

# Constellation Optimization for Coherent Optical Channels Distorted by Nonlinear Phase Noise

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**CHALMERS**

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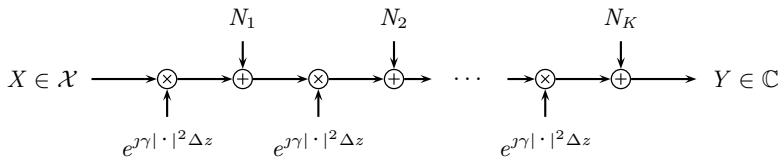
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- **Practical question:** How much can we gain by optimizing the constellation compared to standard QAM?
- **Theoretical question:** How do optimal constellations look like for very strong nonlinearities?

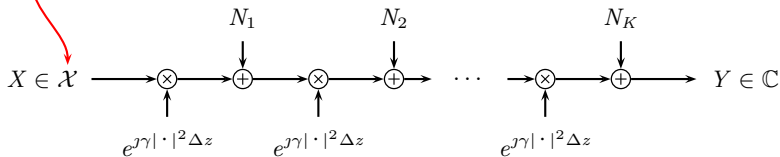
## Discrete-Time Channel Model



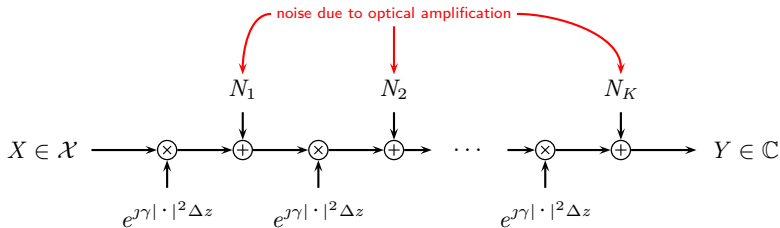


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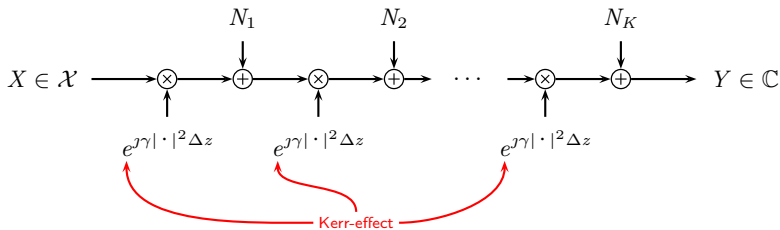
signal constellation, here  $|\mathcal{X}| = 16$



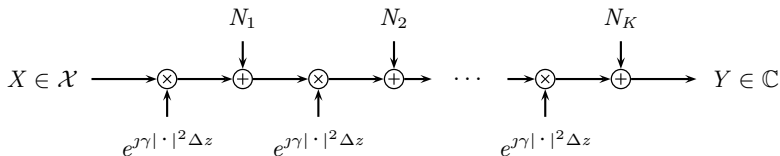
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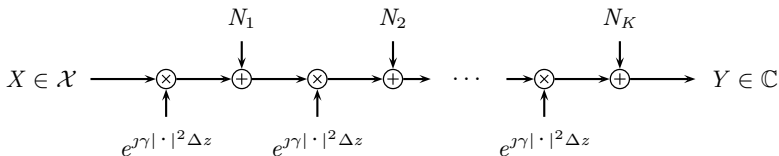


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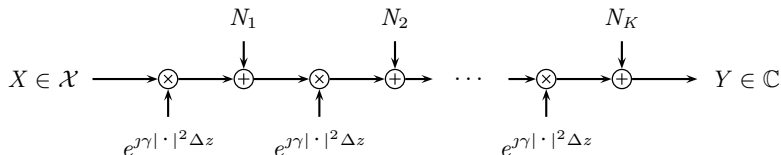
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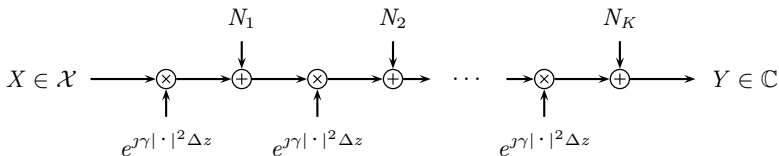
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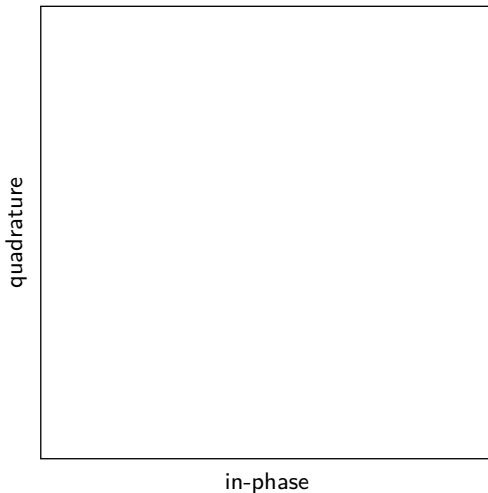
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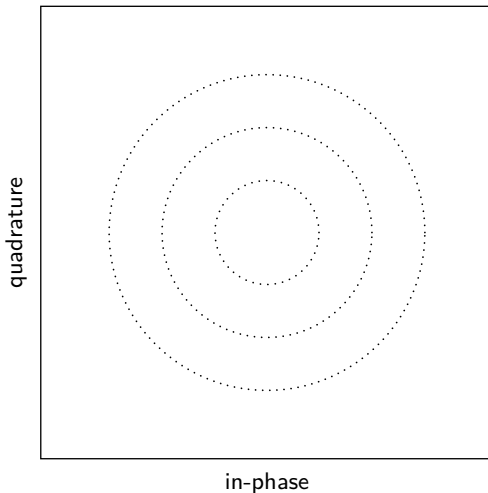
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- $L = 5500$  km, other parameters  $\gamma, \sigma^2$  taken from [Lau and Kahn, 2007]

