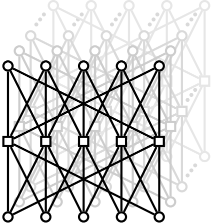
protograph		
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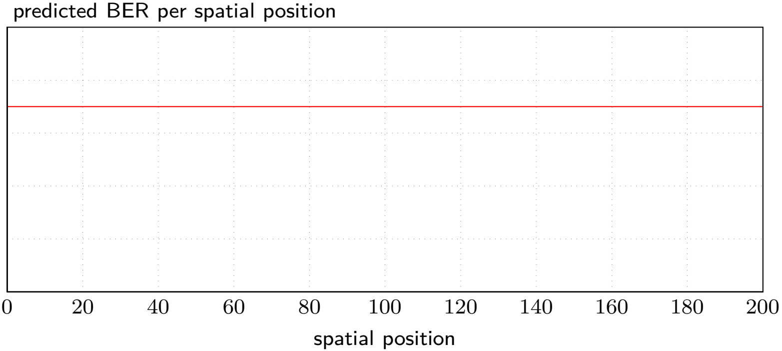
rate	$R(T) = R - R_{\text{loss}}(T)$ $R_{\text{loss}}(T) \to 0$ as $T \to \infty$	R (no rate loss)

Decoding Wave



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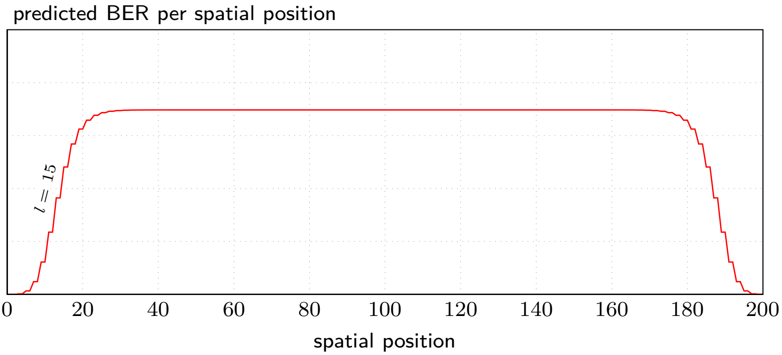

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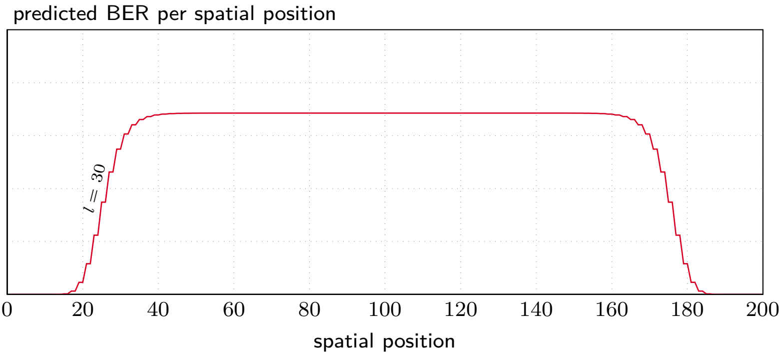

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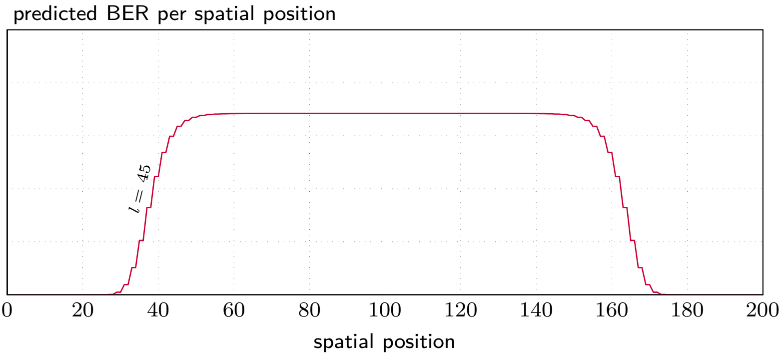

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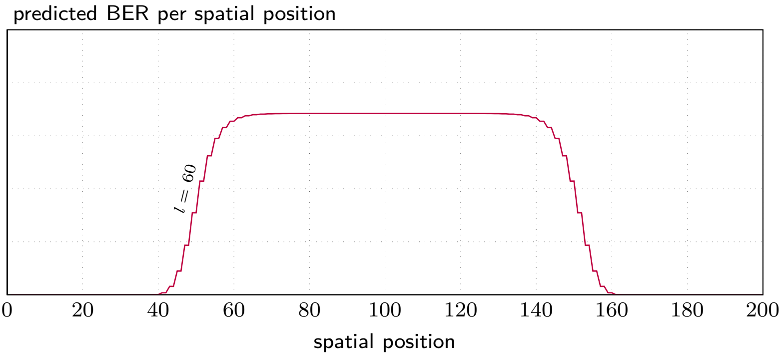

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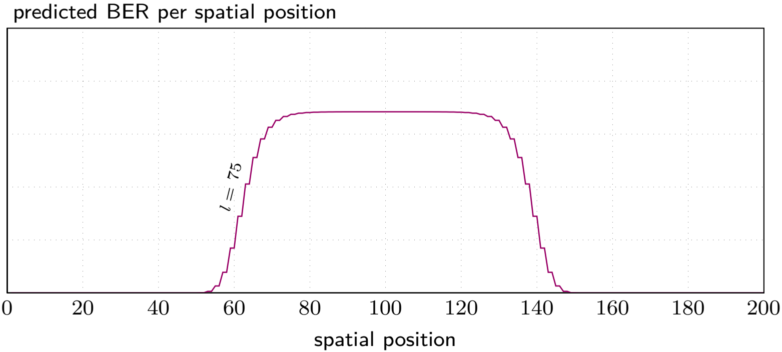

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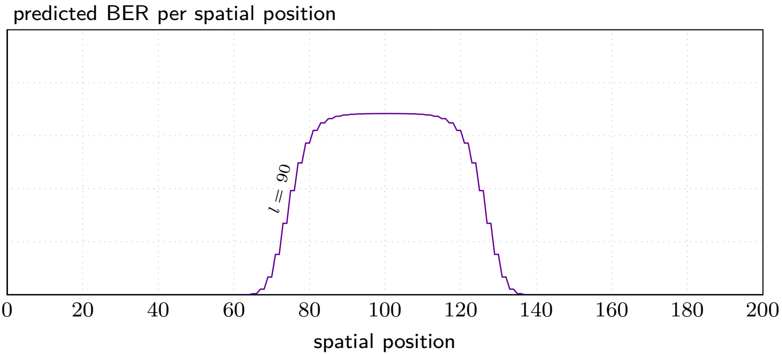

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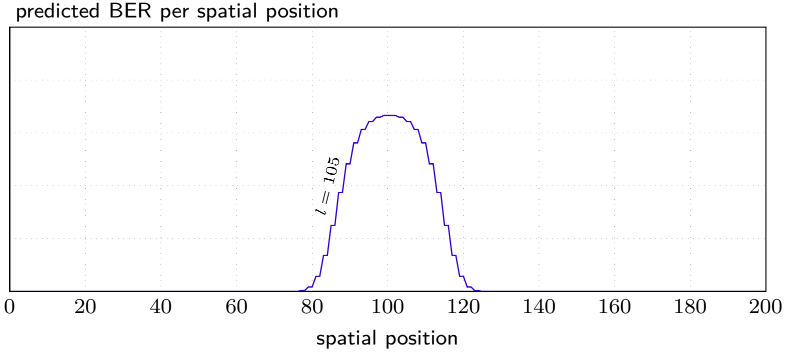

