Agenda

- Welcome and introduction (Chairman)
- Presentation and mention of
  - Faculty Opponent: Rüdiger Urbanke
  - Evaluation Committee: Michael Lentmaier, Gianluigi Liva, Laurent Schmalen
  - Funding sources
  - Contributors to the thesis work
- Errata List
- Short introduction to the thesis work (Faculty Opponent)
- Presentation (25 min.)
- Discussion (60–90 min.)
- Questions and comments from the Evaluation Committee
- Questions from the audience
- Evaluation Committee meeting, decision and lunch (S2 lunch room)

Analysis and Design of Spatially-Coupled Codes with Application to Fiber-Optical Communications

Christian Häger

Department of Signals and Systems, Chalmers University of Technology, Gothenburg, Sweden
christian.haeger@chalmers.se

FORCE
FIBER-OPTIC COMMUNICATIONS
RESEARCH CENTER

PhD Seminar
May 30, 2016

CHALMERS

Analysis and Design of Spatially-Coupled Codes with Application to Fiber-Optical Communications

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Many thanks to Alexandre Graell i Amat, Fredrik Brännström, Alex Alvarado, Erik Agrell, and Henry Pfister

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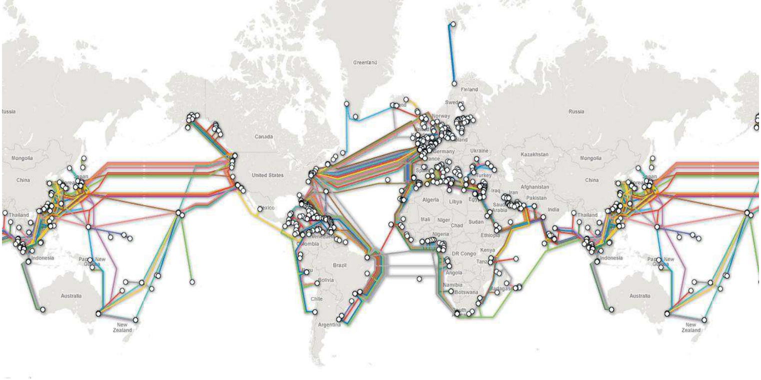
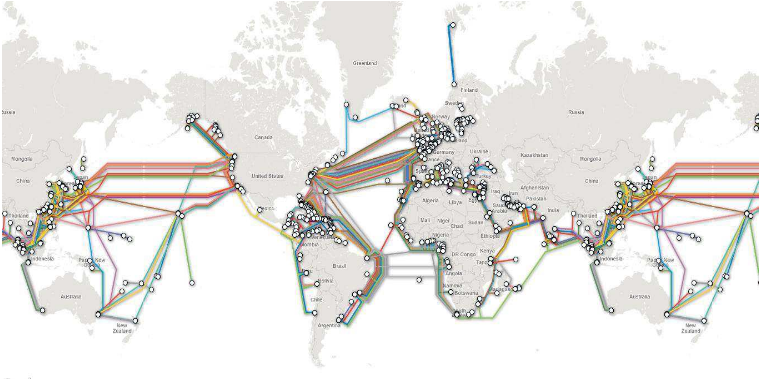
Fiber-Optical Communications

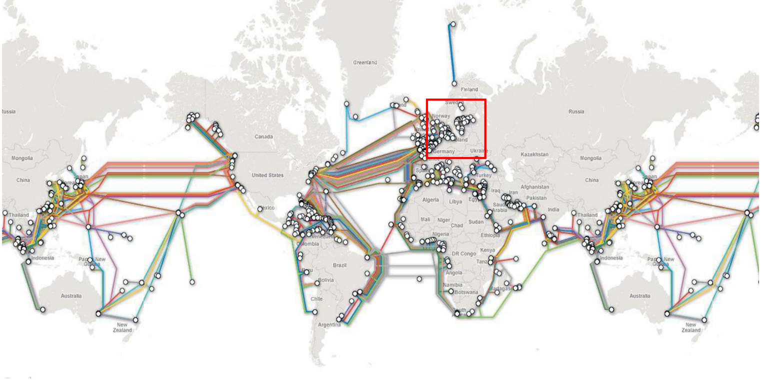


Fiber-Optical Communications



Fiber-optical communications enables data traffic over very long distances connecting cities, countries, and continents.

Fiber-Optical Communications



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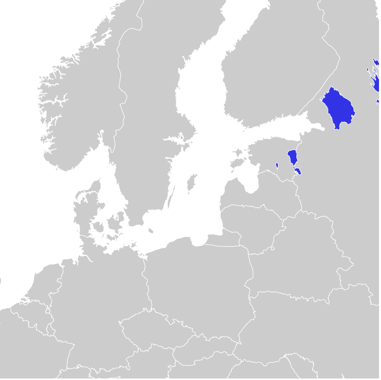

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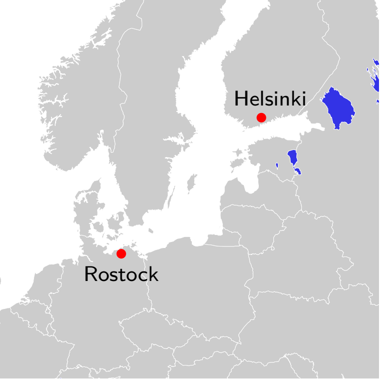

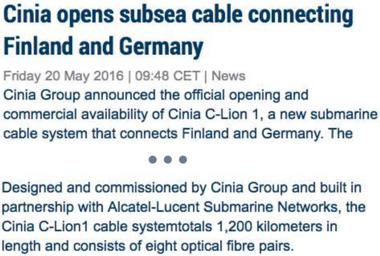

Cinia opens subsea cable connecting Finland and Germany

Friday 20 May 2016 | 09:48 CET | News

Cinia Group announced the official opening and commercial availability of Cinia C-Lion 1, a new submarine cable system that connects Finland and Germany. The

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Designed and commissioned by Cinia Group and built in partnership with Alcatel-Lucent Submarine Networks, the Cinia C-Lion1 cable systemtotals 1,200 kilometers in length and consists of eight optical fibre pairs.

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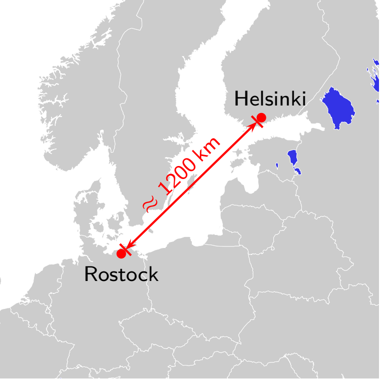

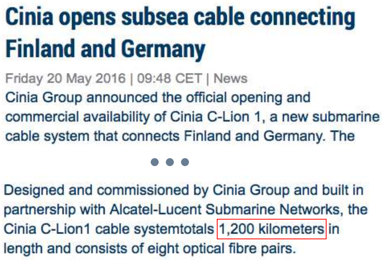

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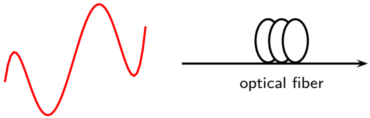

signal transmitted in Rostock

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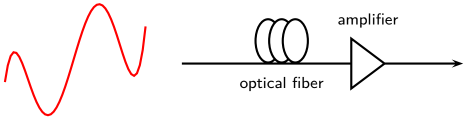

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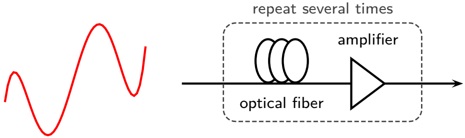

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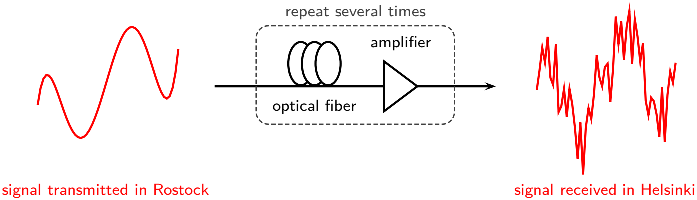

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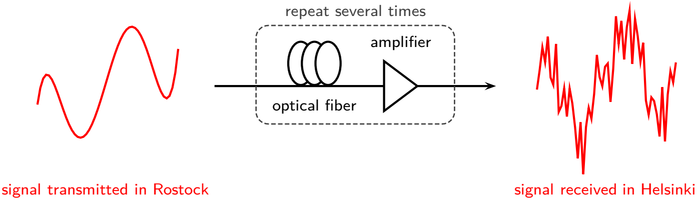
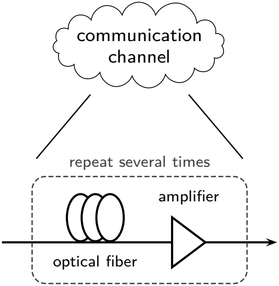


Error-correcting codes are essential in modern fiber-optical communication systems to ensure reliable data transmission.

Error-Correcting Codes



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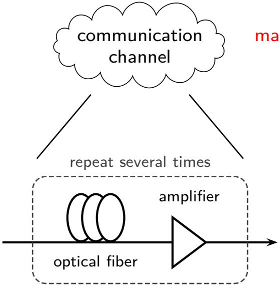


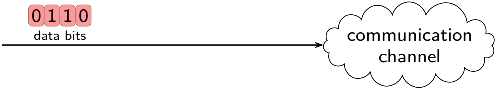
mathematical description of the transmission medium

Error-Correcting Codes

communication channel

mathematical description of the transmission medium

Error-Correcting Codes



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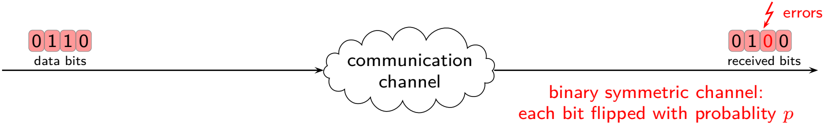
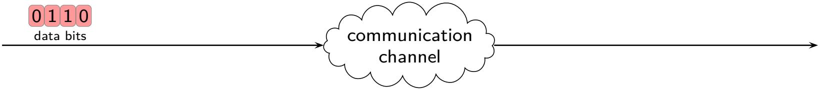


binary symmetric channel:
each bit flipped with probablity p

Error-Correcting Codes



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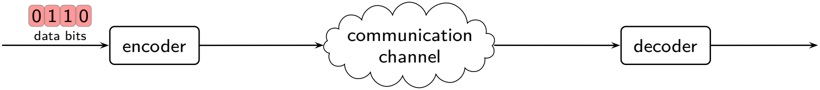

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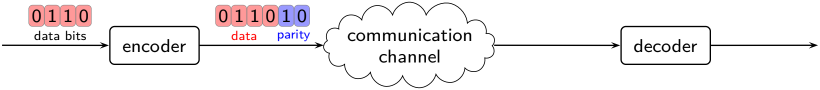

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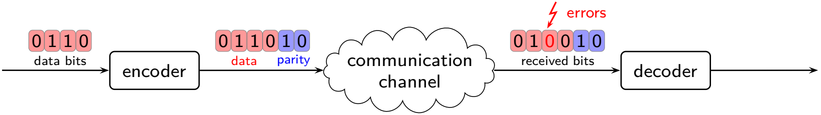

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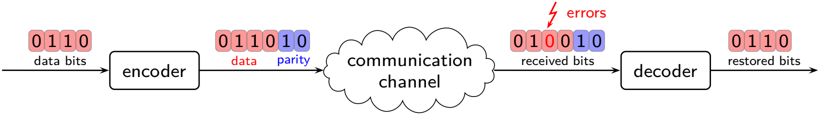

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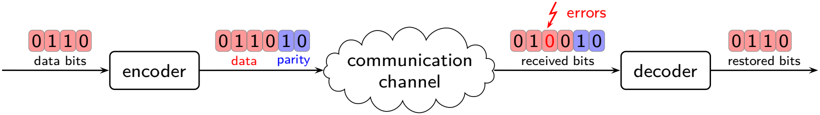
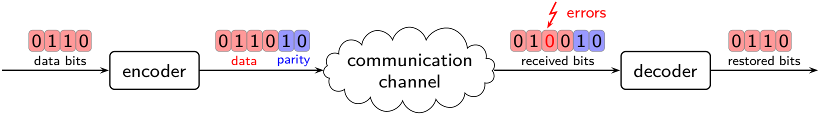


Requirements for Fiber-Optical Communications

Error-Correcting Codes



Requirements for Fiber-Optical Communications

- Very high throughputs (100 Gigabits per second or higher)
- Very high net coding gains (close-to-capacity performance)
- Very low bit error rates (below 10^{-15})

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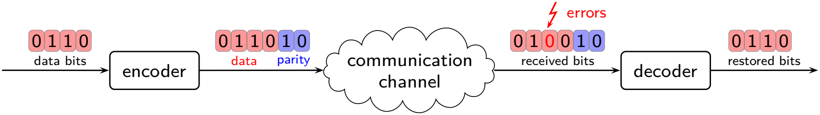