## Amplitude Phase-Shift Keying (APSK), Example

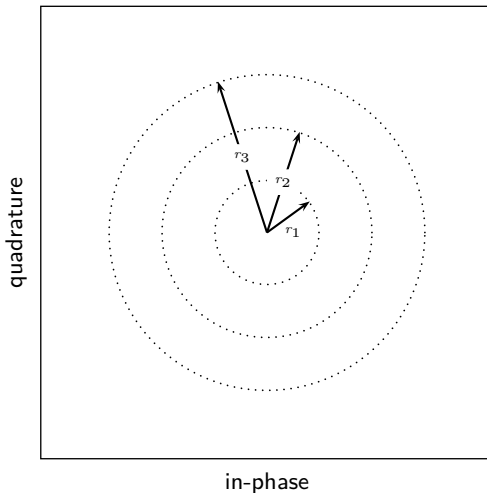




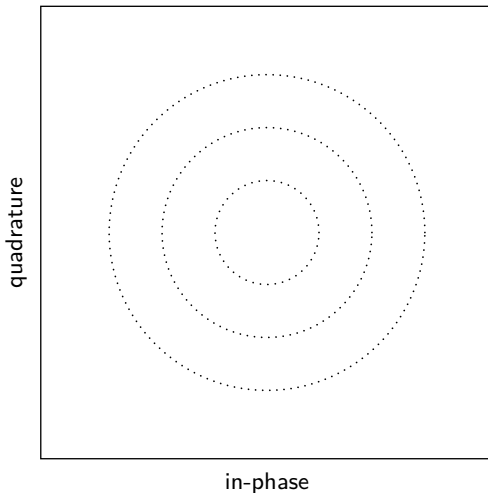
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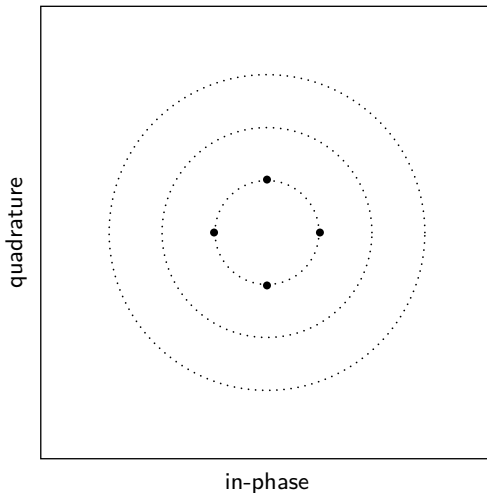


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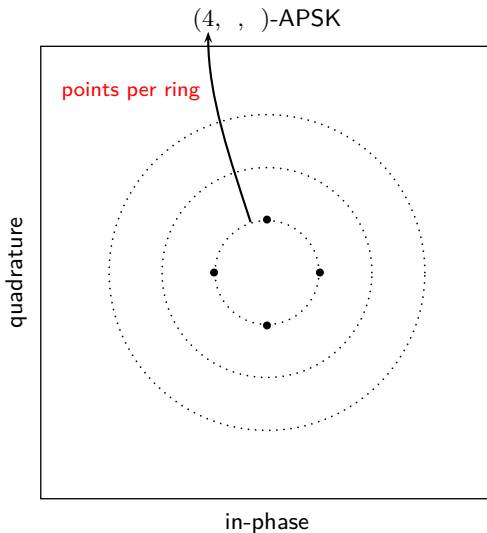


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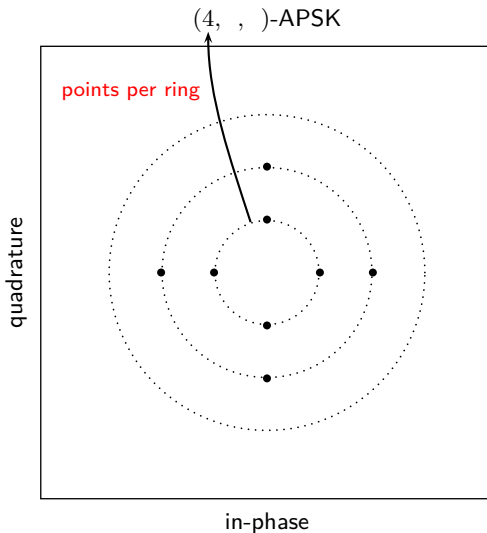
( , , )-APSK



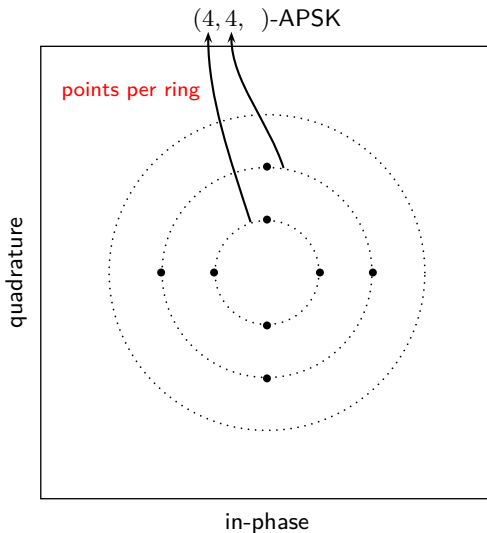
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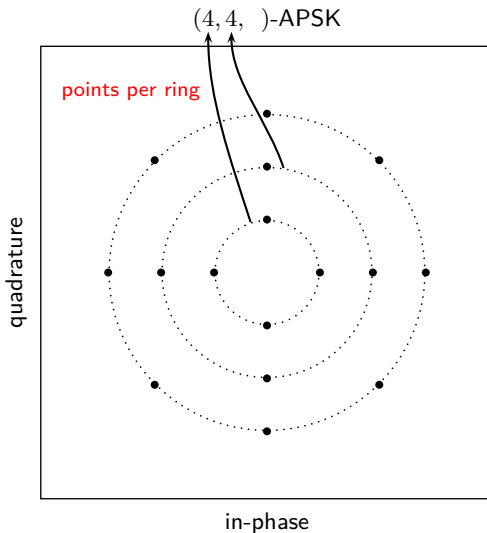
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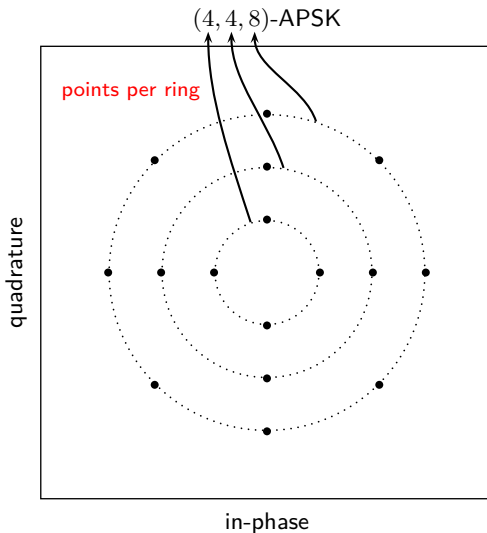


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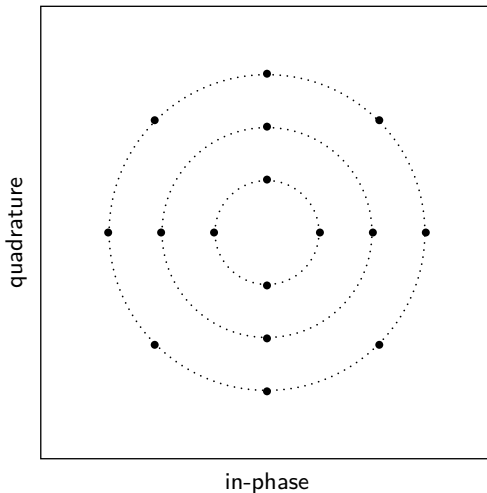