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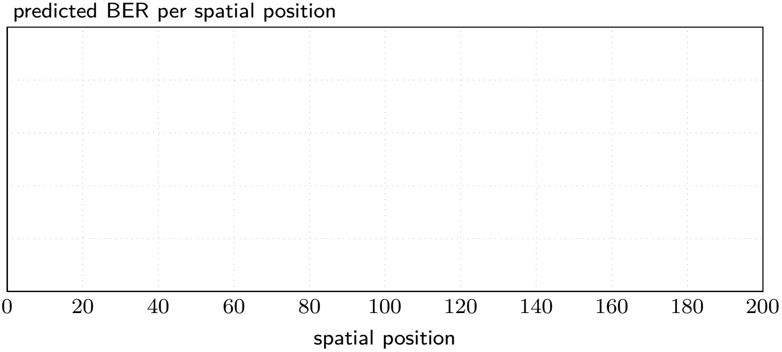
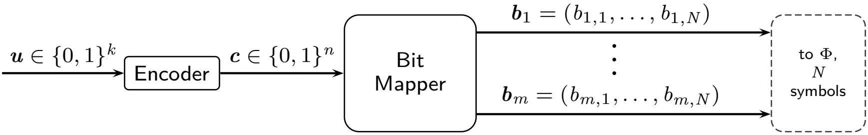


Successful decoding!

Bit Mapper


$\boldsymbol{u} \in \{0,1\}^k$
Encoder
$\boldsymbol{c} \in \{0,1\}^n$
Bit Mapper
$\boldsymbol{b}_1 = (b_{1,1}, \dots, b_{1,N})$
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to Φ,
N
symbols

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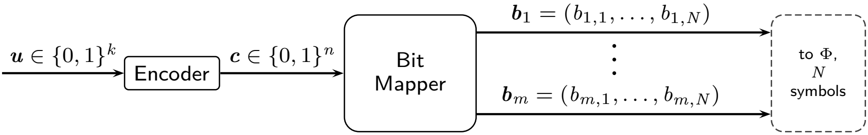

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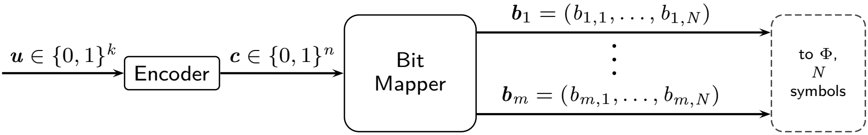

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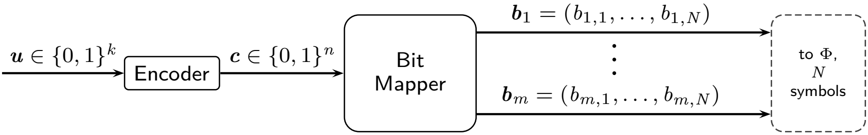

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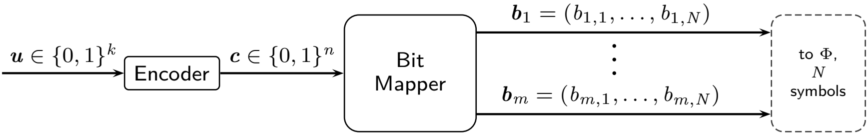

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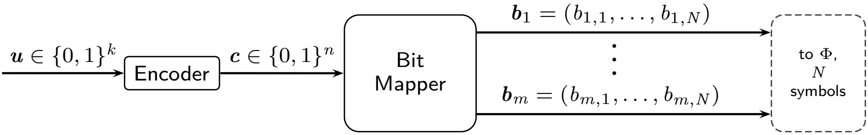

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- $\mathbf{A}$ is optimized based on modified protograph extrinsic transfer function (P-EXIT) analysis (predicts BER for $M \to \infty$)

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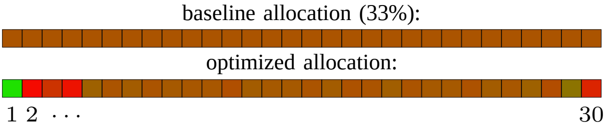

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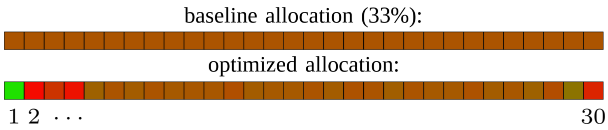
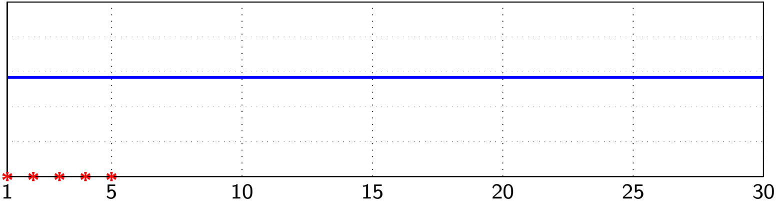


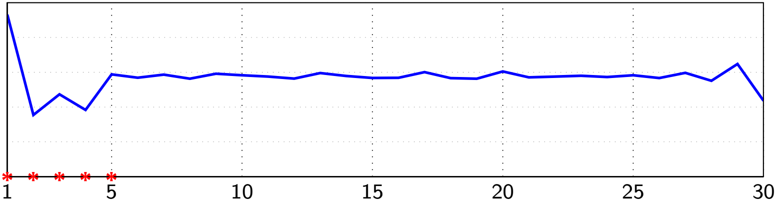
- Locally improved decoding convergence in the first spatial position leads to wave-like decoding behavior, similar to terminated SC-LDPC codes

Decoding Behavior For The Same SNR

predicted BER per spatial position (baseline)



predicted BER per spatial position (optimized)



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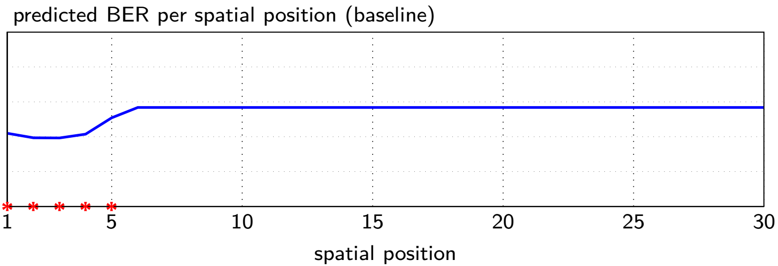