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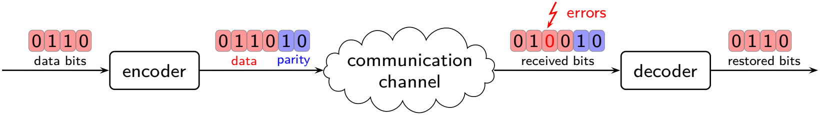

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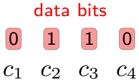
In this talk

1. Basics of spatially-coupled codes
2. Asymptotic analysis and design of deterministic codes `Papers C-F`
3. Designing spectrally-efficient fiber-optical systems `Papers A, B`

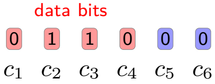
Codes on Graphs

Codes on Graphs



- Parity bits are formed by adding (modulo 2) subsets of data bits:

Codes on Graphs



data bits

0 1 1 0 0 0

c_1 c_2 c_3 c_4 c_5 c_6

- Parity bits are formed by adding (modulo 2) subsets of data bits:

$$c_1 + c_2 + c_3 = c_5$$
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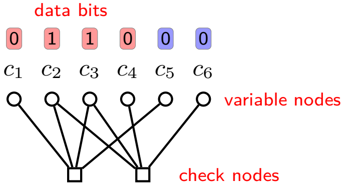

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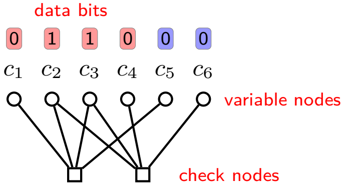

code ≜ set of all bit assignments such that all parity-checks are satisfied

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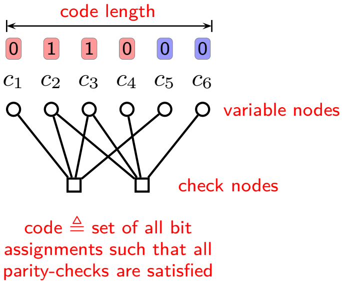
code length
0
1
1
0
0
0
c_1
c_2
c_3
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variable nodes
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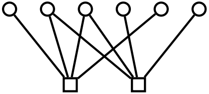

low-density parity-check (LDPC) code
[Gallager, 1962]

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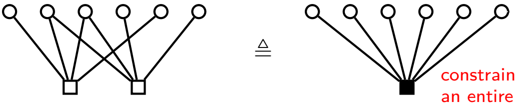

constraint node representing an entire (component) code

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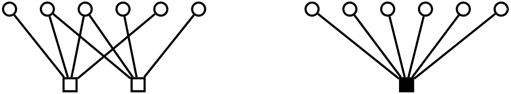

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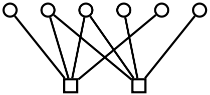

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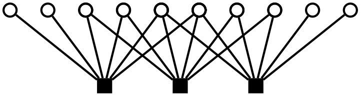

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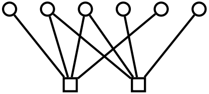

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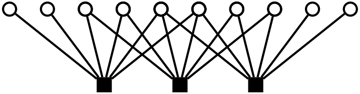

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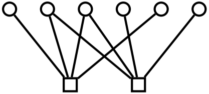

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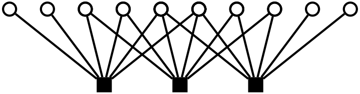

generalized LDPC code
[Tanner, 1981]

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Spatially-Coupled Codes

[Felström and Zigangirov, 1999], [Lentmaier et al., 2005], [Kudekar et al., 2011], ...

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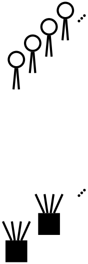

Start with a regular ("uncoupled") code/graph

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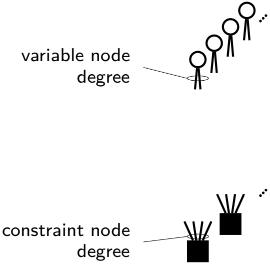

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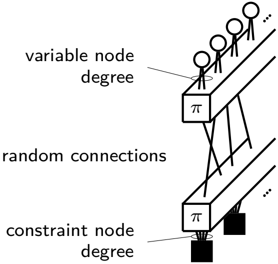

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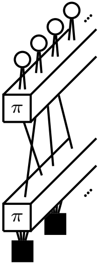

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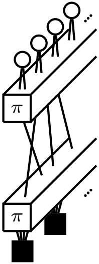

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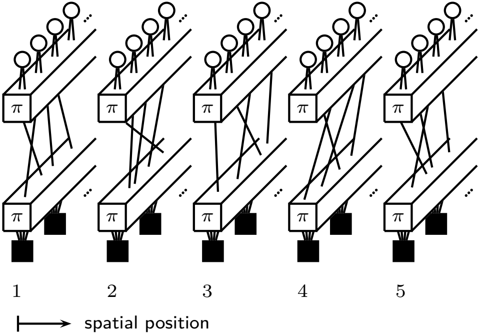

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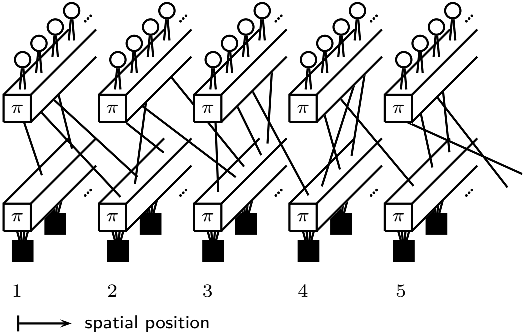
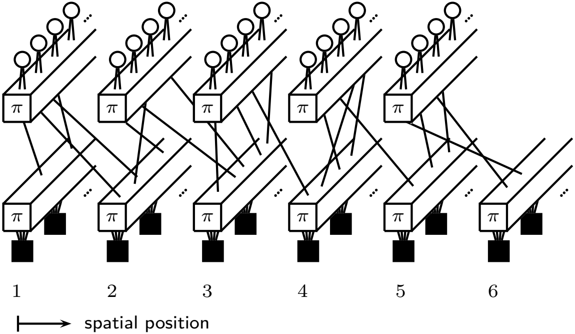


Spatially-Coupled Codes

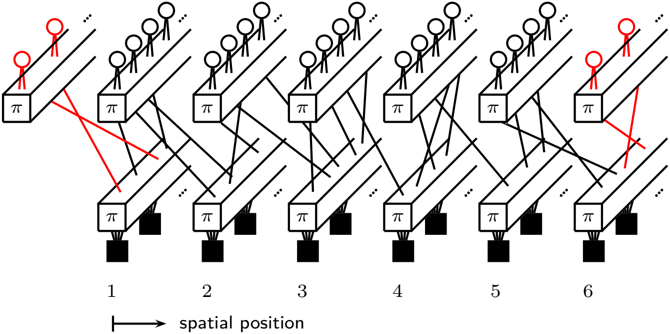
[Felström and Zigangirov, 1999], [Lentmaier et al., 2005], [Kudekar et al., 2011], . . .



Spatially-Coupled Codes

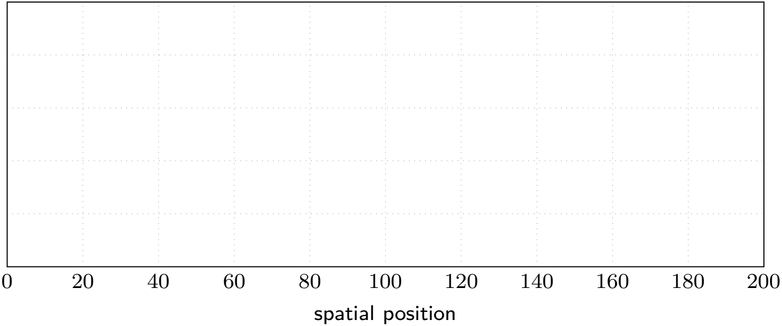
[Felström and Zigangirov, 1999], [Lentmaier et al., 2005], [Kudekar et al., 2011], . . .

known variable nodes $\Longrightarrow$ slight graph irregularity at the boundaries $\Longrightarrow$ better protection



Decoding Wave Effect

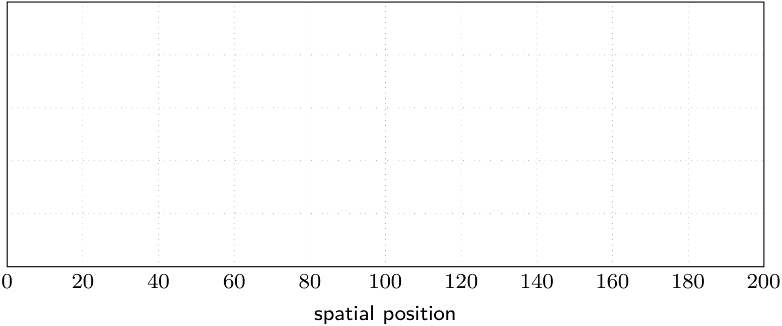
predicted bit error rate per spatial position



Decoding Wave Effect

- Apply (suboptimal) iterative decoding, exchanging messages between variable and constraint nodes

predicted bit error rate per spatial position



Decoding Wave Effect

- Apply (suboptimal) iterative decoding, exchanging messages between variable and constraint nodes

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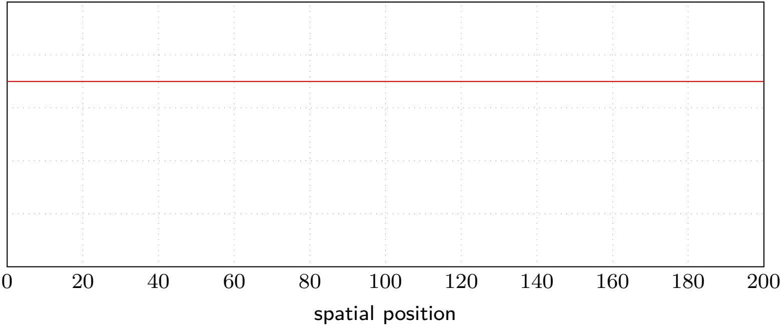

Decoding Wave Effect

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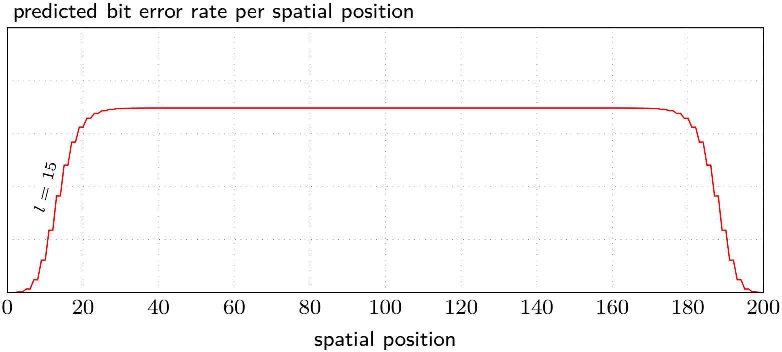

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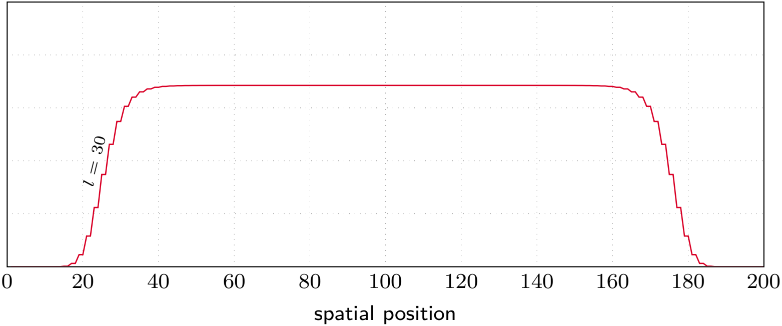

Decoding Wave Effect

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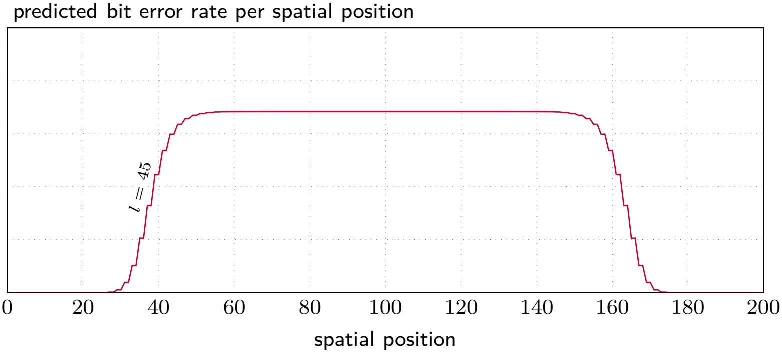