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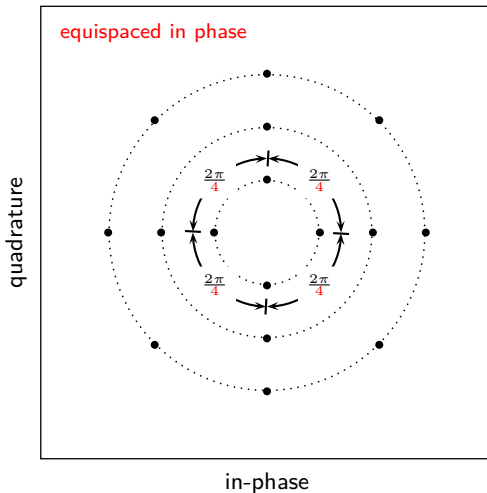
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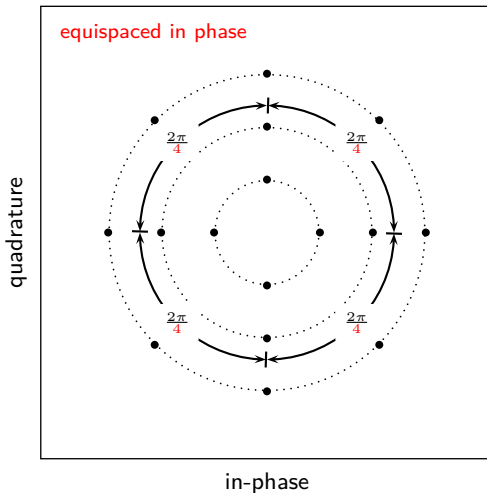
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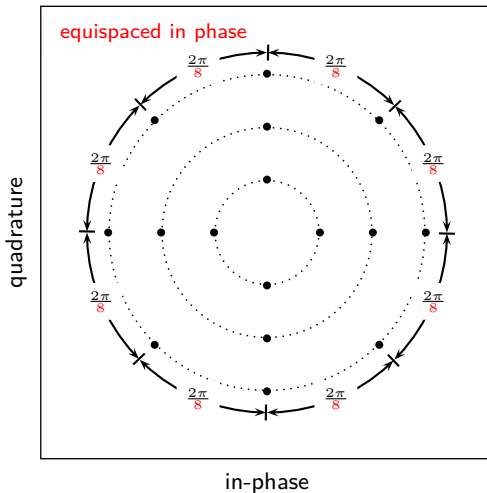
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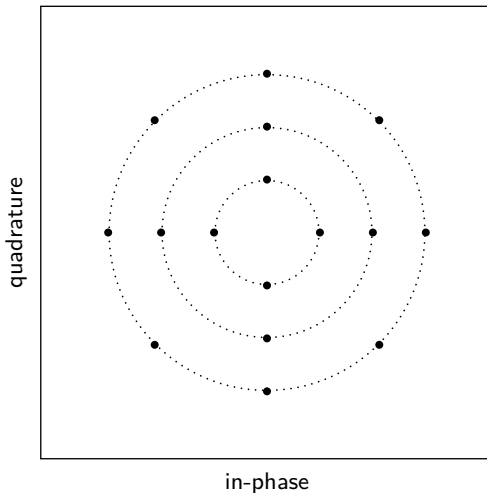
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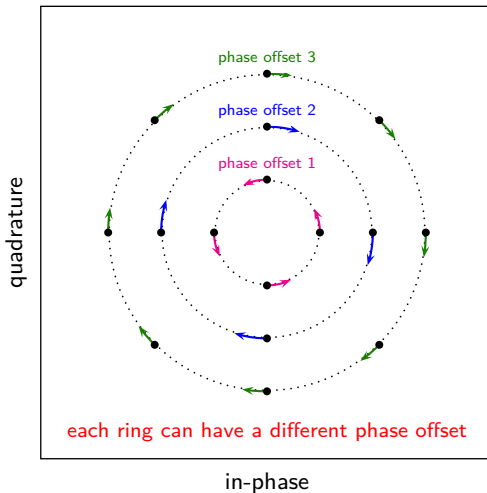
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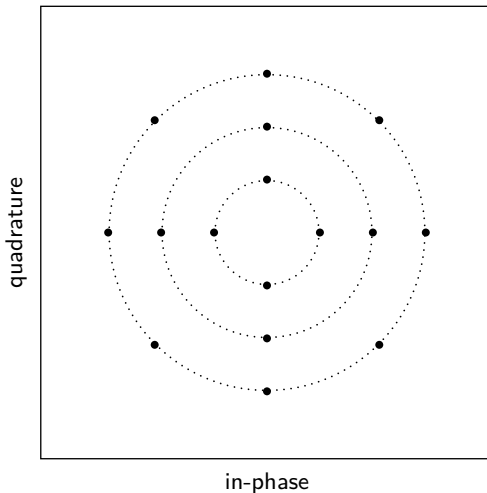
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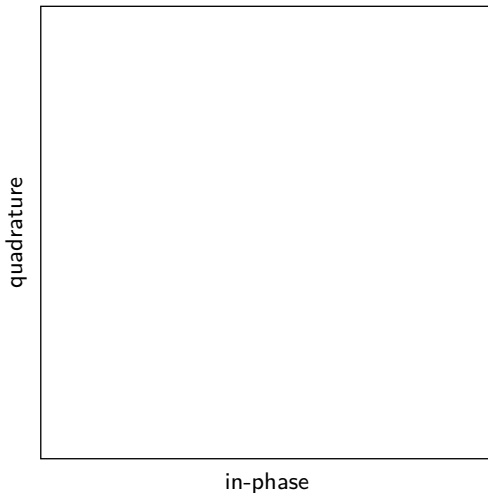
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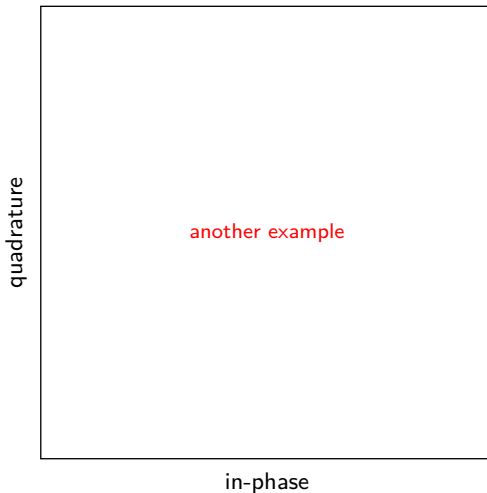




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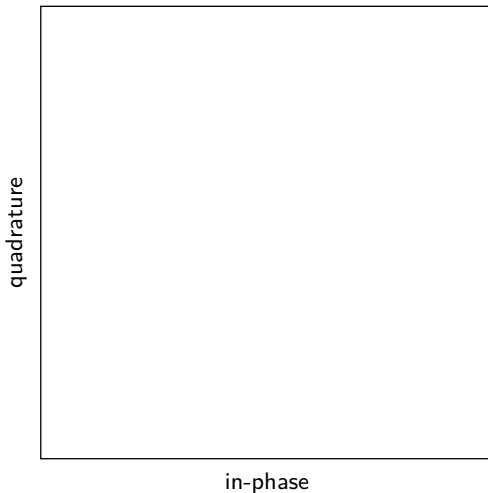