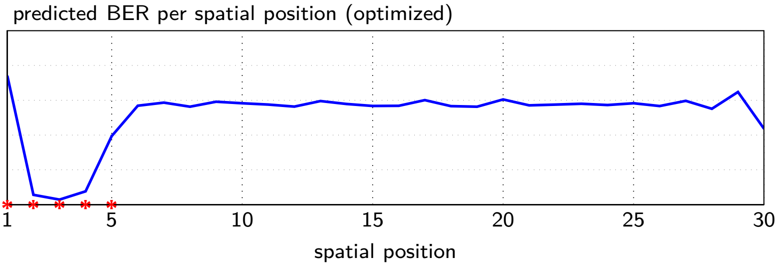

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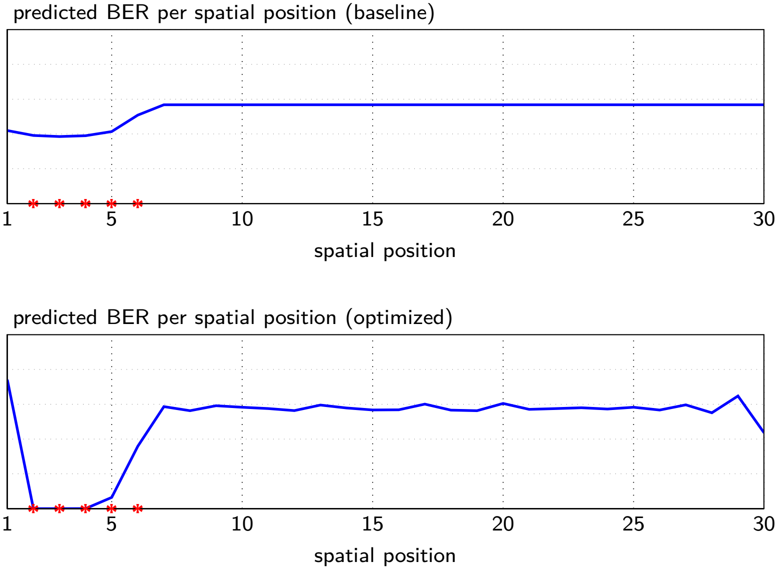

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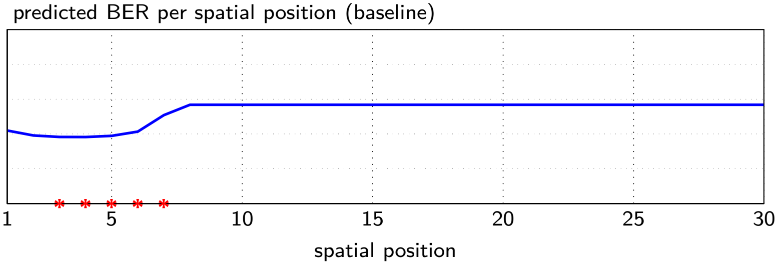
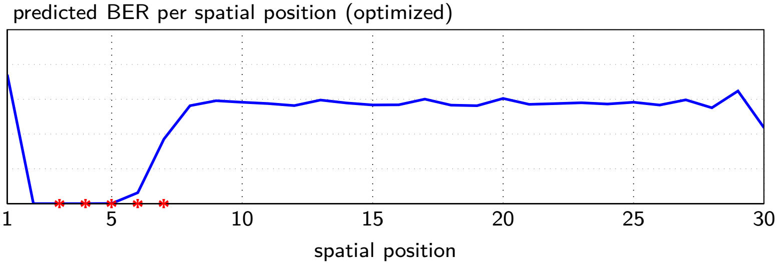
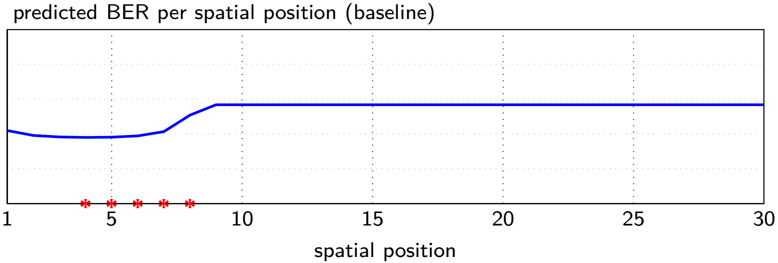
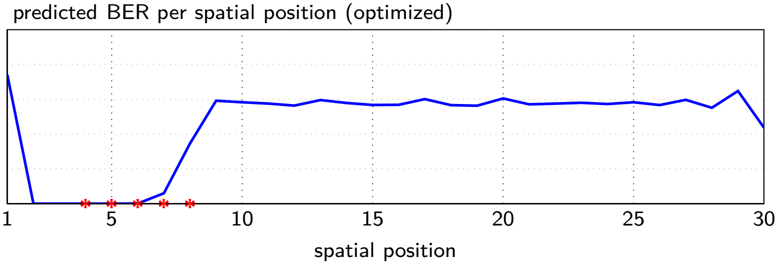




Decoding Behavior For The Same SNR





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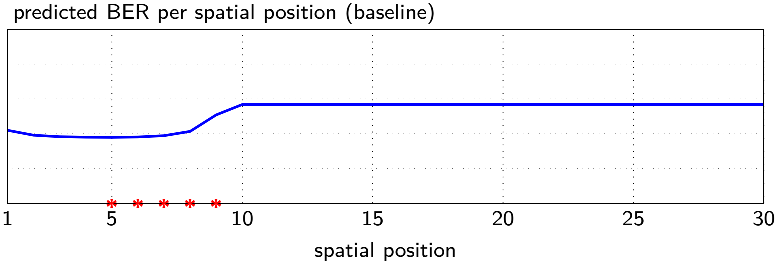

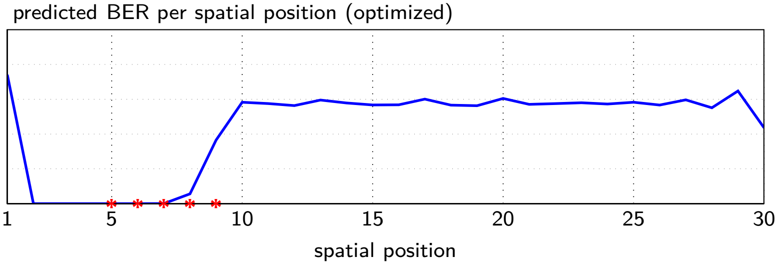

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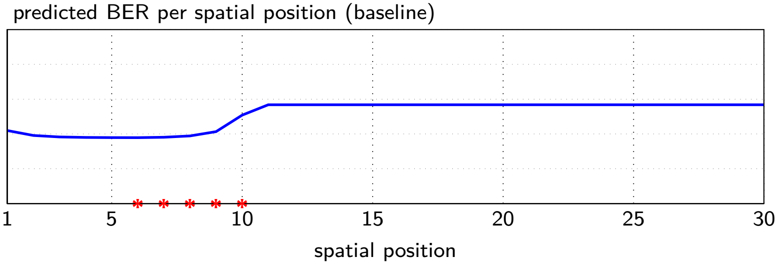

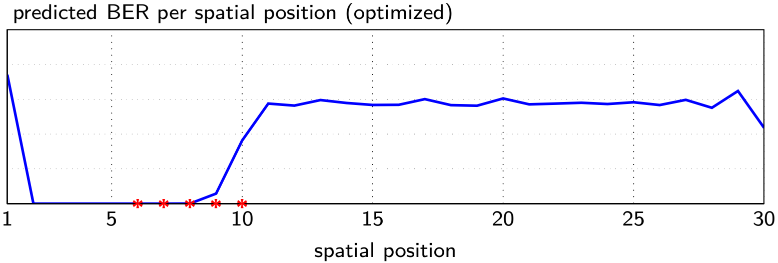

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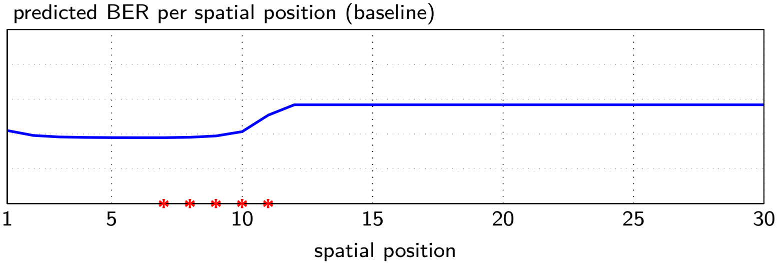