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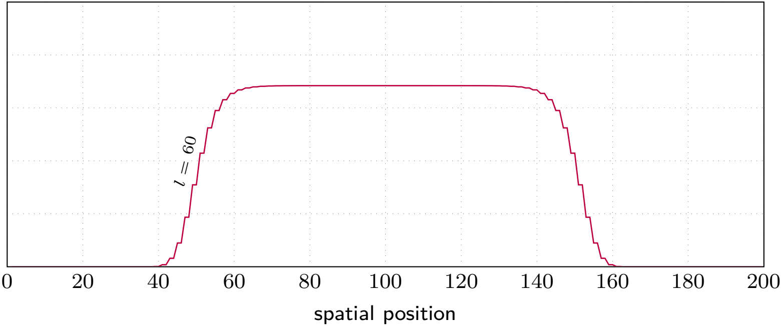

Decoding Wave Effect

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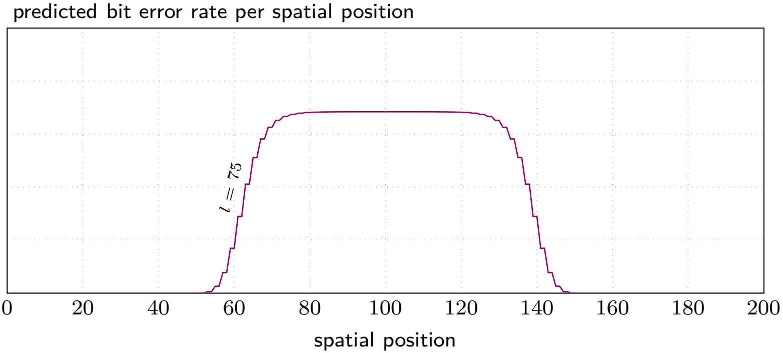
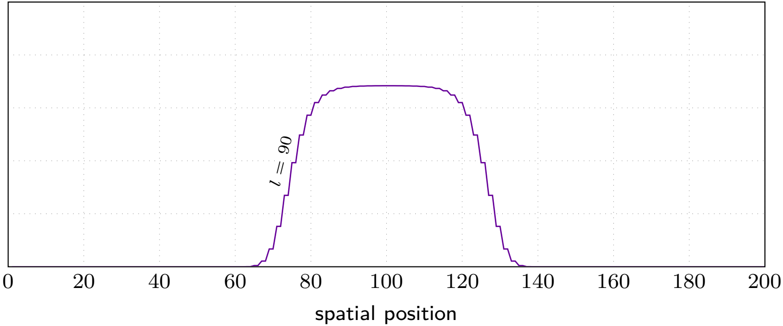


Decoding Wave Effect

- Apply (suboptimal) iterative decoding, exchanging messages between variable and constraint nodes

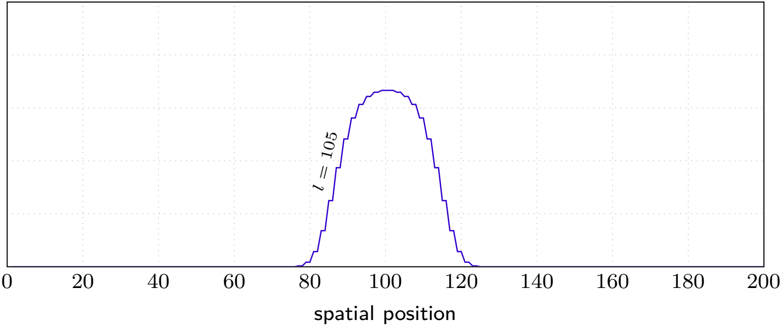
predicted bit error rate per spatial position



Decoding Wave Effect

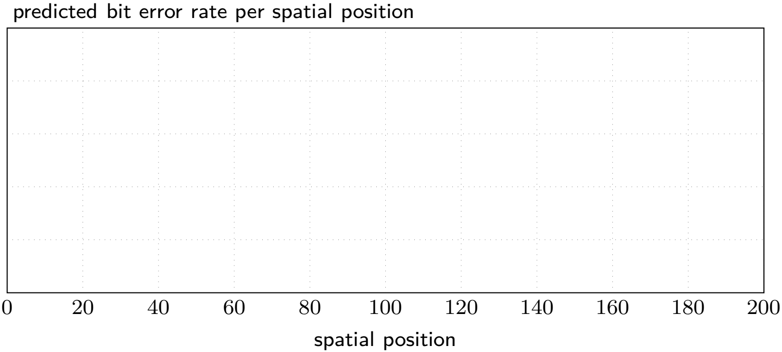
- Apply (suboptimal) iterative decoding, exchanging messages between variable and constraint nodes

predicted bit error rate per spatial position



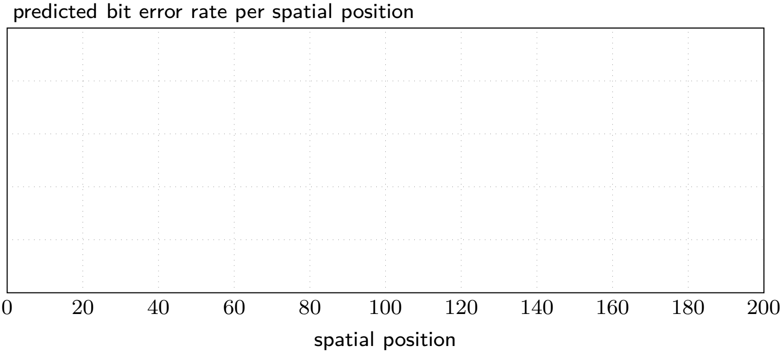
Decoding Wave Effect

- Apply (suboptimal) iterative decoding, exchanging messages between variable and constraint nodes
- Successful decoding



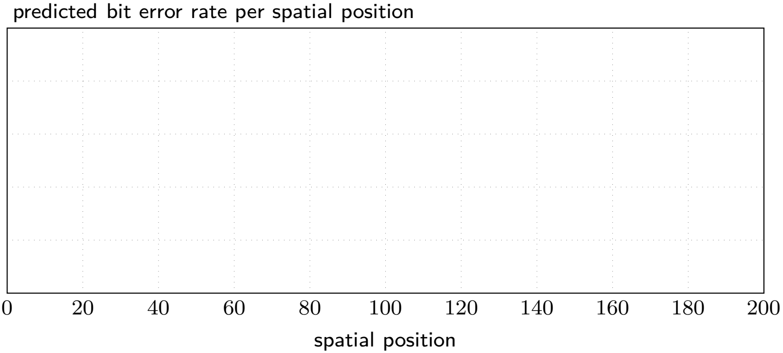
Decoding Wave Effect

- Apply (suboptimal) iterative decoding, exchanging messages between variable and constraint nodes
- Successful decoding even for cases where decoding of "uncoupled" regular codes fails



Decoding Wave Effect

- Apply (suboptimal) iterative decoding, exchanging messages between variable and constraint nodes
- Successful decoding even for cases where decoding of "uncoupled" regular codes fails
- Performance can be as good as under optimal decoding [Kudekar et al., 2011], [Yedla et al., 2014]



Decoding Wave Effect

Summary

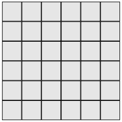
Spatial coupling is a tool to construct codes on graphs that have excellent performance under iterative decoding.

Introduction: Product Codes and Staircase Codes

Code proposals for fiber-optical communication systems are often very structured (i.e., deterministic) and not random-like (for example [Justesen et al., 2010], [Smith et al., 2012], [Jian et al., 2013]).

Introduction: Product Codes and Staircase Codes

rectangular array [Elias, 1954]



Introduction: Product Codes and Staircase Codes

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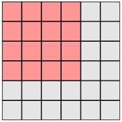

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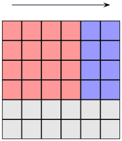

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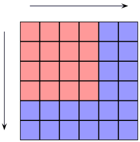

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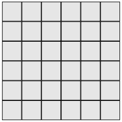

each row/column is a codeword in some component code

Introduction: Product Codes and Staircase Codes

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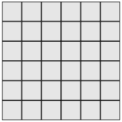

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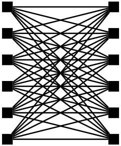
Tanner graph



Introduction: Product Codes and Staircase Codes

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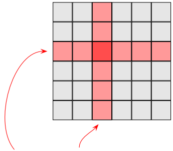

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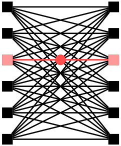

edge = degree-2 variable node

Introduction: Product Codes and Staircase Codes

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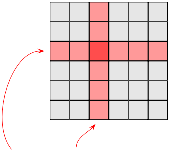

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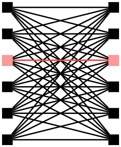

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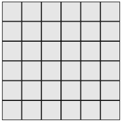

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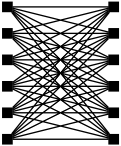

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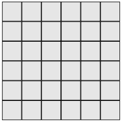
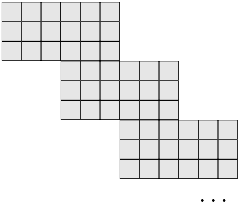


staircase array [Smith et al., 2012]



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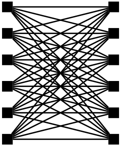

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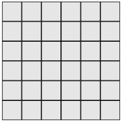

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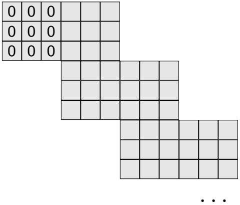

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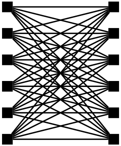

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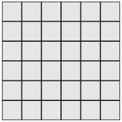

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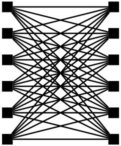

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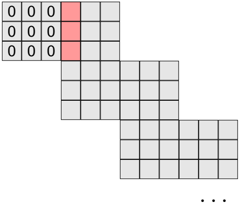

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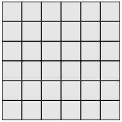

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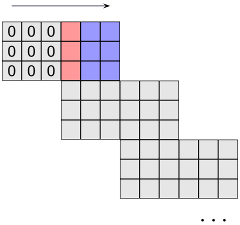

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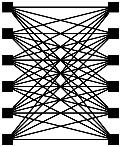

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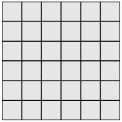

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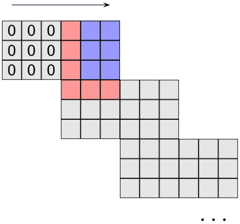

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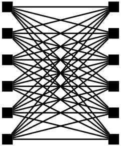

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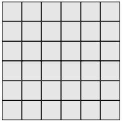

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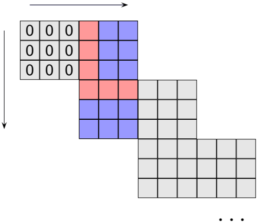

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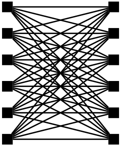

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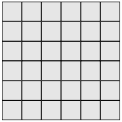
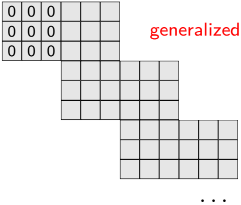
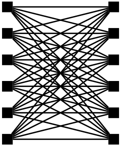




generalized product code (GPC)

Tanner graph



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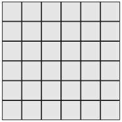
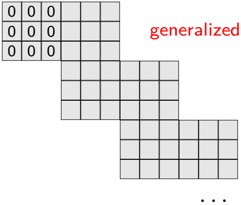


staircase array [Smith et al., 2012]



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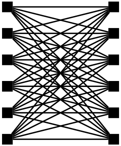

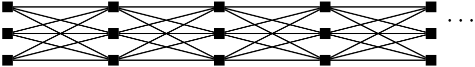

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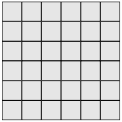

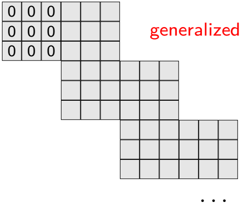

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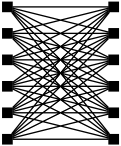
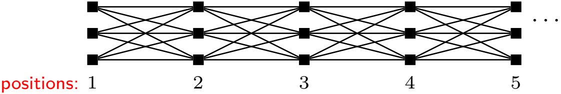
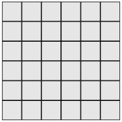


spatially-coupled code



Introduction: Product Codes and Staircase Codes

rectangular array [Elias, 1954]



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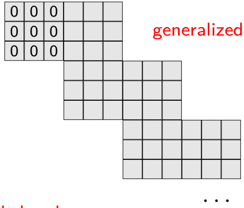
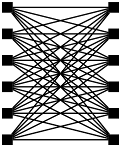


generalized product code (GPC)

Tanner graph



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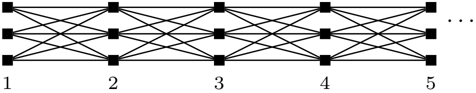
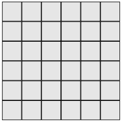


positions:

- Deterministic codes with fixed and structured Tanner graph

Introduction: Product Codes and Staircase Codes

rectangular array [Elias, 1954]