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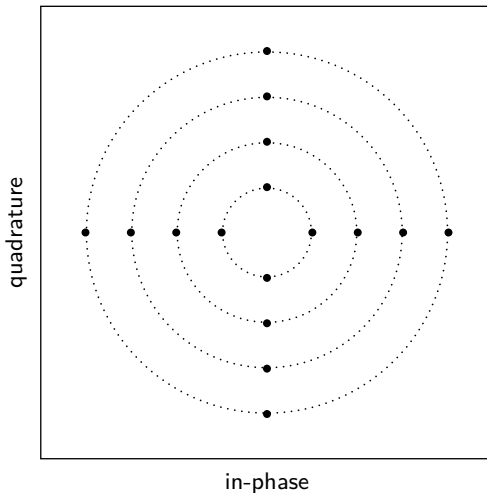
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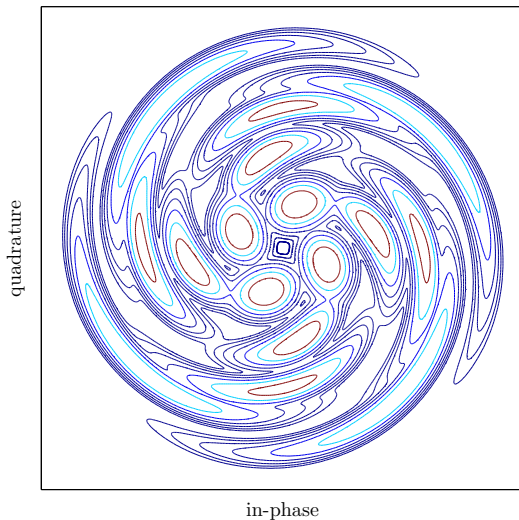
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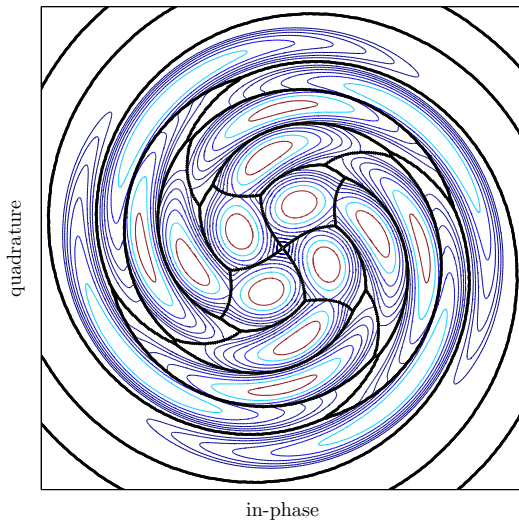
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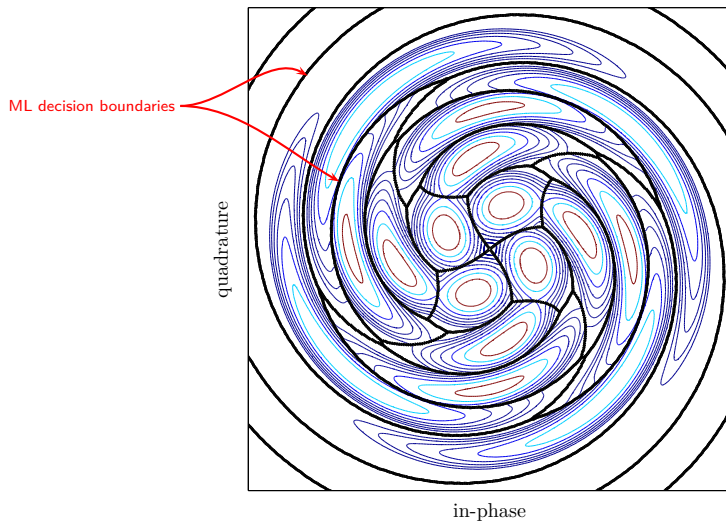
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- Probability density function (PDF)  $f_{Y|X=x_i}(y)$  **known**

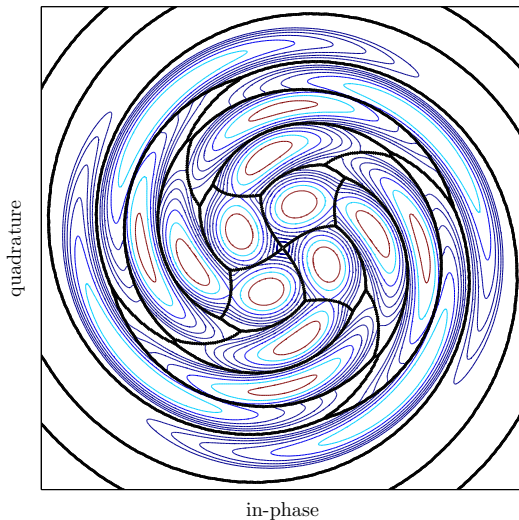


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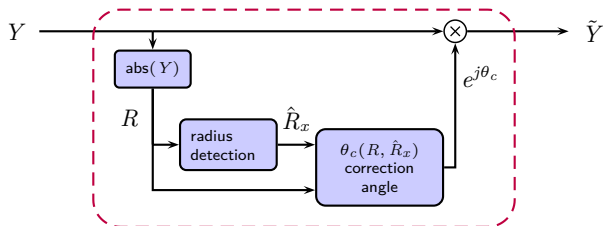
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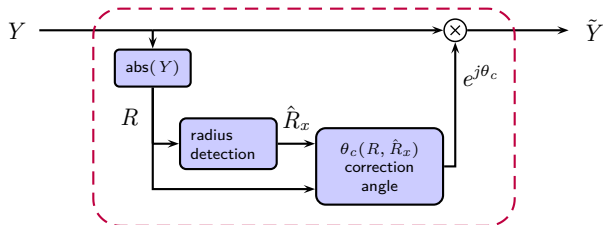
ML detection possible,  
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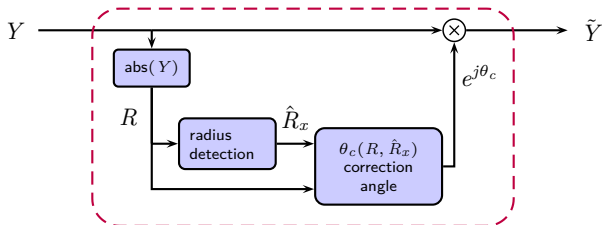
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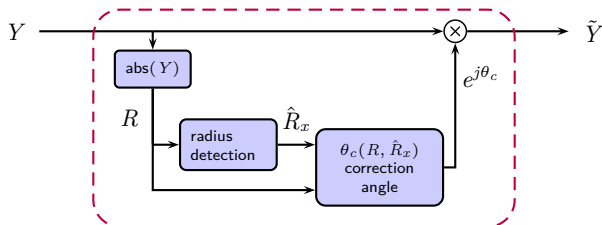