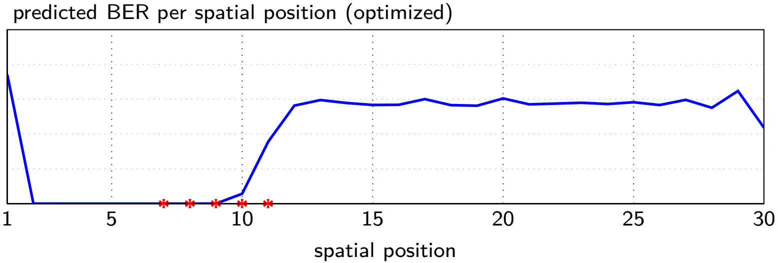

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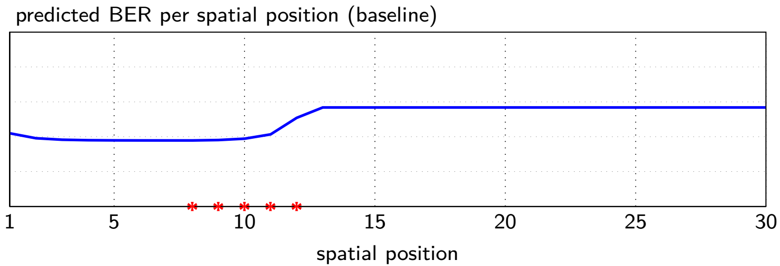

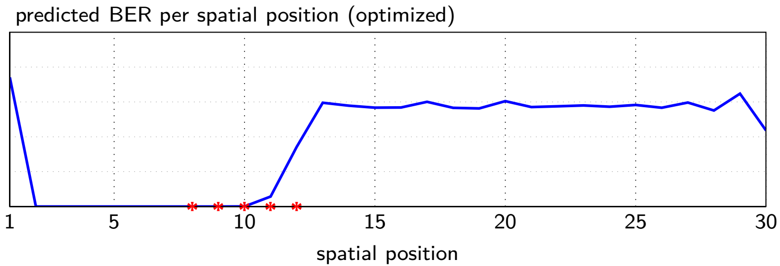

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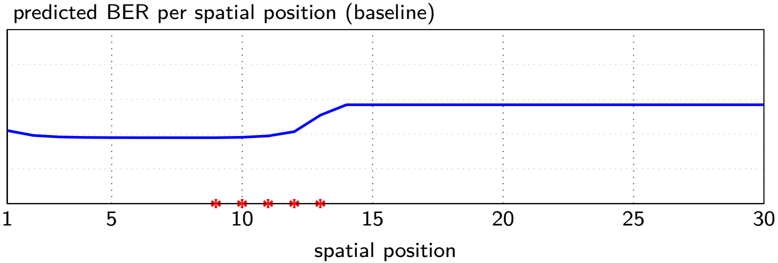
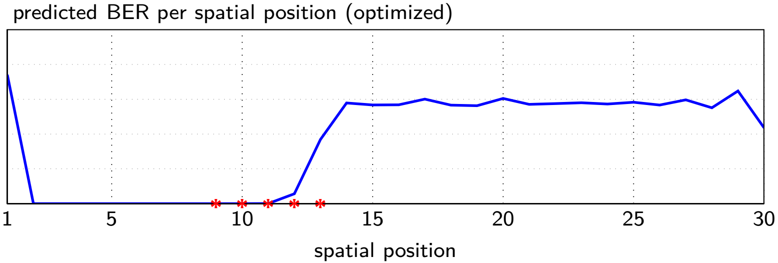
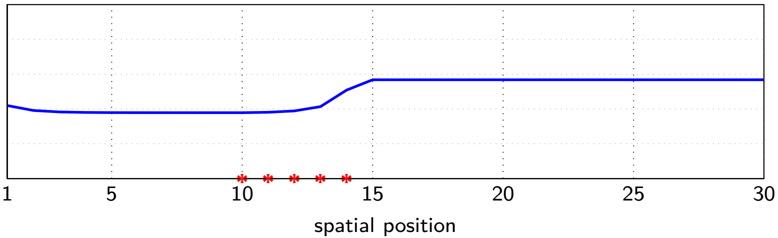


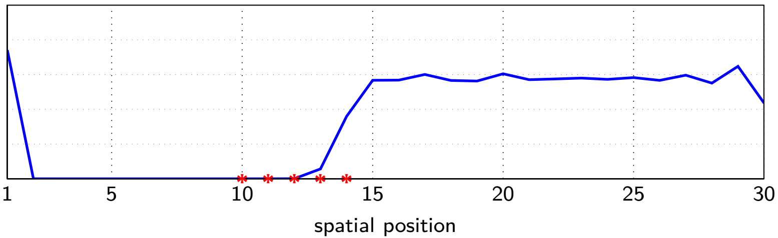


Decoding Behavior For The Same SNR

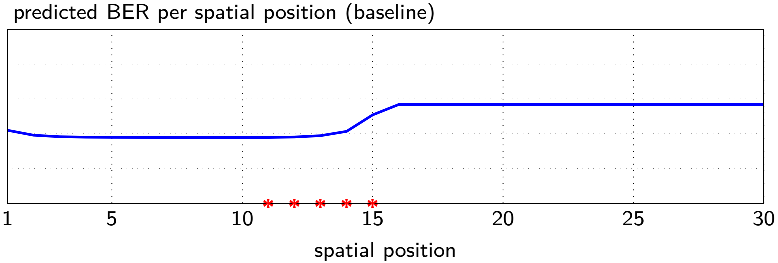
predicted BER per spatial position (baseline)

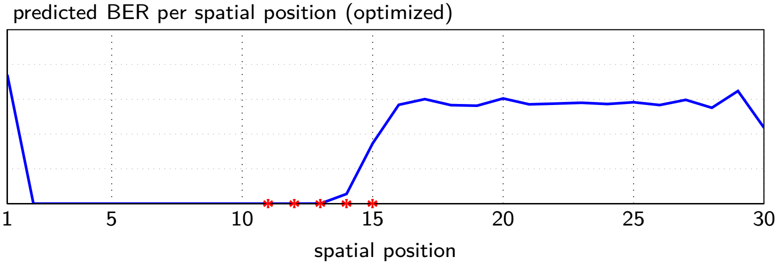


predicted BER per spatial position (optimized)



Decoding Behavior For The Same SNR





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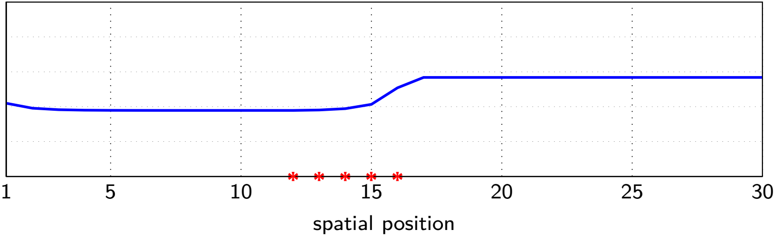