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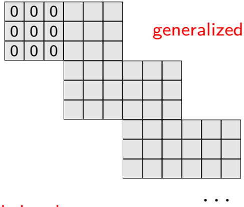
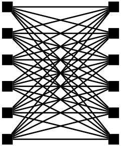
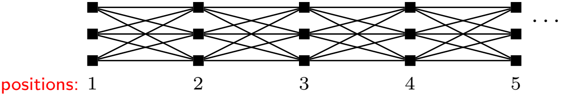


generalized product code (GPC)

Tanner graph

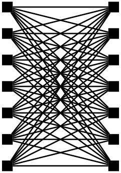


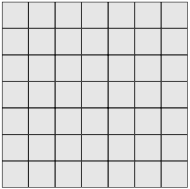
spatially-coupled code



- Deterministic codes with fixed and structured Tanner graph
- GPCs with iterative bounded-distance decoding are very appealing due to low-complexity hardware implementation

Iterative Bounded-Distance Decoding





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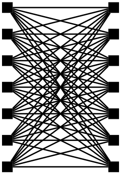

0	1	0	1	0	1	0
0	1	0	1	1	0	1
0	1	0	1	0	1	0
1	1	1	0	1	1	0
0	0	1	0	0	0	1
1	0	0	0	1	1	1
0	1	0	0	0	1	1

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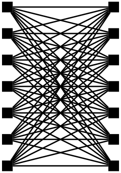
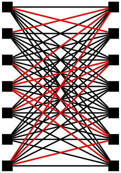


0	?	0	?	0	1	?
?	1	0	1	1	0	1
0	1	0	?	0	?	?
1	1	1	0	1	1	0
0	0	1	0	0	0	1
1	0	?	?	1	1	?
0	1	0	?	0	1	1

- Codeword transmission over binary erasure channel with erasure probability p

Iterative Bounded-Distance Decoding



0	?	0	?	0	1	?
?	1	0	1	1	0	1
0	1	0	?	0	?	?
1	1	1	0	1	1	0
0	0	1	0	0	0	1
1	0	?	?	1	1	?
0	1	0	?	0	1	1

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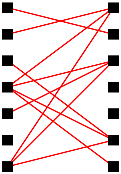

0	?	0	?	0	1	?
?	1	0	1	1	0	1
0	1	0	?	0	?	?
1	1	1	0	1	1	0
0	0	1	0	0	0	1
1	0	?	?	1	1	?
0	1	0	?	0	1	1

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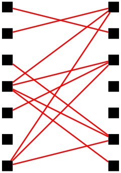
Iterative Bounded-Distance Decoding

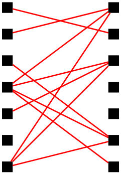


residual graph

0	?	0	?	0	1	?
?	1	0	1	1	0	1
0	1	0	?	0	?	?
1	1	1	0	1	1	0
0	0	1	0	0	0	1
1	0	?	?	1	1	?
0	1	0	?	0	1	1

- Codeword transmission over binary erasure channel with erasure probability p
- Each constraint node corresponds to t-erasure correcting component code

Iterative Bounded-Distance Decoding


residual graph

0	?	0	?	0	1	?
?	1	0	1	1	0	1
0	1	0	?	0	?	?
1	1	1	0	1	1	0
0	0	1	0	0	0	1
1	0	?	?	1	1	?
0	1	0	?	0	1	1

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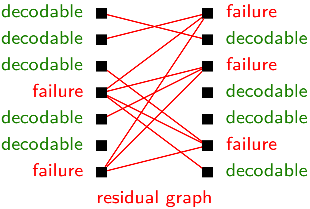
- ℓ iterations of bounded-distance decoding $=$ peeling of vertices with degree $\leq t$ (in parallel)

Iterative Bounded-Distance Decoding

1st iteration $(t = 2)$



0	?	0	?	0	1	?
?	1	0	1	1	0	1
0	1	0	?	0	?	?
1	1	1	0	1	1	0
0	0	1	0	0	0	1
1	0	?	?	1	1	?
0	1	0	?	0	1	1

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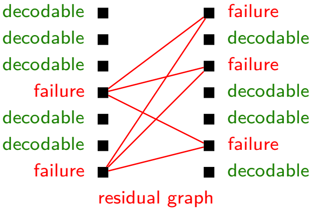

0	1	0	?	0	1	?
0	1	0	1	1	0	1
0	1	0	?	0	1	?
1	1	1	0	1	1	0
0	0	1	0	0	0	1
1	0	0	?	1	1	?
0	1	0	0	0	1	1

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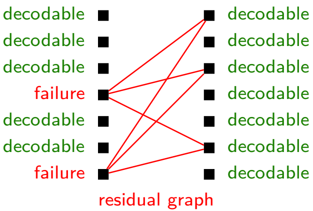
Iterative Bounded-Distance Decoding

2nd iteration $(t = 2)$



0	1	0	?	0	1	?
0	1	0	1	1	0	1
0	1	0	?	0	1	?
1	1	1	0	1	1	0
0	0	1	0	0	0	1
1	0	0	?	1	1	?
0	1	0	0	0	1	1

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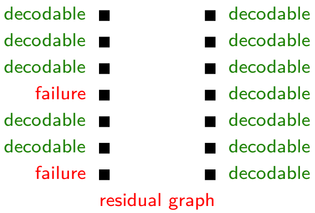

decodable ■ ■ decodable
decodable ■ ■ decodable
decodable ■ ■ decodable
failure ■ ■ decodable
decodable ■ ■ decodable
decodable ■ ■ decodable
failure ■ ■ decodable
residual graph

0	1	0	1	0	1	0
0	1	0	1	1	0	1
0	1	0	1	0	1	0
1	1	1	0	1	1	0
0	0	1	0	0	0	1
1	0	0	0	1	1	1
0	1	0	0	0	1	1

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Staircase Code Optimization

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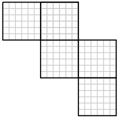
Problem Formulation

For staircase code with fixed code rate R, find “good” component codes



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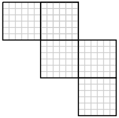

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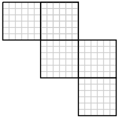

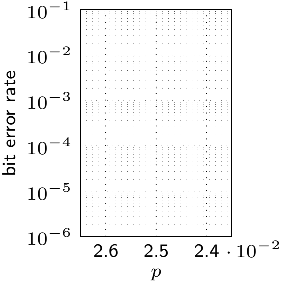

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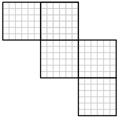

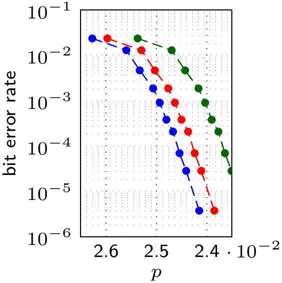

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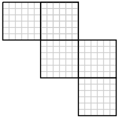

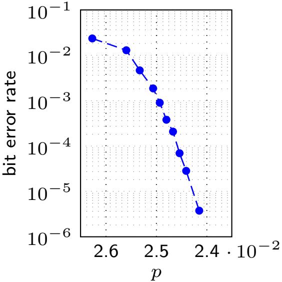

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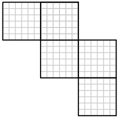

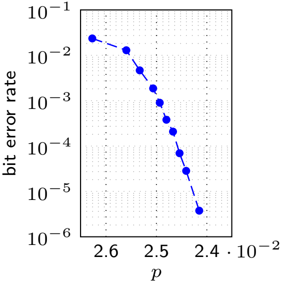

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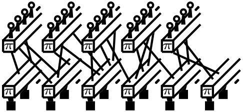

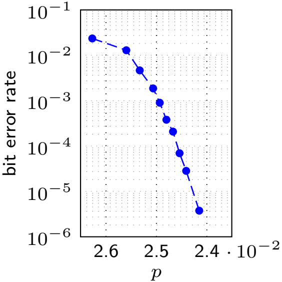

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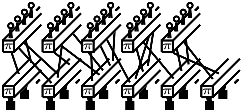

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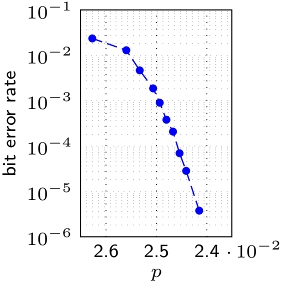

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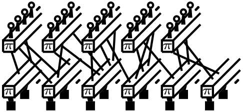

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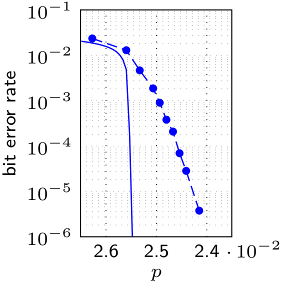

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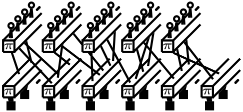

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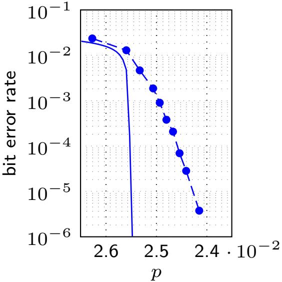

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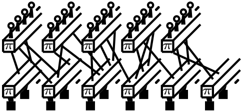

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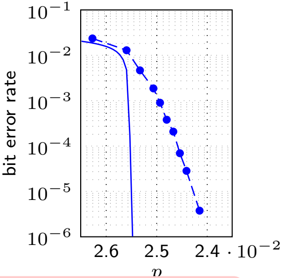
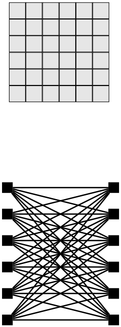


Fundamental question

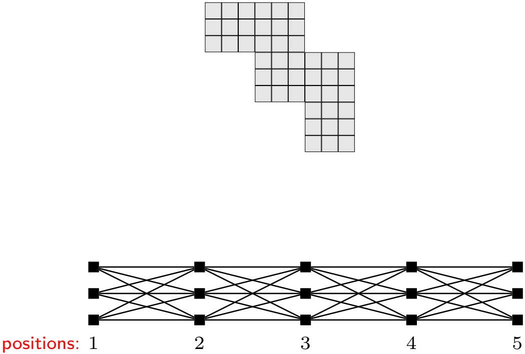
Is it possible to directly analyze staircase codes (and other deterministic GPCs) without the detour to random-like codes? Papers D-F

Parametrized Construction of Generalized Product Codes

product codes



staircase codes



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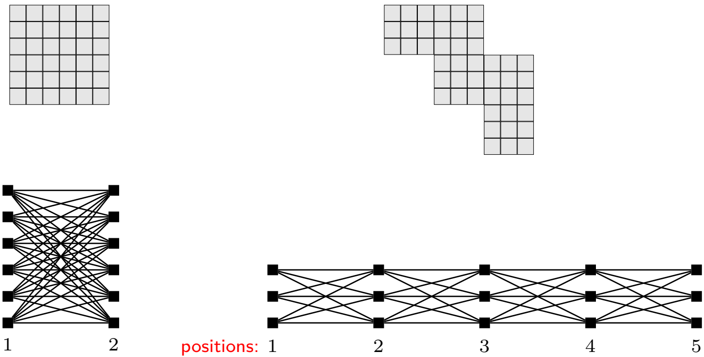

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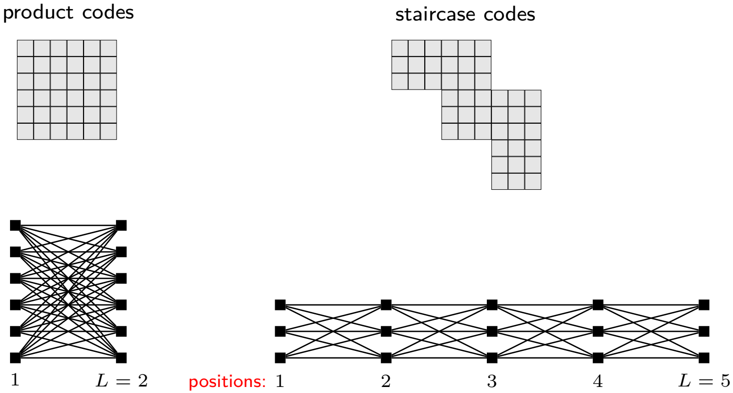

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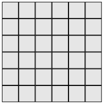
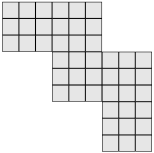
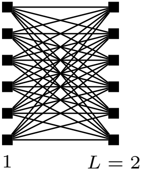
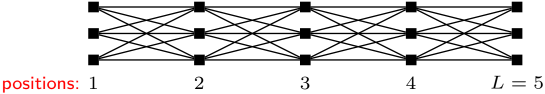


staircase codes



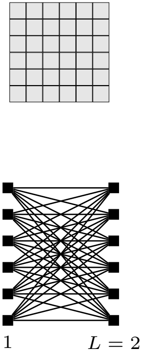




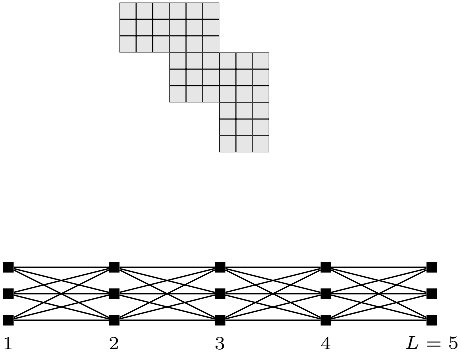
$\boldsymbol{\eta}$: symmetric $L \times L$ matrix that defines graph connectivity