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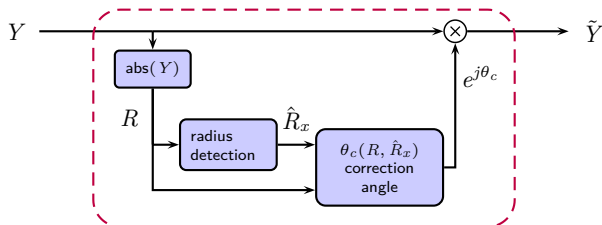
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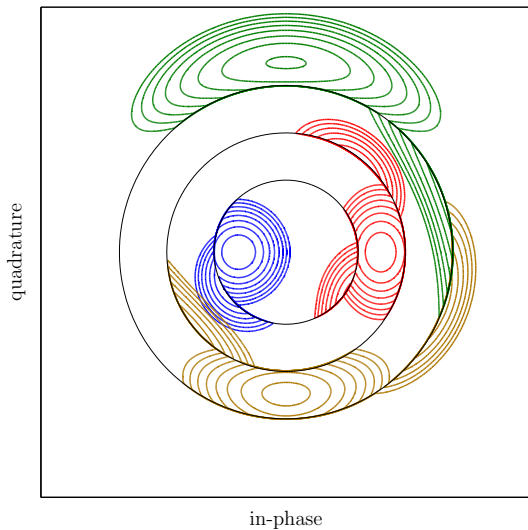
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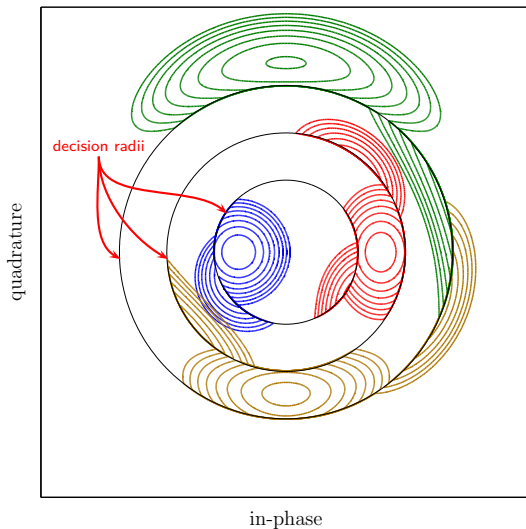
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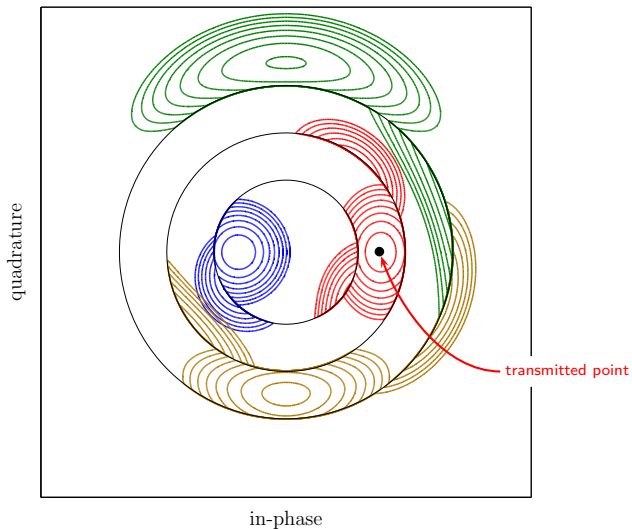


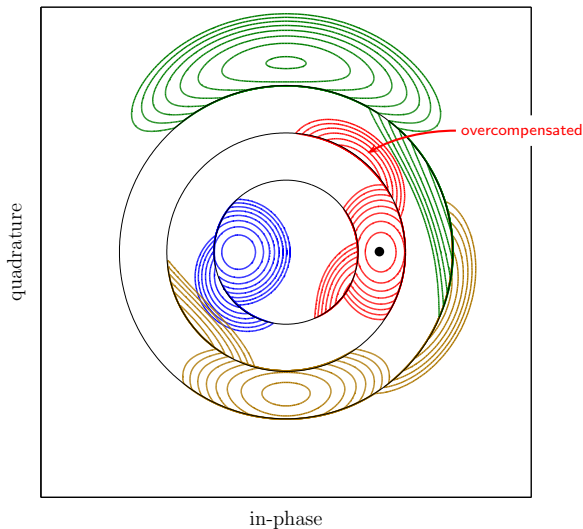
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- Note: the PDF of  $\tilde{Y}$  is defined piecewise

# PDF of $\tilde{Y}$ for (4,4,4,4)-APSK at $P = -4$ dBm

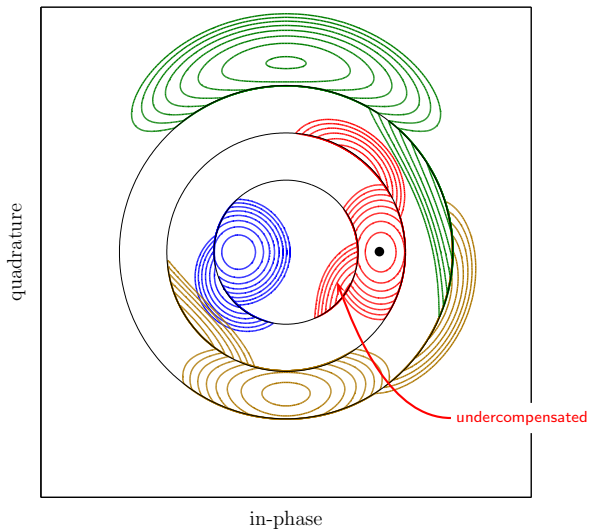
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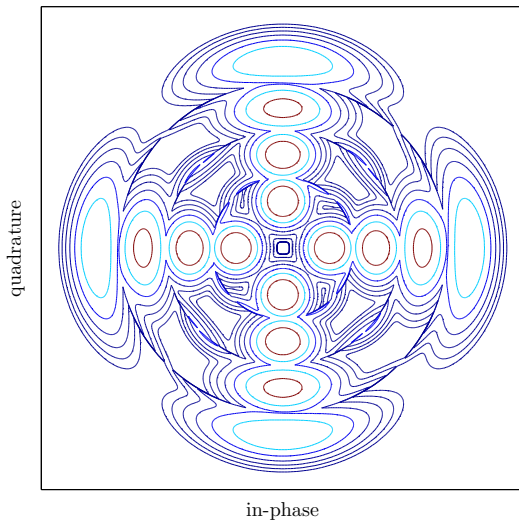
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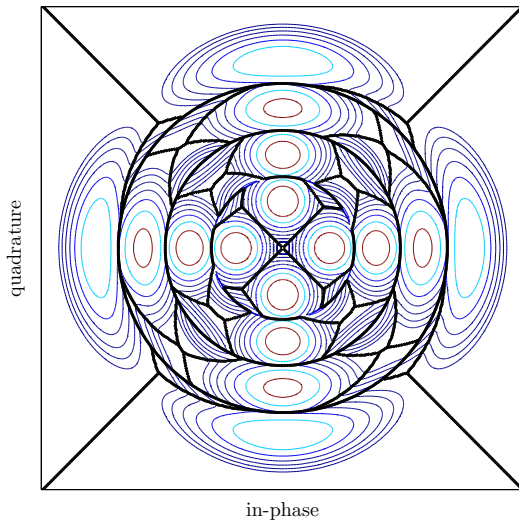
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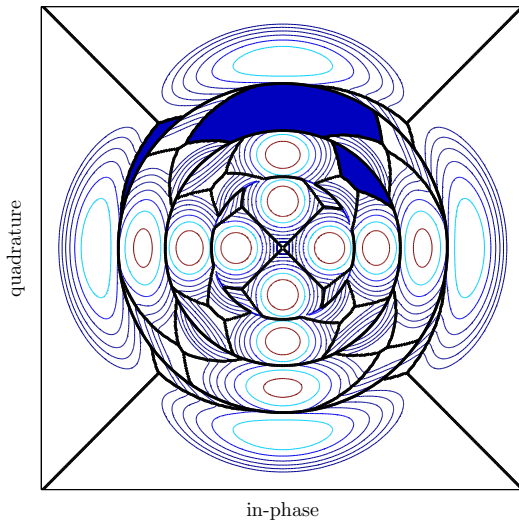