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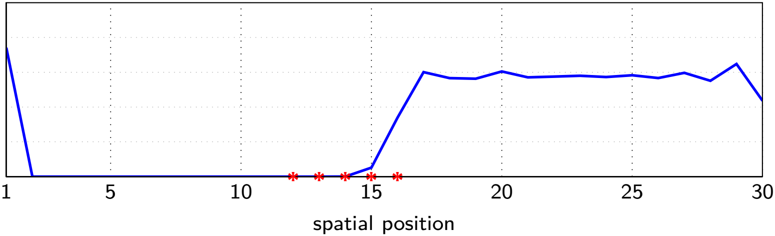
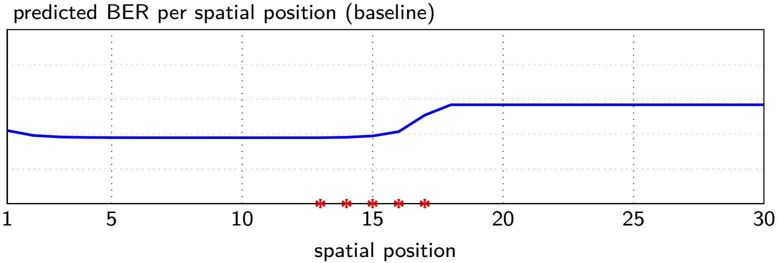
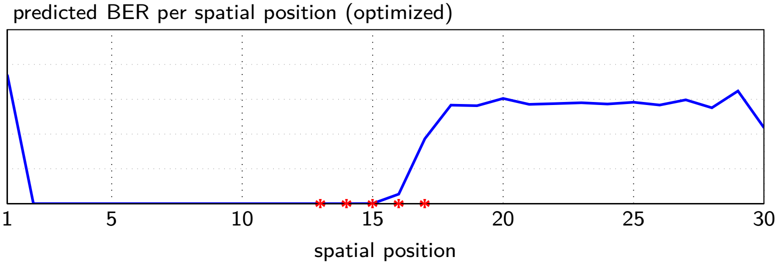


Decoding Behavior For The Same SNR





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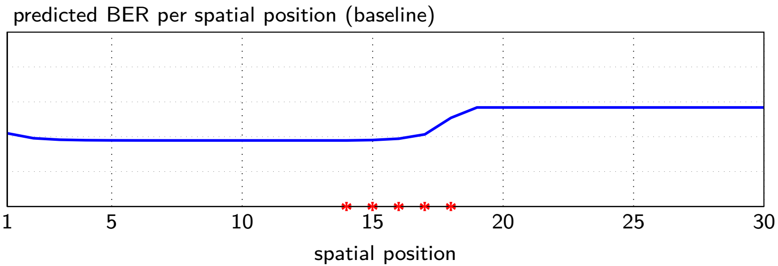

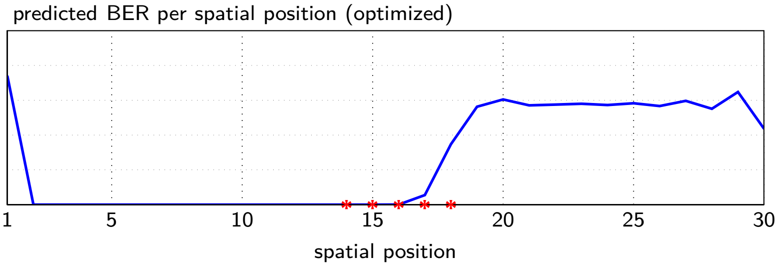

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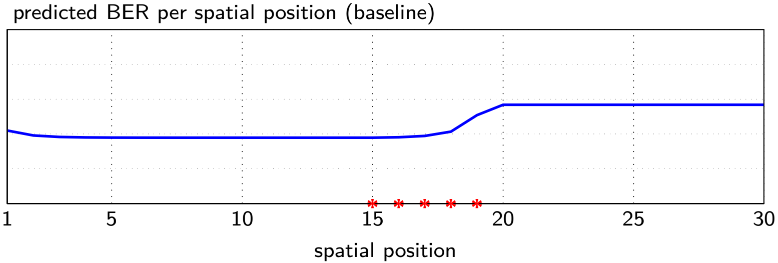
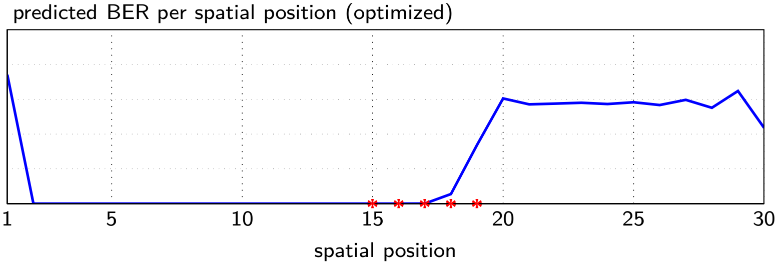
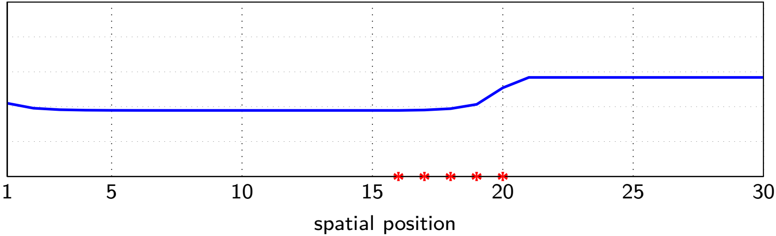


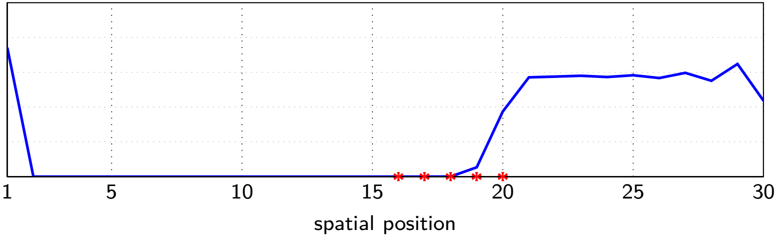


Decoding Behavior For The Same SNR

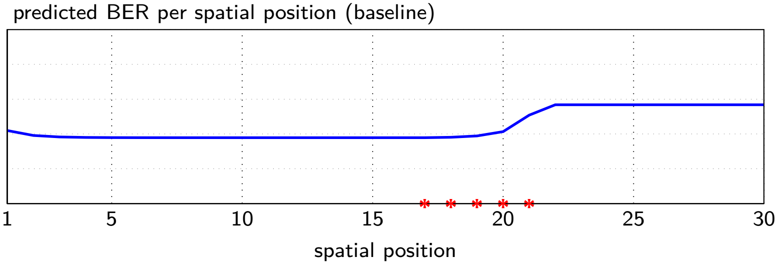
predicted BER per spatial position (baseline)

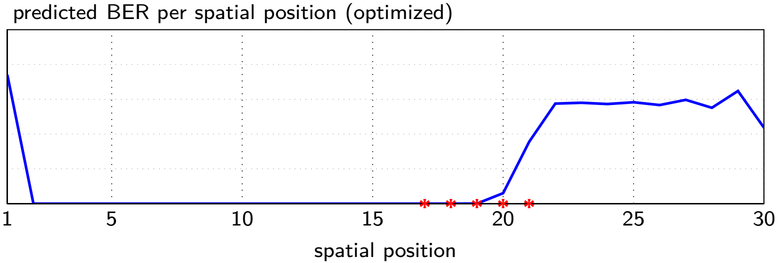


predicted BER per spatial position (optimized)