Parametrized Construction of Generalized Product Codes

product codes



staircase codes

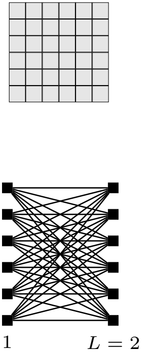


$$\boldsymbol{\eta} = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}$$

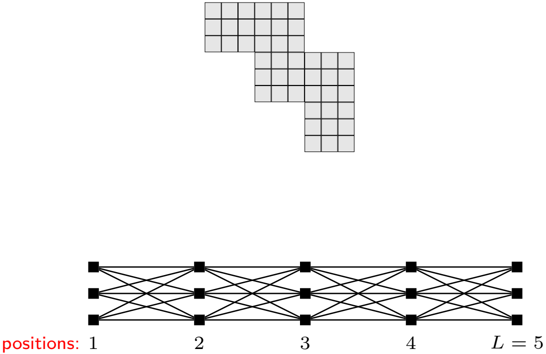
$\boldsymbol{\eta}$: symmetric $L \times L$ matrix that defines graph connectivity

Parametrized Construction of Generalized Product Codes

product codes



staircase codes



$$\boldsymbol{\eta} = \left(\begin{smallmatrix} 0 & 1 \\ 1 & 0 \end{smallmatrix}\right)$$

$$\boldsymbol{\eta} = \begin{pmatrix} 0 & 1 & 0 & 0 & 0 \\ 1 & 0 & 1 & 0 & 0 \\ 0 & 1 & 0 & 1 & 0 \\ 0 & 0 & 1 & 0 & 1 \\ 0 & 0 & 0 & 1 & 0 \end{pmatrix}$$

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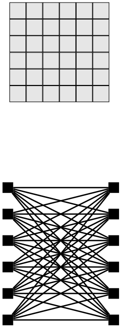
product codes



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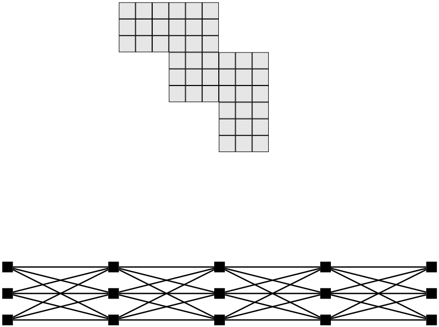

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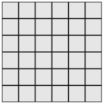

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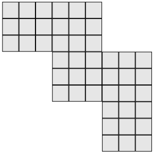
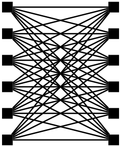
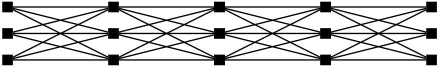


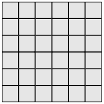


n: “problem size”, proportional to the number of constraint nodes



Parametrized Construction of Generalized Product Codes

product codes



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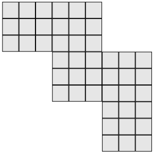

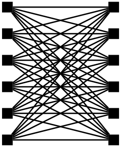

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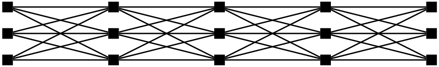

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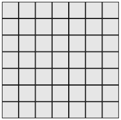
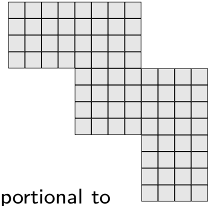
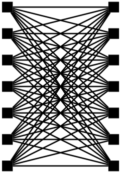


staircase codes





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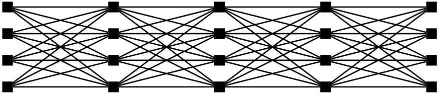

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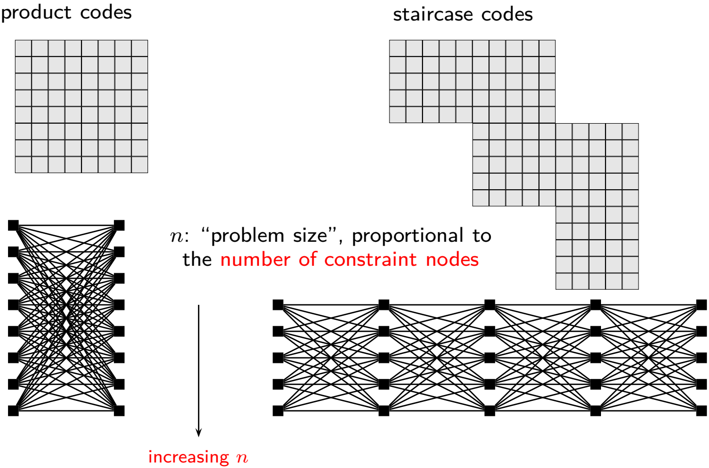

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Density Evolution

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- What happens asymptotically for $n \to \infty$?

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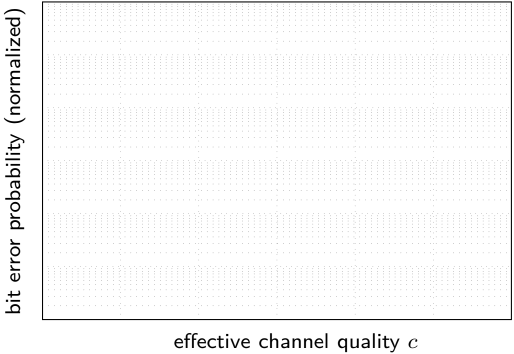

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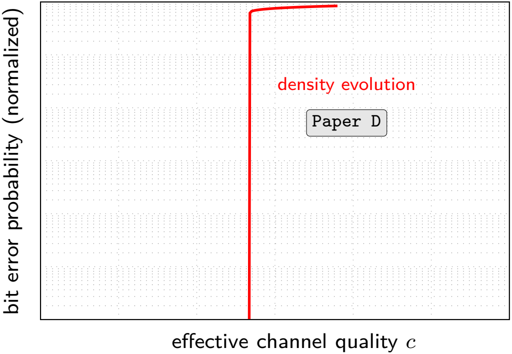

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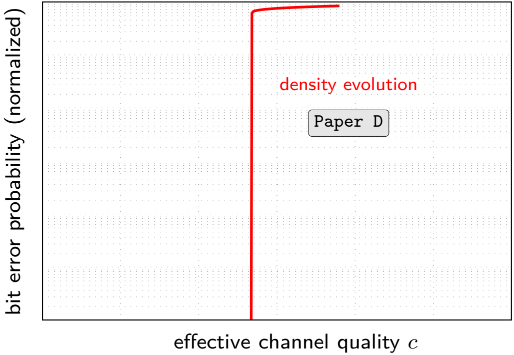

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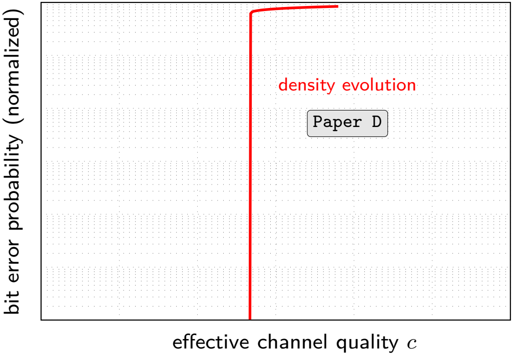

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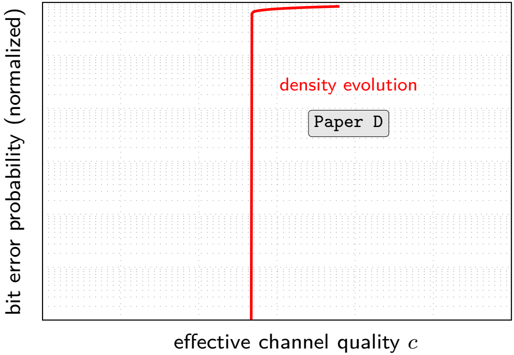

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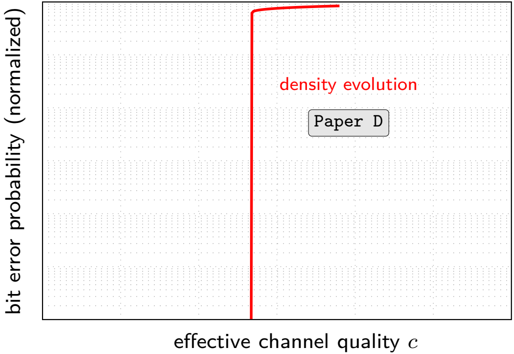

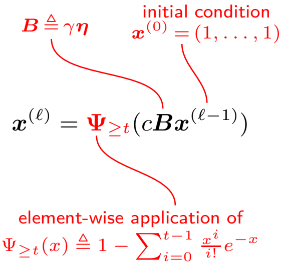

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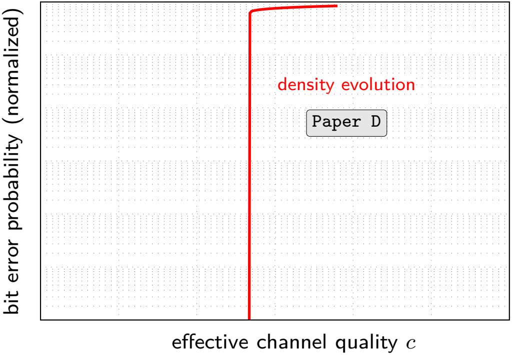

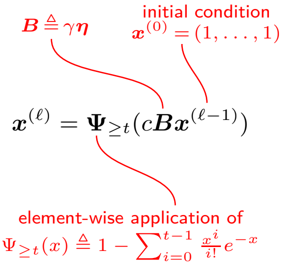

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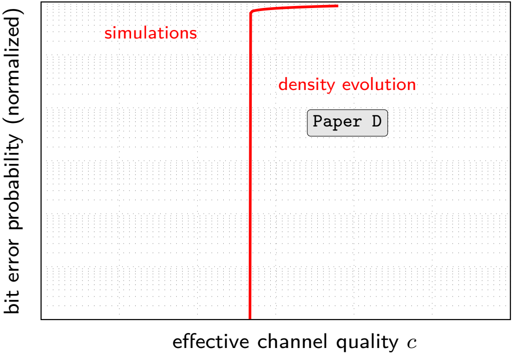

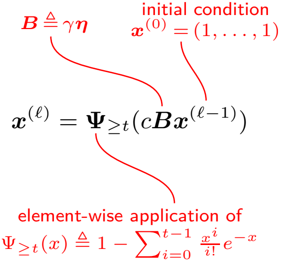

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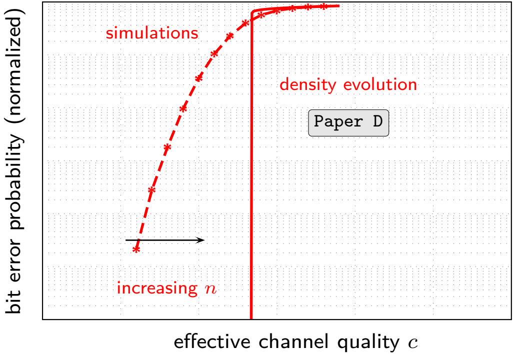

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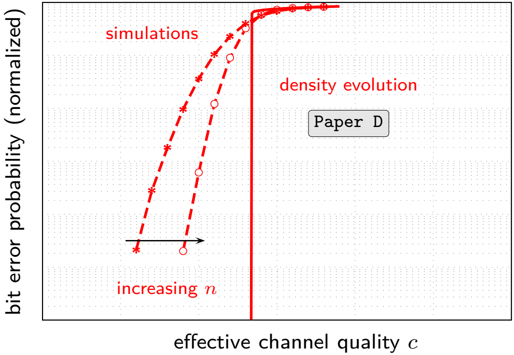

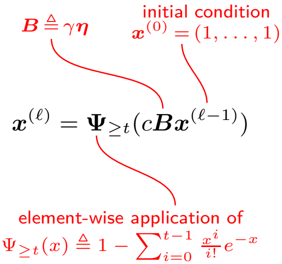

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Comparison of Deterministic and Random-Like Codes

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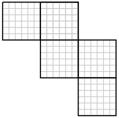
Deterministic



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$$\frac{1}{2}\begin{pmatrix} 0 & 1 & 0 & 0 & 0 & 0 \\ 1 & 0 & 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 & 0 & 1 \\ 0 & 0 & 0 & 0 & 1 & 0 \end{pmatrix}$$

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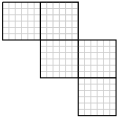

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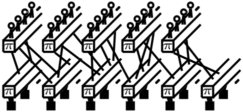
Random-Like [Jian et al., 2012]