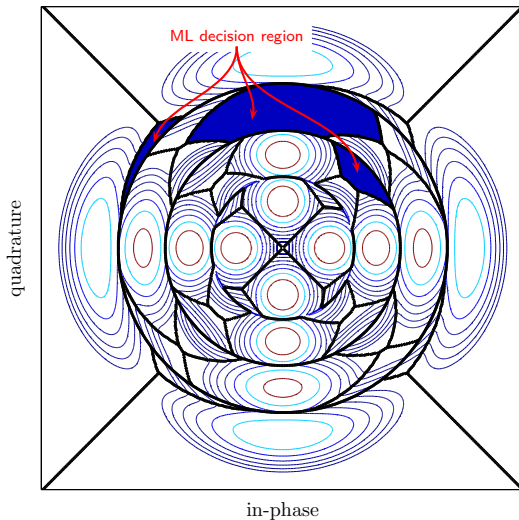
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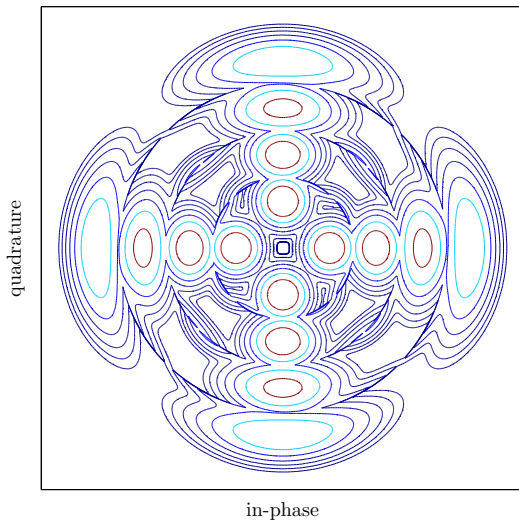
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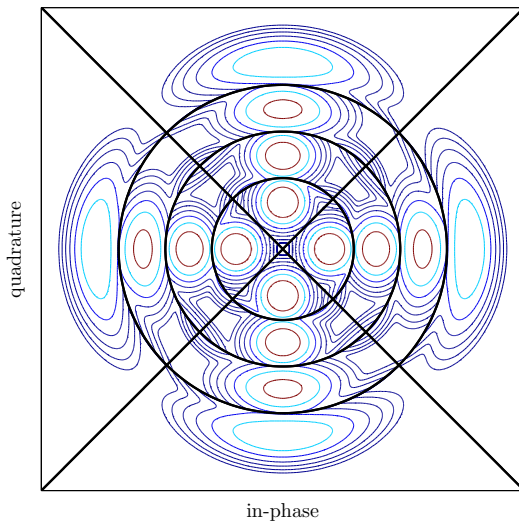
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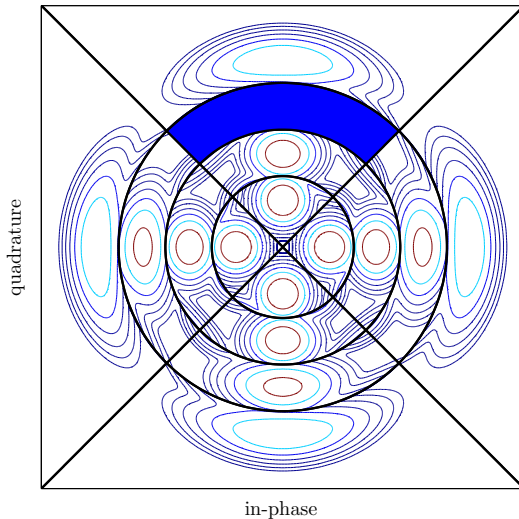


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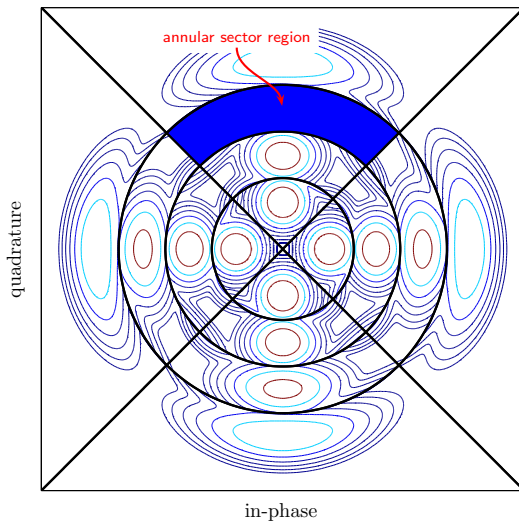




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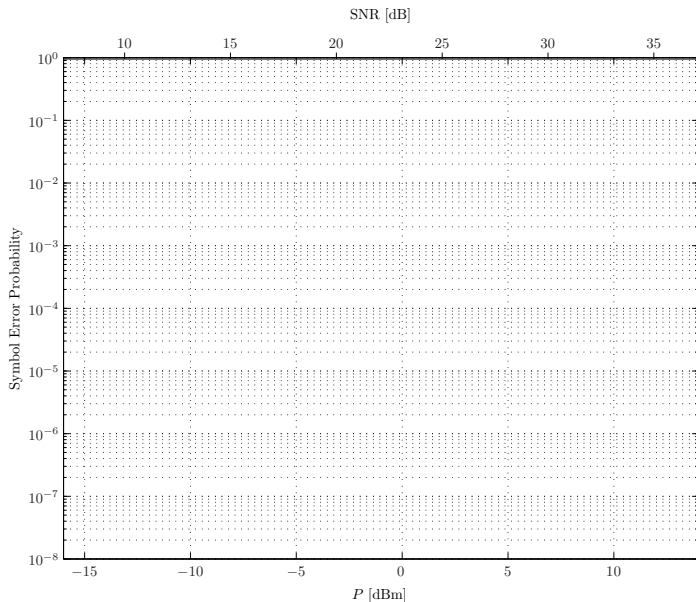
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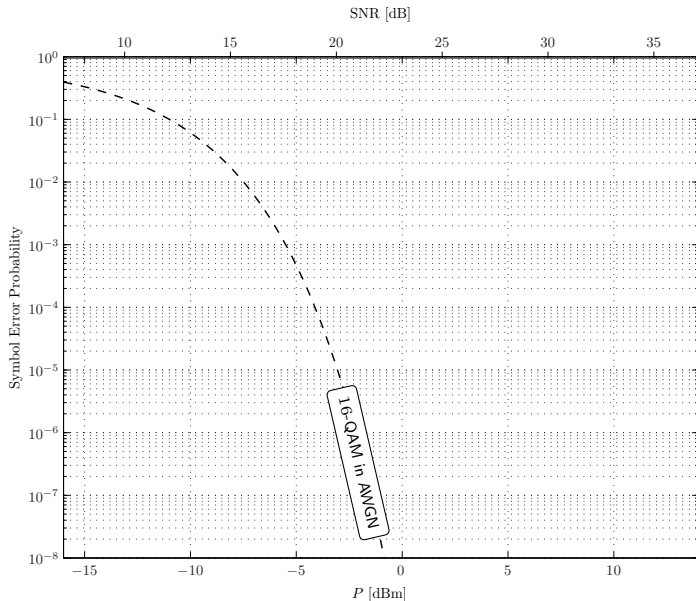
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- What phase offset? **Two-stage detection is insensitive to a phase offset.**