Decoding Behavior For The Same SNR





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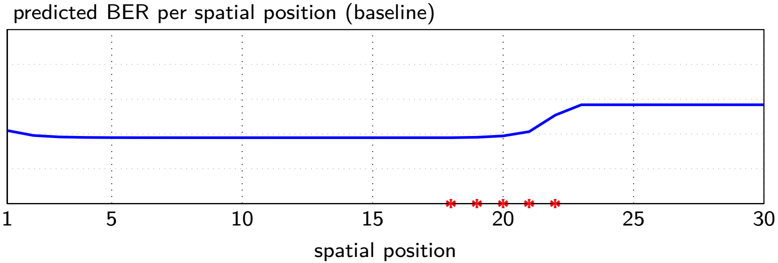

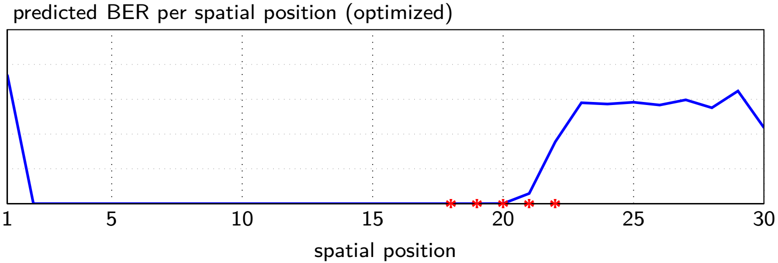

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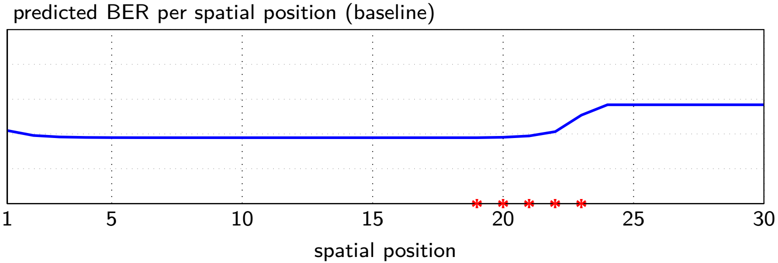

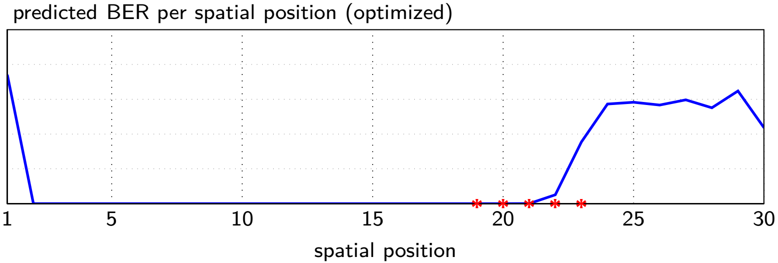

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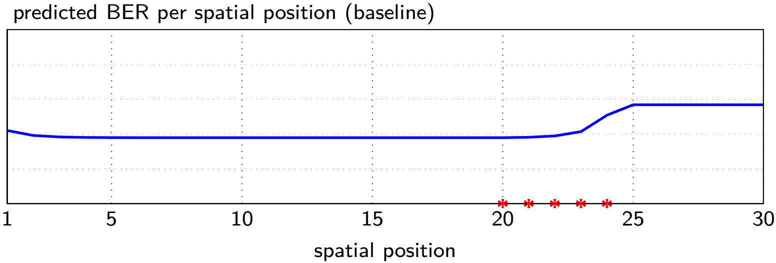

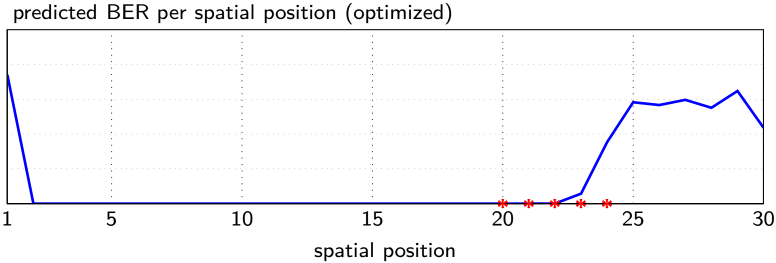

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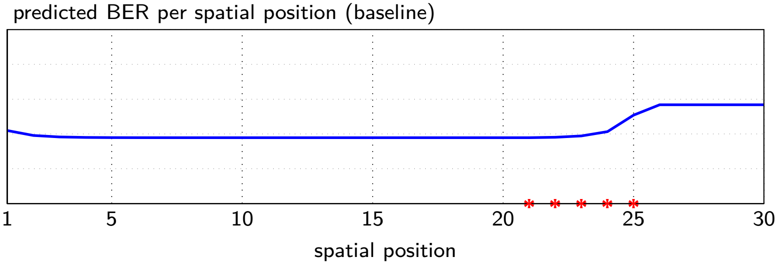

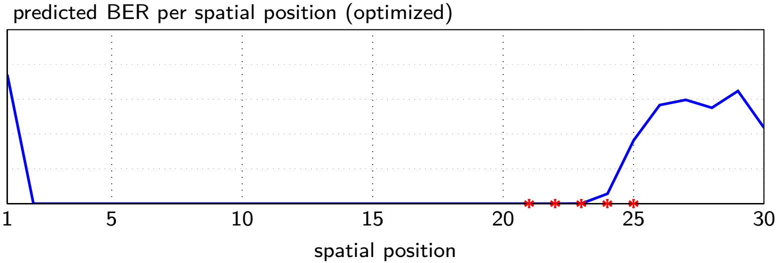

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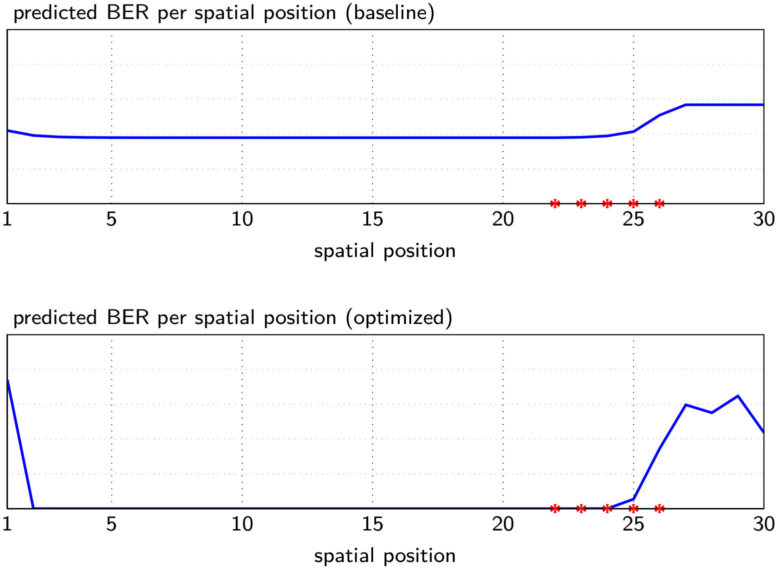

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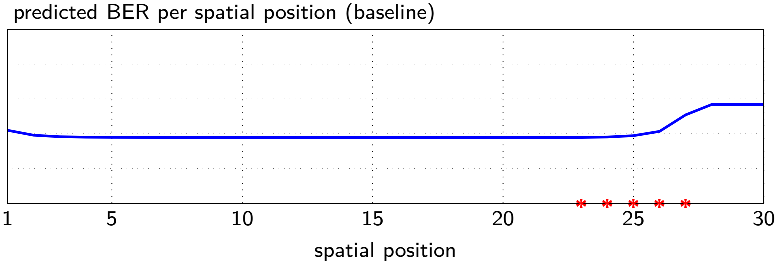