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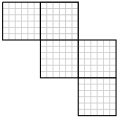

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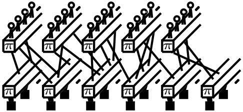

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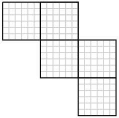

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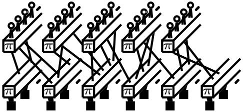

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- Equations have the same form $\implies$ similar performance

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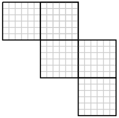

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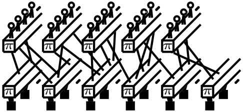

capacity-achieving at high rates over the binary symmetric channel

$$\frac{1}{4}\begin{pmatrix} 1 & 1 & 0 & 0 & 0 & 0 \\ 1 & 2 & 1 & 0 & 0 & 0 \\ 0 & 1 & 2 & 1 & 0 & 0 \\ 0 & 0 & 1 & 2 & 1 & 0 \\ 0 & 0 & 0 & 1 & 2 & 1 \\ 0 & 0 & 0 & 0 & 1 & 1 \end{pmatrix}$$

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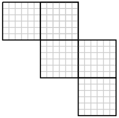

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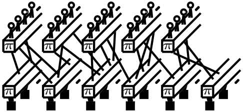

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- Equations have the same form $\Longrightarrow$ similar performance
- The performance of random-like codes (over the binary erasure channel) can be "emulated" with deterministic codes `Paper F`

Design and Analysis of Deterministic Codes

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- Several deterministic codes (including spatially-coupled versions) have been proposed for fiber-optical communications
- Rigorous asymptotic performance analysis over the binary erasure channel under iterative bounded-distance decoding possible
- Future work: extension to binary symmetric channel

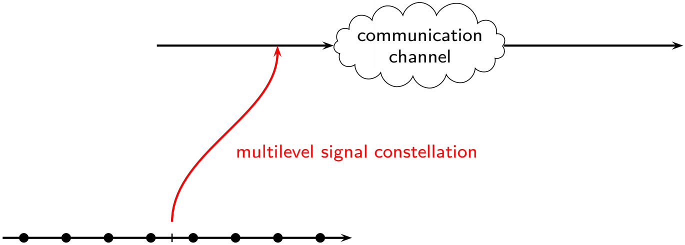
Spectrally-Efficient Communication

Large interest in analyzing and designing spectrally-efficient fiber-optical systems ([Essiambre et al., 2010], [Smith and Kschischang, 2010], [Schmalen et al., 2013], [Beygi et al., 2014], . . .)

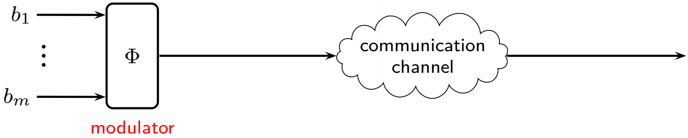
Spectrally-Efficient Communication

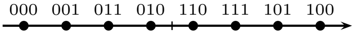
communication
channel

Spectrally-Efficient Communication



Spectrally-Efficient Communication





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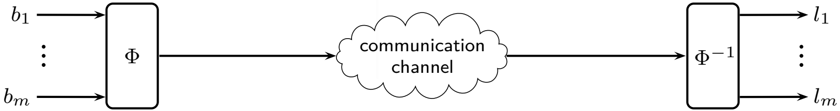
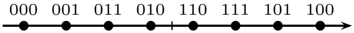


modulator

demodulator:
log-likelihood ratios (LLR)
("soft" information)



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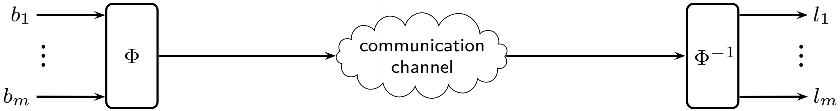

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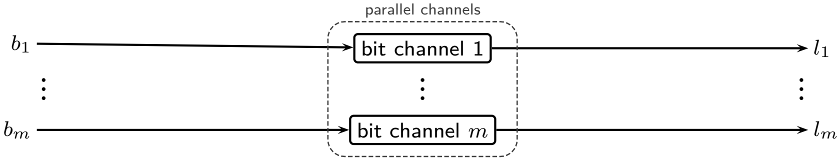

- Approximate setup: parallel channels with different qualities (constellation size determines the number of channels)

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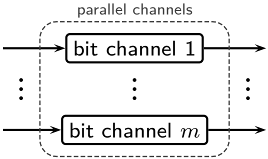

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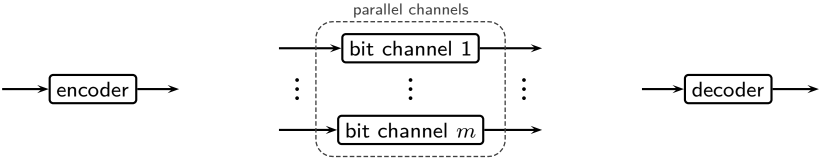
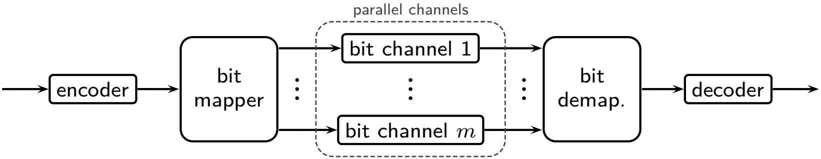


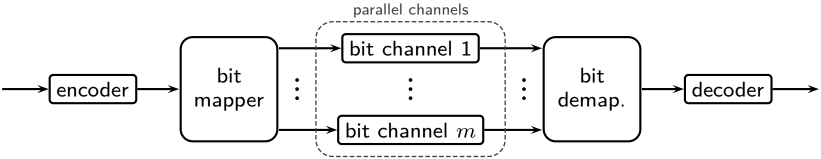
- Approximate setup: parallel channels with different qualities (constellation size determines the number of channels)
- Fix one binary encoder/decoder pair

Spectrally-Efficient Communication



- Approximate setup: parallel channels with different qualities (constellation size determines the number of channels)
- Fix one binary encoder/decoder pair
- Bit mapper determines the allocation of coded bits to the channels

Spectrally-Efficient Communication



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Problem Formulation ([Richter et al., 2007], [Cheng et al., 2012], . . .)

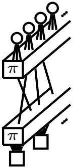
Optimize the bit mapper for a given code and signal constellation

Protograph LDPC Codes

Protograph LDPC Codes

- Compact representation of a large random-like graph [Thorpe, 2005]

Protograph LDPC Codes



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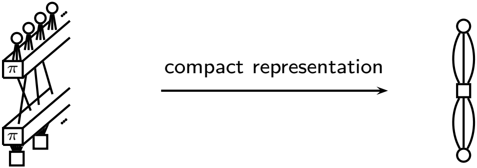
Protograph LDPC Codes



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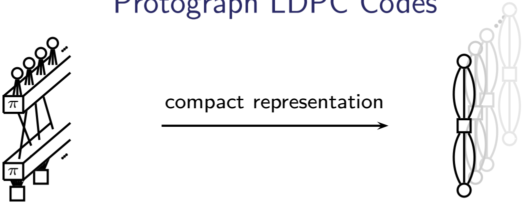
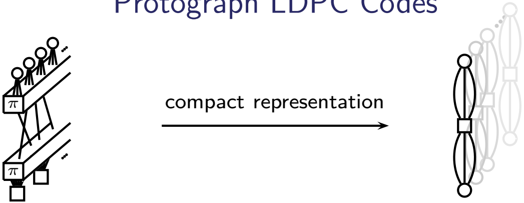
Protograph LDPC Codes



compact representation

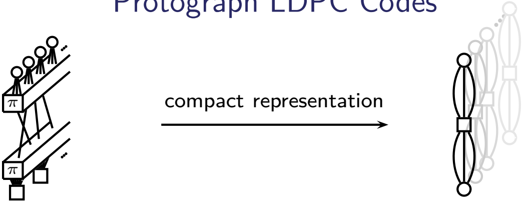
- <span>Compact representation</span> of a large random-like graph [Thorpe, 2005]

Protograph LDPC Codes



- Compact representation of a large random-like graph [Thorpe, 2005]
- We propose a bit mapper optimization technique that is more flexible than previous approaches in [Divsalar and Jones, 2005], [Jin et al., 2010], [Van Nguyen et al., 2011]

Protograph LDPC Codes



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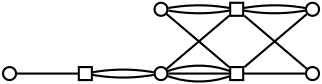
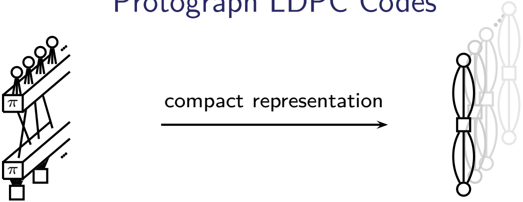
AR4JA codes [Divsalar et al., 2005]



Paper A

Protograph LDPC Codes



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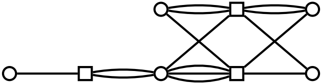
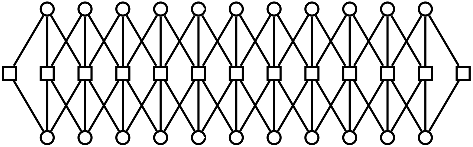


Paper A

spatially-coupled LDPC codes

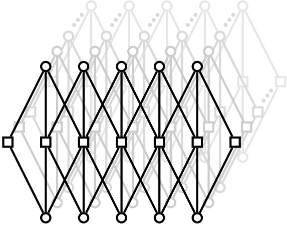


Paper A, B

Terminated

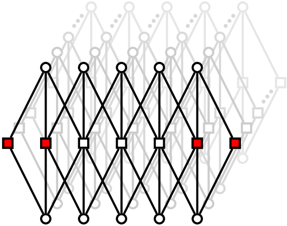
Terminated

protograph



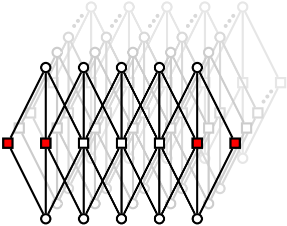
Terminated

protograph

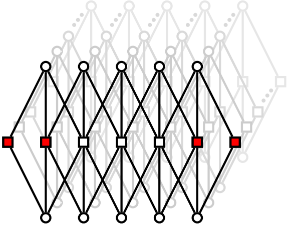


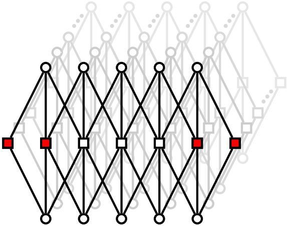
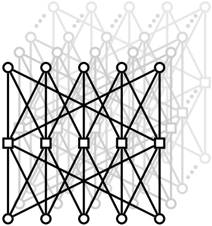
graph irregularity yes (boundaries)

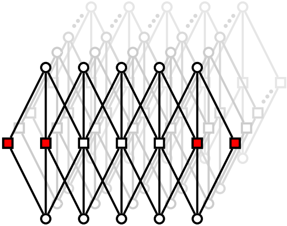
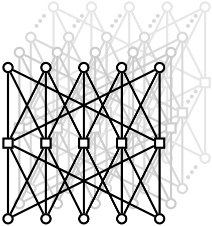
Terminated

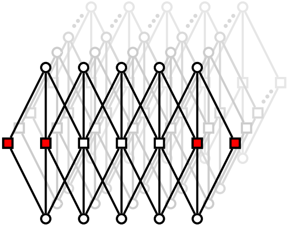
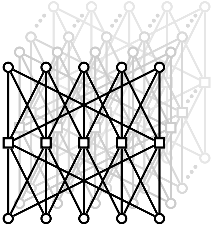
protograph	
graph irregularity	yes (boundaries)
wave effect	yes (capacity-approaching)

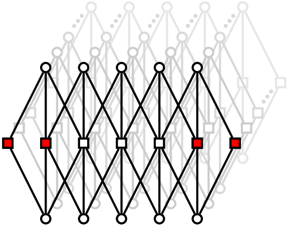
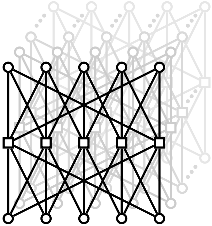
Terminated

protograph	
graph irregularity	yes (boundaries)
wave effect	yes (capacity-approaching)
rate loss	yes

	Terminated	Tail-biting
protograph		
graph irregularity	yes (boundaries)	
wave effect	yes (capacity-approaching)	
rate loss	yes	

	Terminated	Tail-biting
protograph		
graph irregularity	yes (boundaries)	no
wave effect	yes (capacity-approaching)	
rate loss	yes	

	Terminated	Tail-biting
protograph		
graph irregularity	yes (boundaries)	no
wave effect	yes (capacity-approaching)	no (comparable to regular LDPC)
rate loss	yes	

	Terminated	Tail-biting
protograph		
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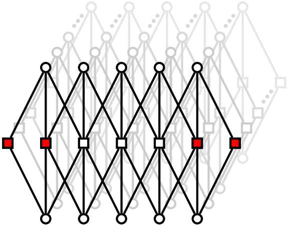
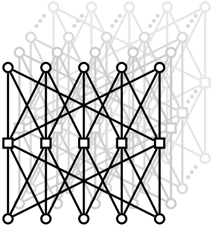
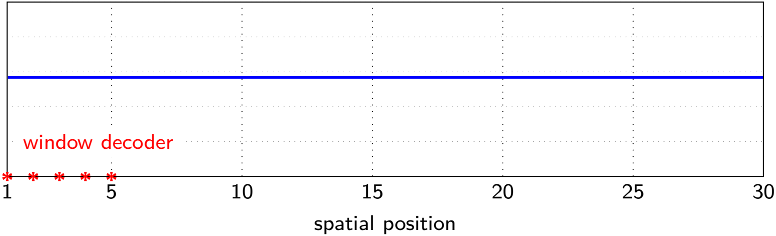
	Terminated	Tail-biting
protograph		
graph irregularity	yes (boundaries)	no
wave effect	yes (capacity-approaching)	no (comparable to regular LDPC)
rate loss	yes	no

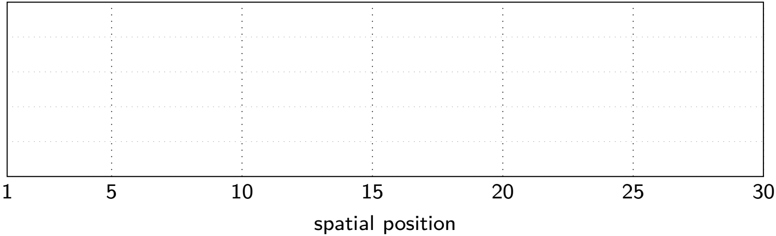
Idea: Use unequal error protection of a multilevel signal constellation to induce wave-like decoding behavior for tail-biting codes.