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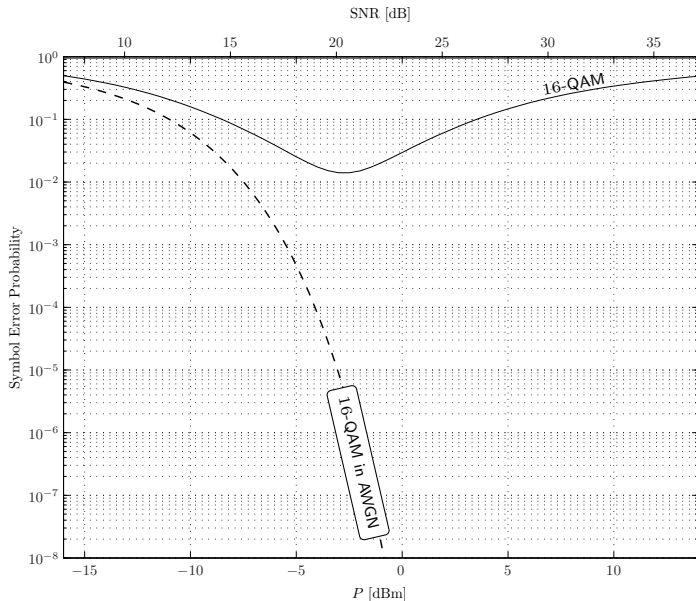
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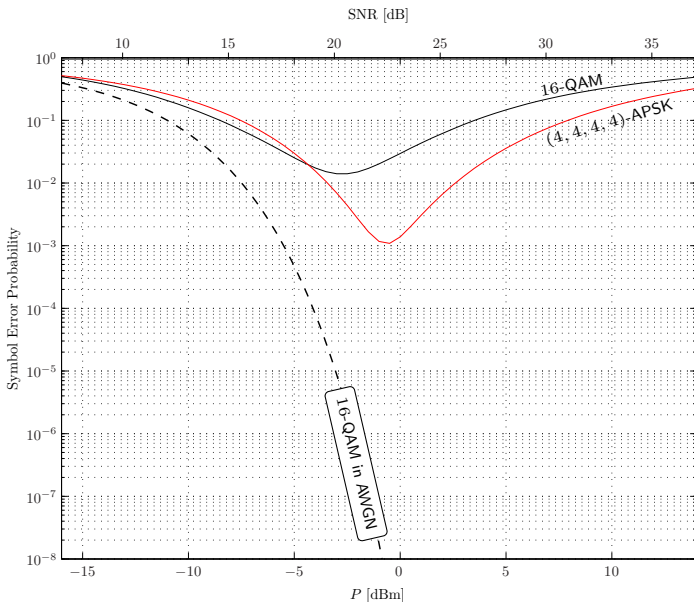
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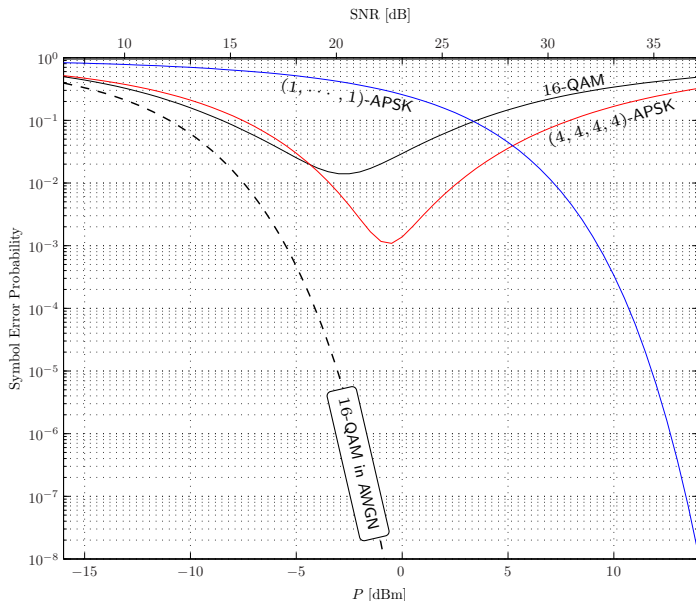
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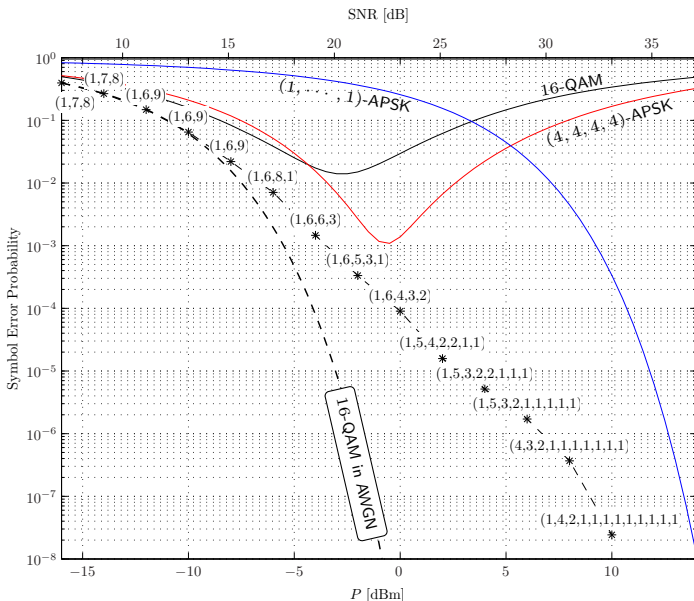
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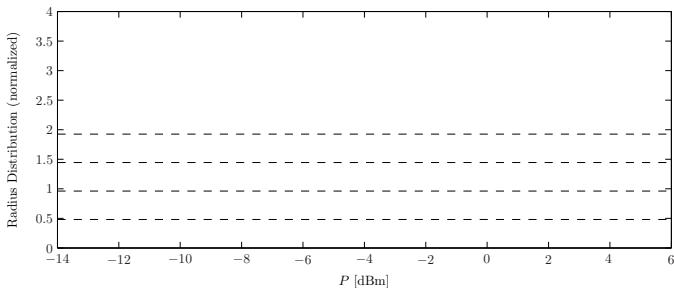
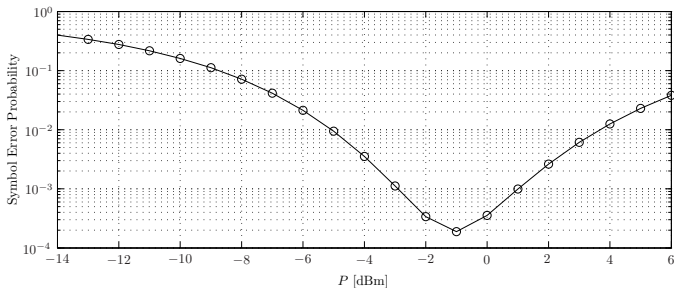