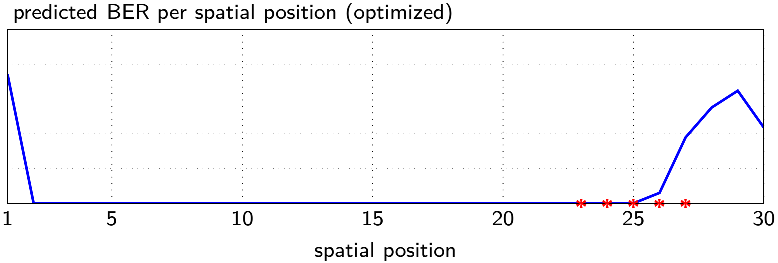

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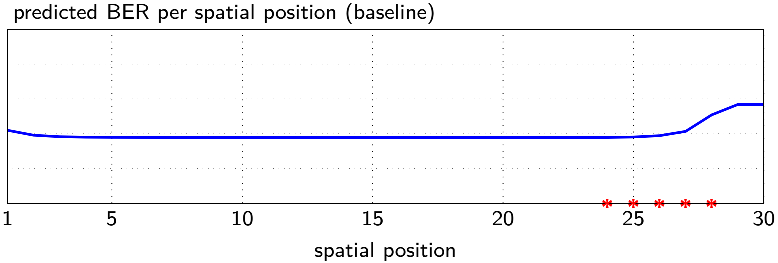

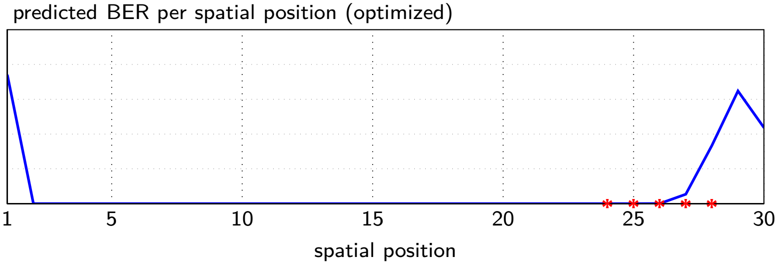

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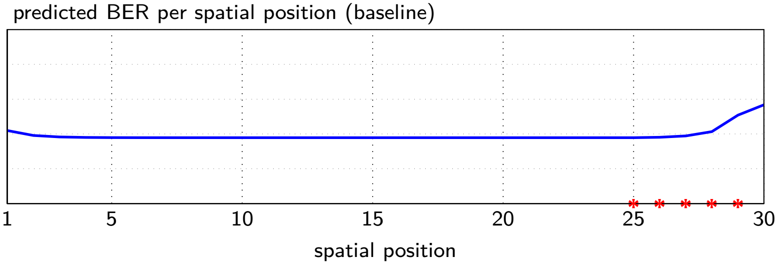

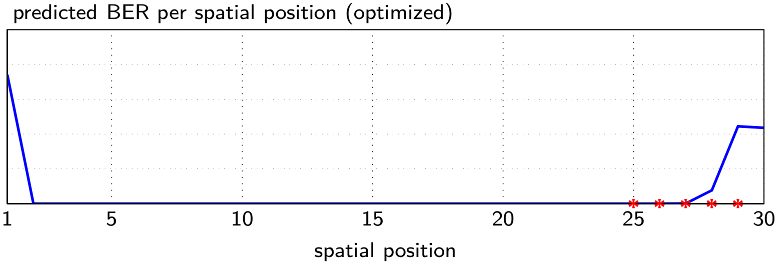

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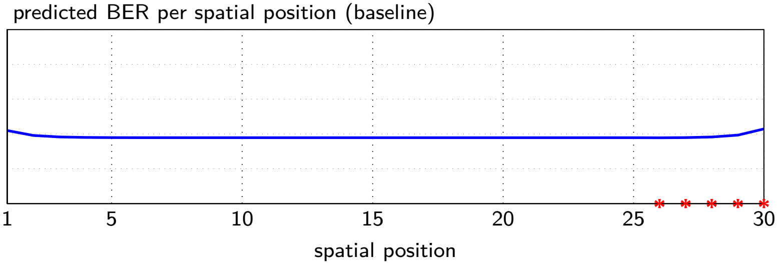
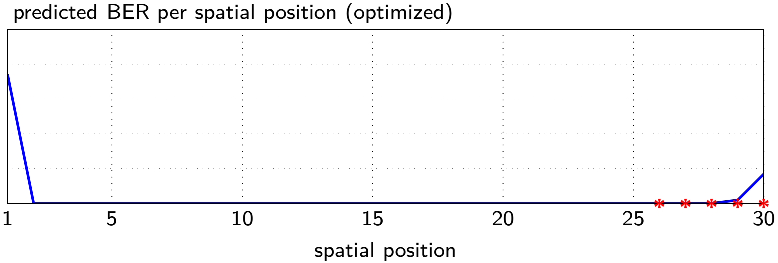
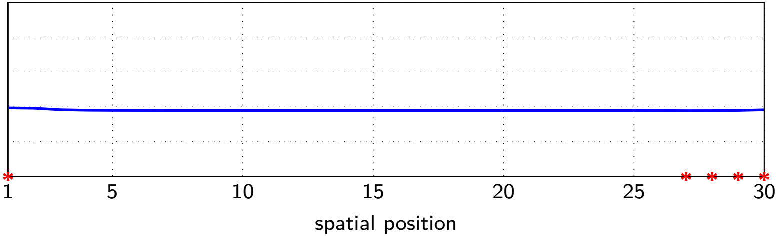


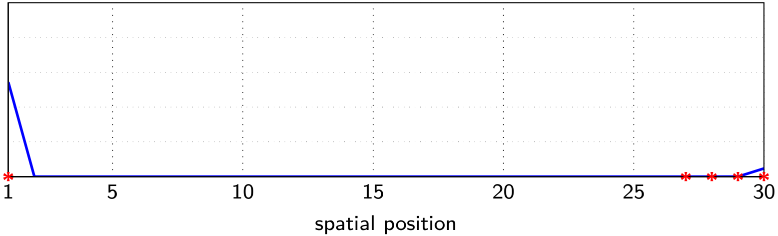


Decoding Behavior For The Same SNR

predicted BER per spatial position (baseline)



predicted BER per spatial position (optimized)



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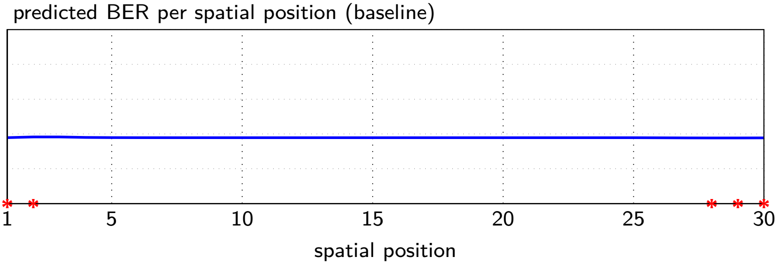