Optimization Result: Decoding Behavior

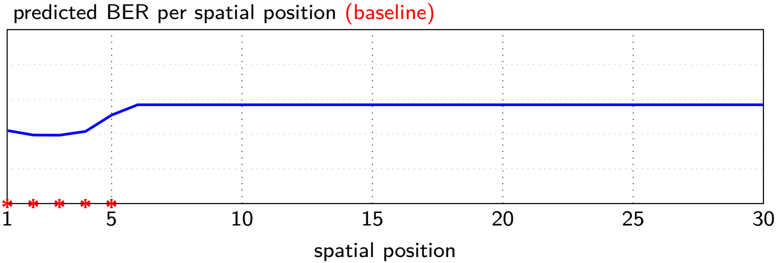
predicted BER per spatial position (baseline)

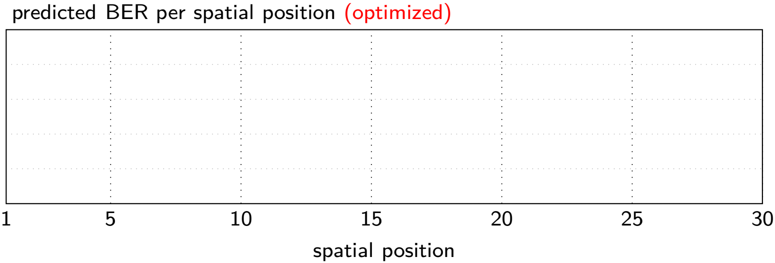


predicted BER per spatial position (optimized)



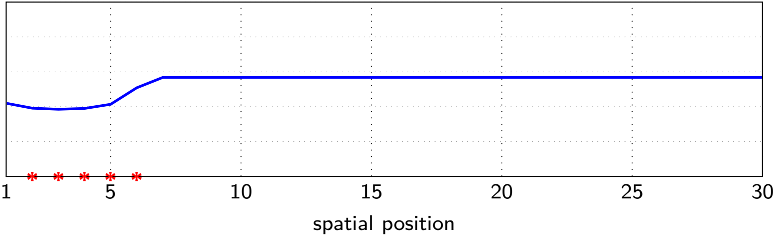
Optimization Result: Decoding Behavior



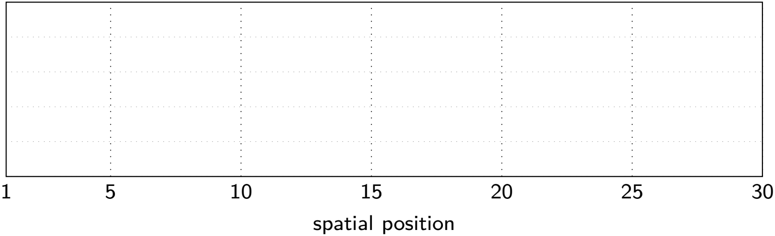


Optimization Result: Decoding Behavior

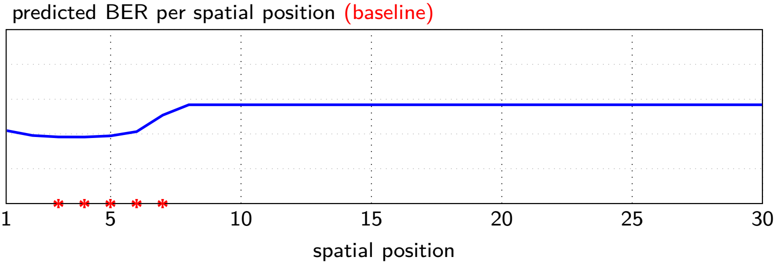
predicted BER per spatial position (baseline)

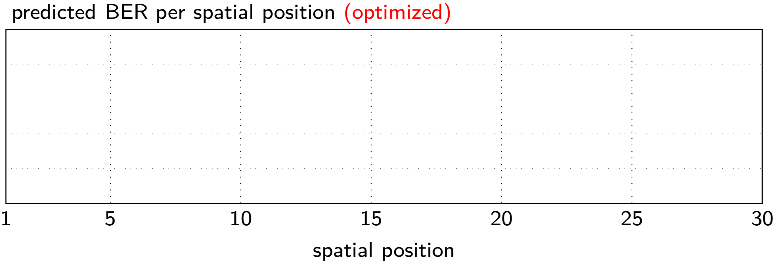


predicted BER per spatial position (optimized)



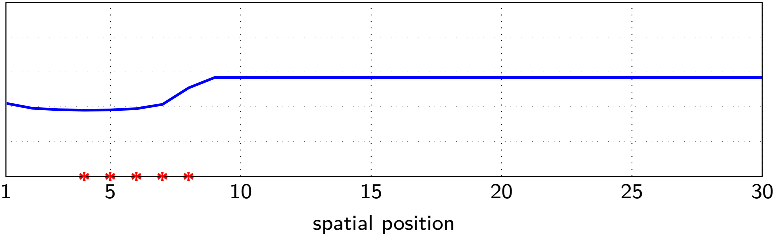
Optimization Result: Decoding Behavior



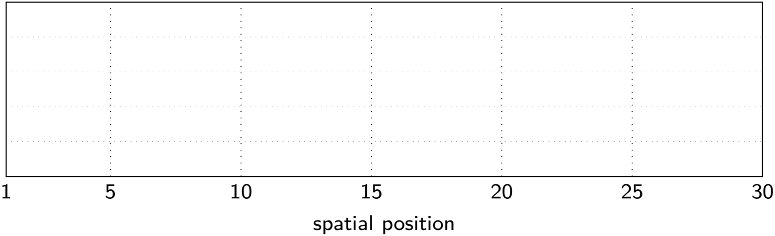


Optimization Result: Decoding Behavior

predicted BER per spatial position (baseline)



predicted BER per spatial position (optimized)



Optimization Result: Decoding Behavior

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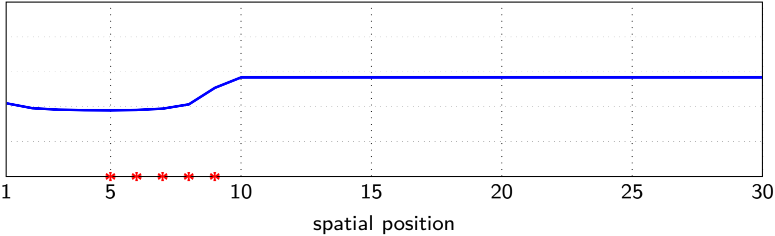
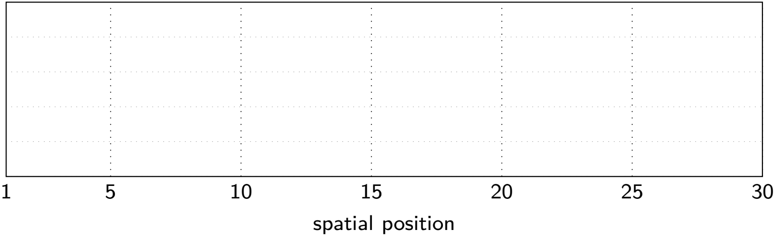
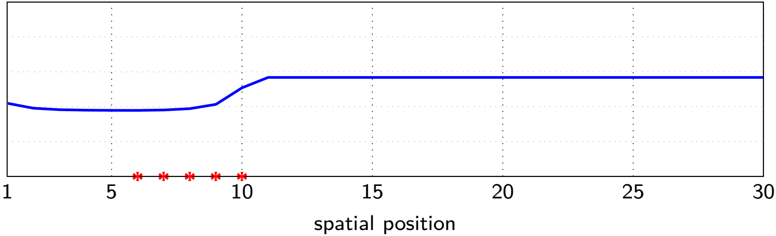


predicted BER per spatial position (optimized)

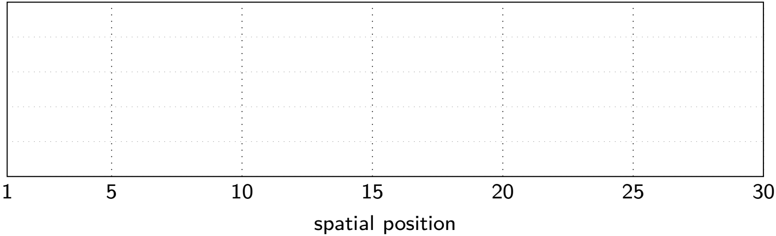


Optimization Result: Decoding Behavior

predicted BER per spatial position (baseline)



predicted BER per spatial position (optimized)



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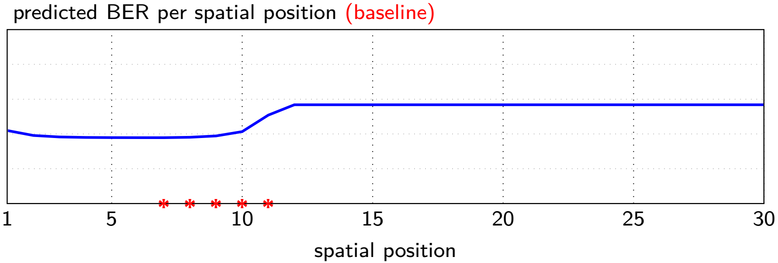

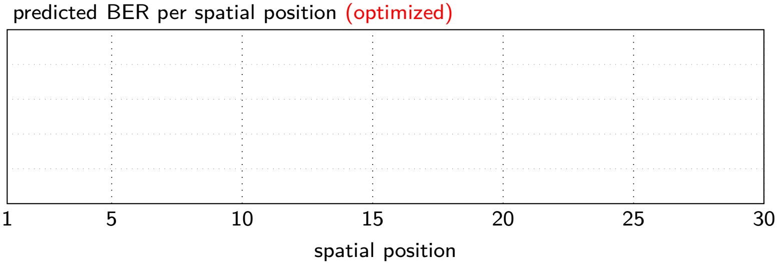

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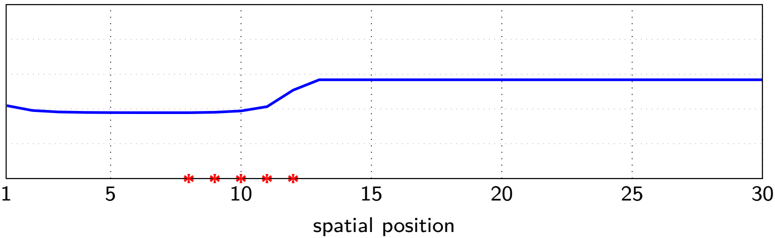
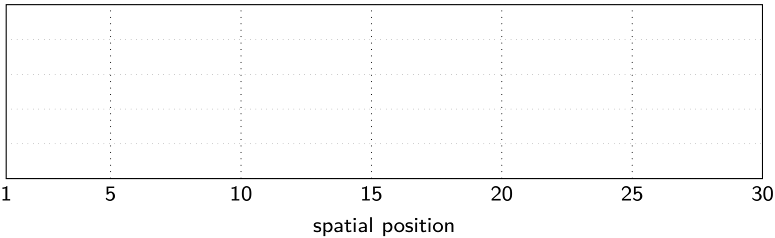
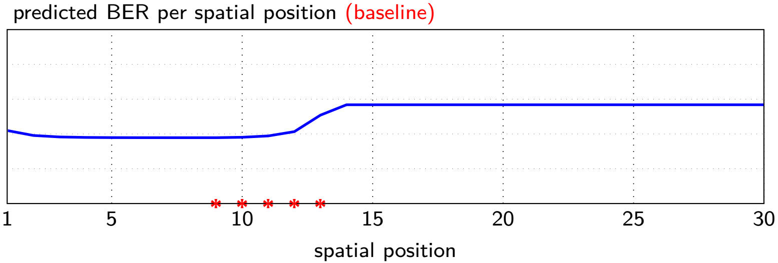
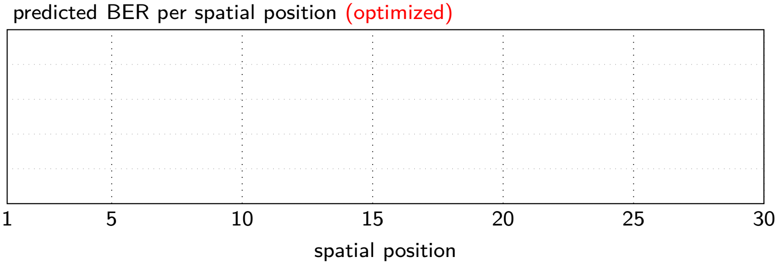


predicted BER per spatial position (optimized)