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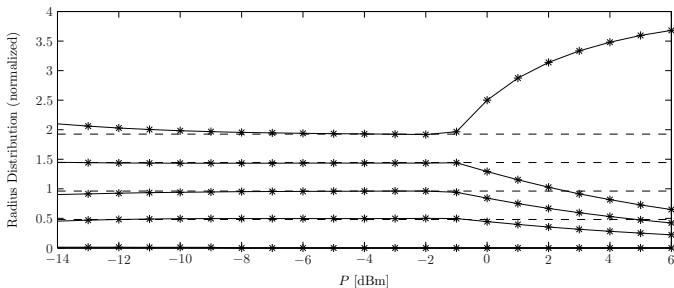
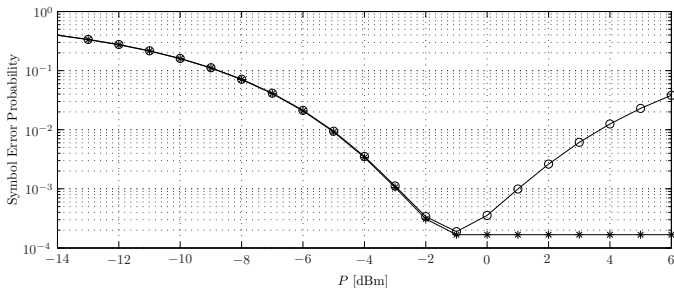




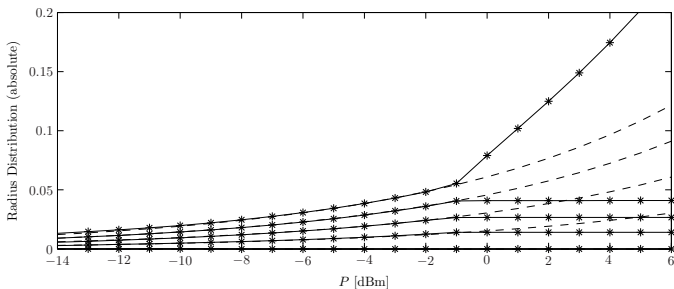
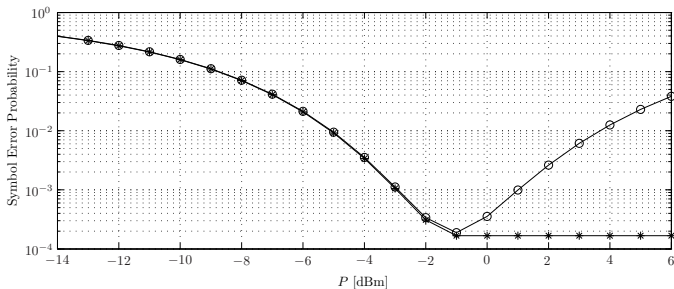
# Radius Optimization for (1,6,5,3,1)-APSK



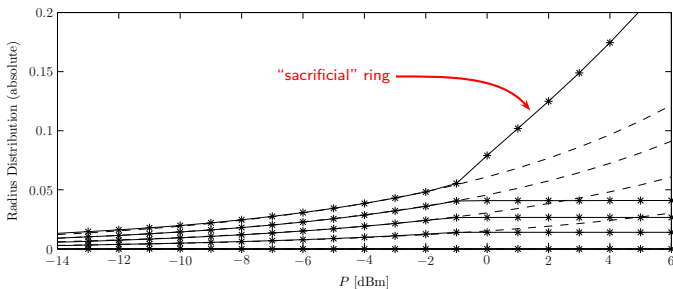
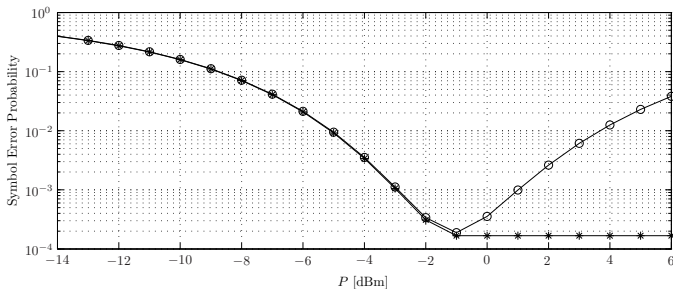
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# Thank you!



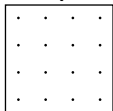
## References

-  Ho, K.-P. (2005).  
*Phase-modulated Optical Communication Systems*.  
Springer.
-  Lau, A. P. T. and Kahn, J. M. (2007).  
Signal Design and Detection in Presence of Nonlinear Phase Noise.  
*J. Lightw. Technol.*, 25(10):3008–3016.
-  Mecozzi, A. (1994).  
Limits to long-haul coherent transmission set by the Kerr nonlinearity and noise of the in-line amplifiers.  
*J. Lightw. Technol.*, 12(11):1993–2000.
-  Turitsyn, K. S., Derevyanko, S. A., Yurkevich, I. V., and Turitsyn, S. K. (2003).  
Information Capacity of Optical Fiber Channels with Zero Average Dispersion.  
*Phys. Rev. Lett.*, 91(20):203901.
-  Yousefi, M. I. and Kschischang, F. R. (2011).  
On the per-sample capacity of nondispersive optical fibers.  
*IEEE Trans. Inf. Theory*, 57(11):7522–7541.

# Power Dependent Phase Noise

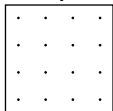
## Power Dependent Phase Noise

16-QAM

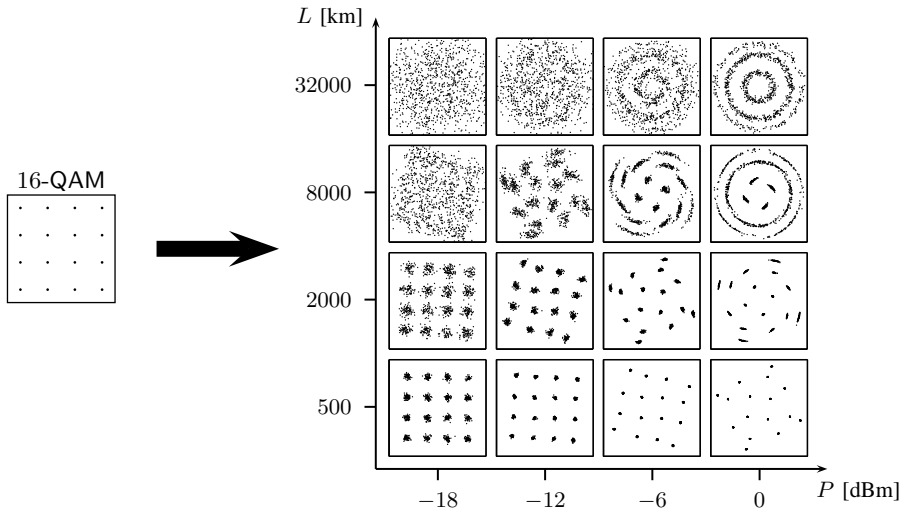


## Power Dependent Phase Noise

16-QAM



## Power Dependent Phase Noise





## Power Dependent Phase Noise