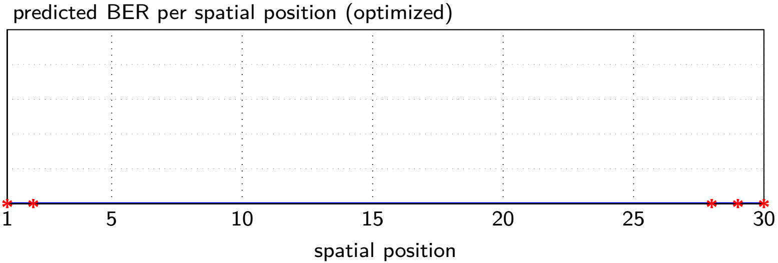

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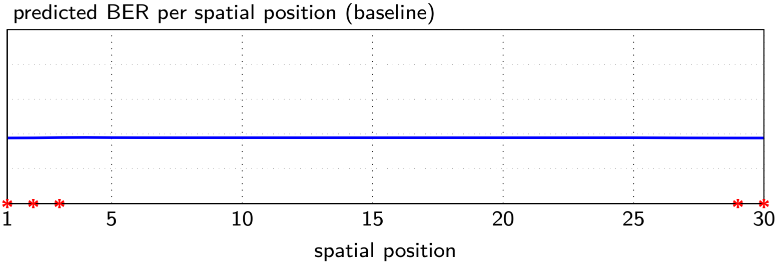

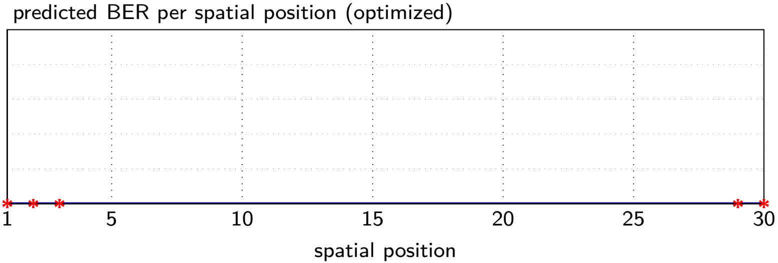

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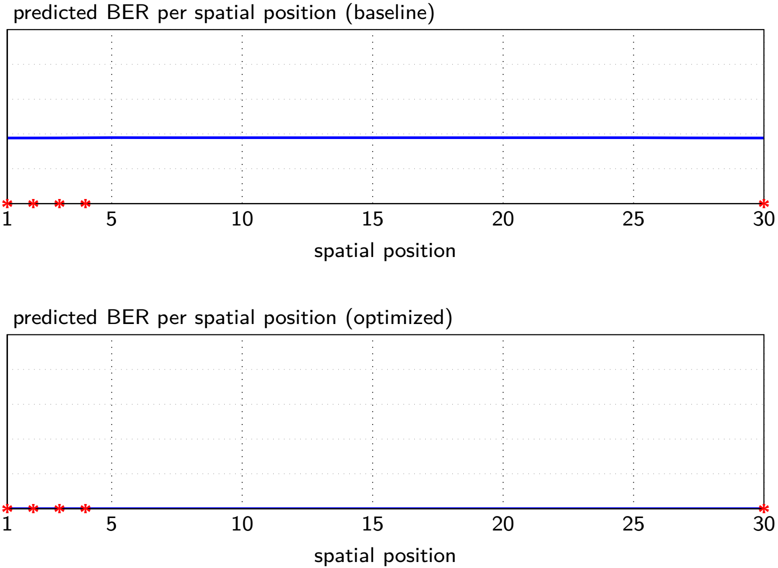

Simulation Results

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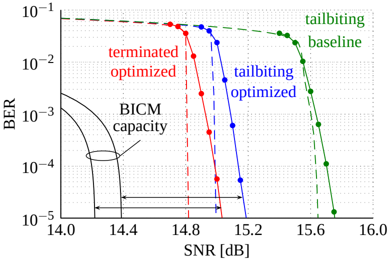

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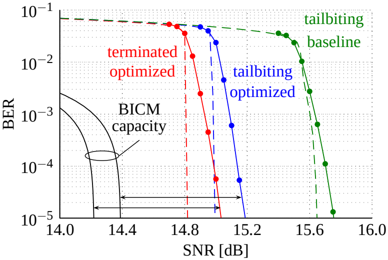

- Gain of ≈ 0.55 dB at a BER of 10^{-5} for AWGN channel

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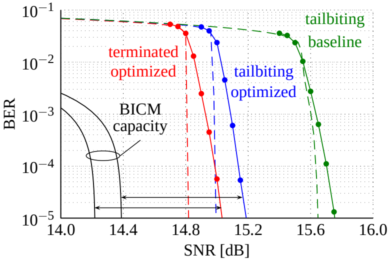

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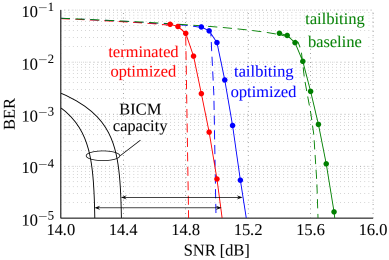

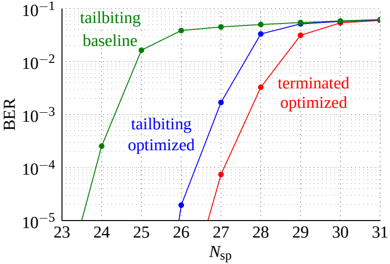

- Gain of ≈ 0.55 dB at a BER of 10^{-5} for AWGN channel
- Approximately the same gap to the BICM capacity for both optimized systems
- Nonlinear propagation at 40 Gbaud with 70 km spans (split-step Fourier simulation)

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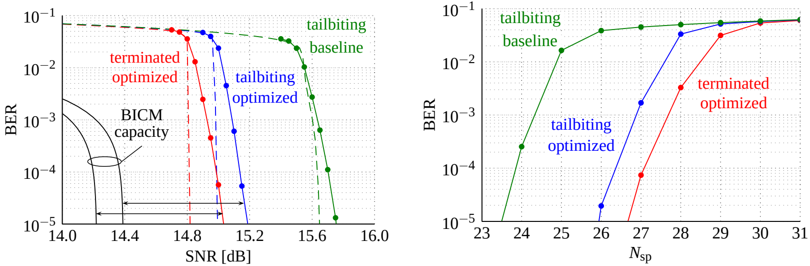

- Gain of ≈ 0.55 dB at a BER of 10^{-5} for AWGN channel
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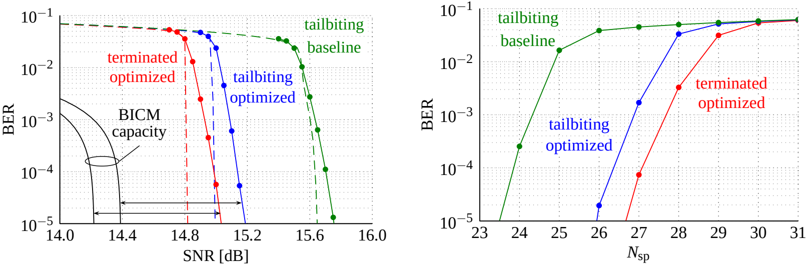

- Gain of ≈ 0.55 dB at a BER of 10^{-5} for AWGN channel
- Approximately the same gap to the BICM capacity for both optimized systems
- Nonlinear propagation at 40 Gbaud with 70 km spans (split-step Fourier simulation)
- 0.55 dB gain translates into ≈ 3 span increase (13%)
- Terminated code enables longer reach, at the expense of 1.2% decrease in spectral efficiency

Conclusions

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2. With optimized bit allocation, terminated and tailbiting codes are competitive, i.e., spectral efficiency can be traded for transmission reach, at similar gap to capacity.

Thank you!

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