Optimization Result: Decoding Behavior





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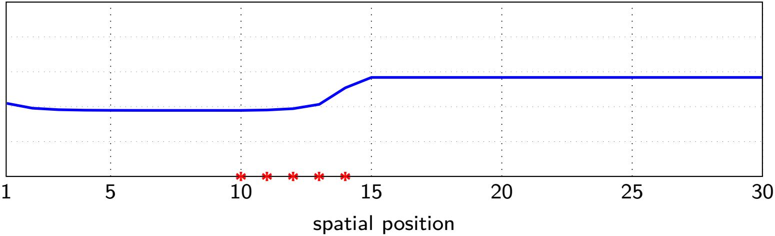
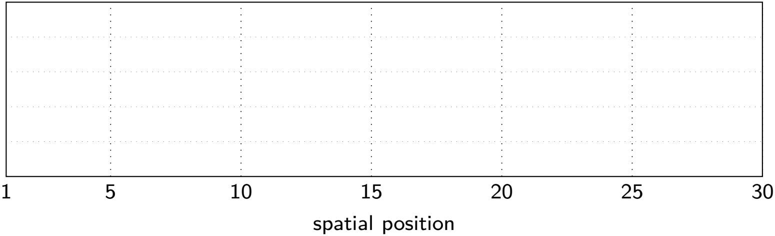


predicted BER per spatial position (optimized)



Optimization Result: Decoding Behavior

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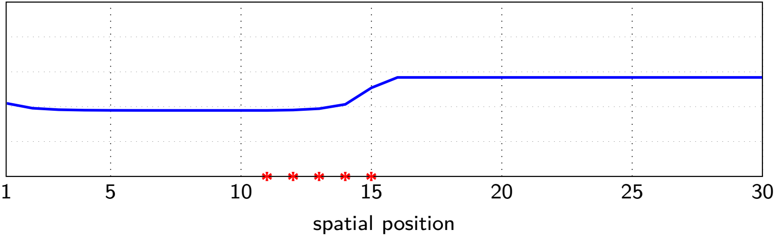
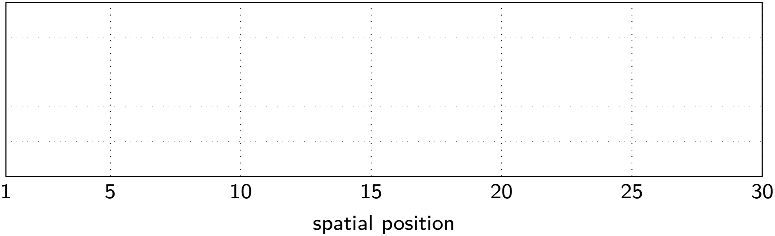


predicted BER per spatial position (optimized)



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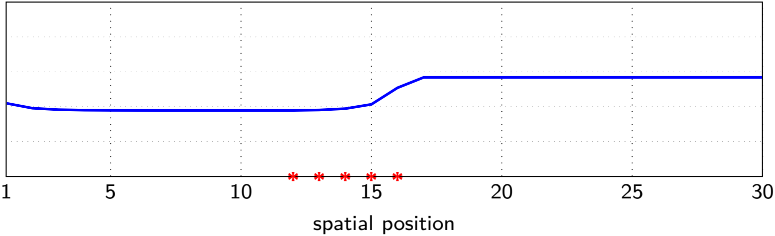

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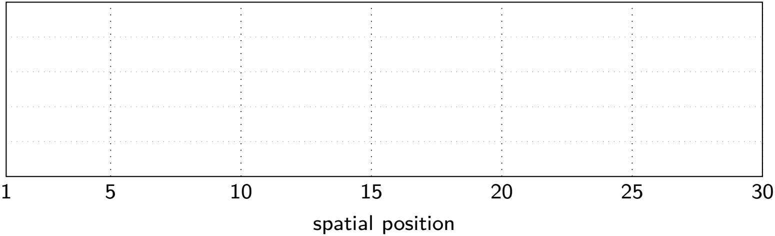

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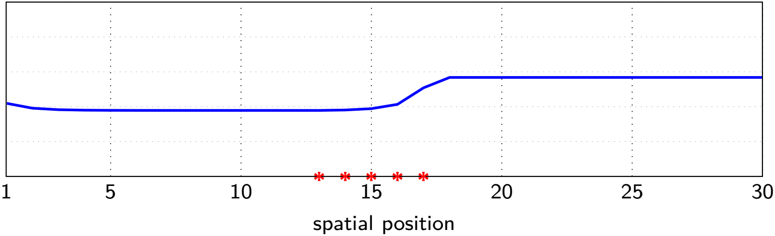
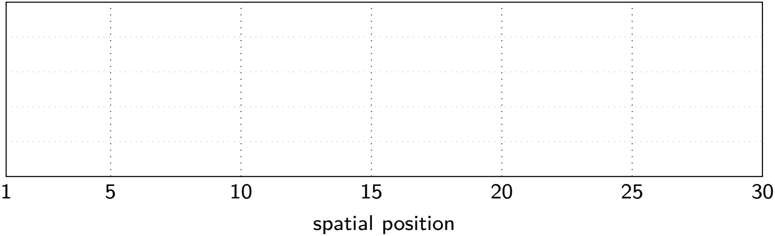


predicted BER per spatial position (optimized)



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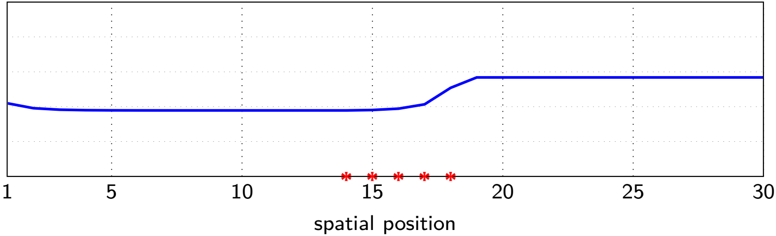
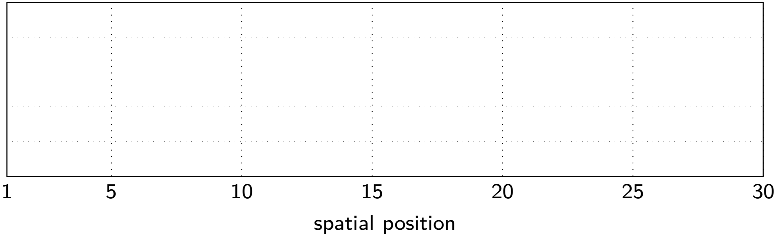


predicted BER per spatial position (optimized)



Optimization Result: Decoding Behavior

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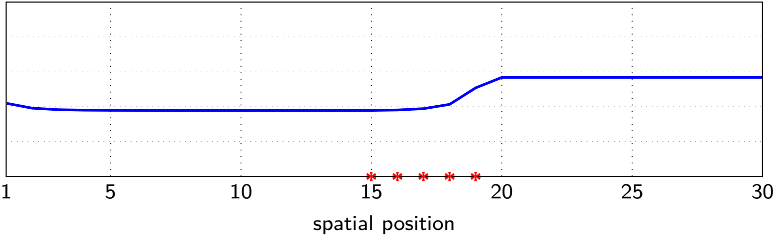
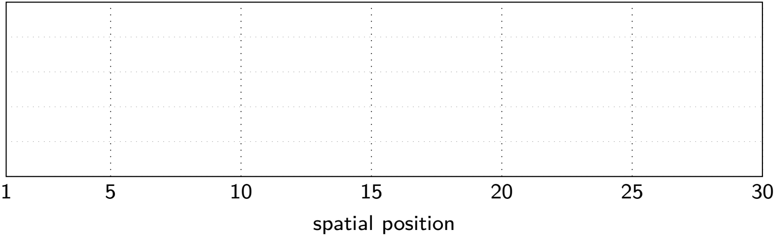


predicted BER per spatial position (optimized)



Optimization Result: Decoding Behavior

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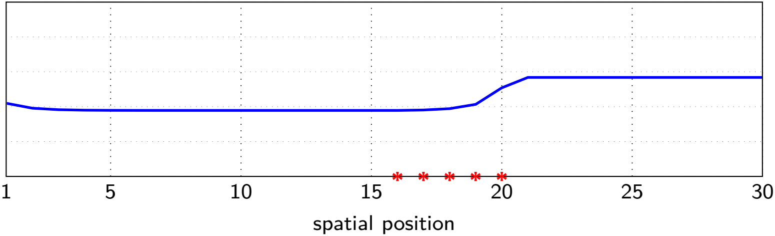
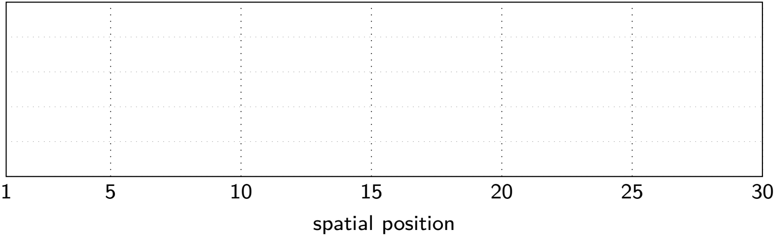


predicted BER per spatial position (optimized)



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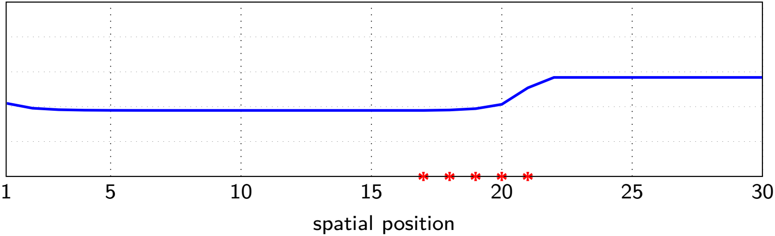

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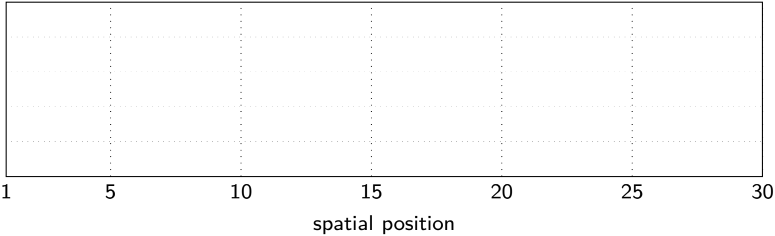

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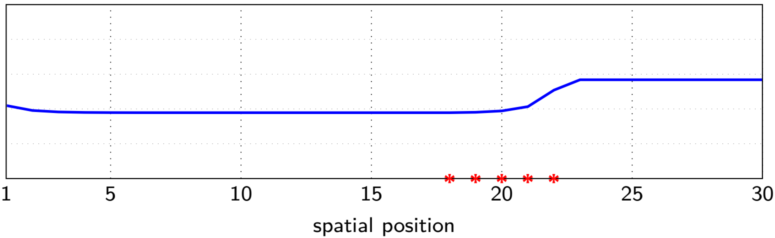
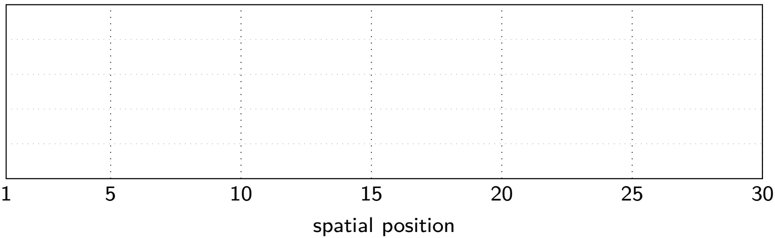
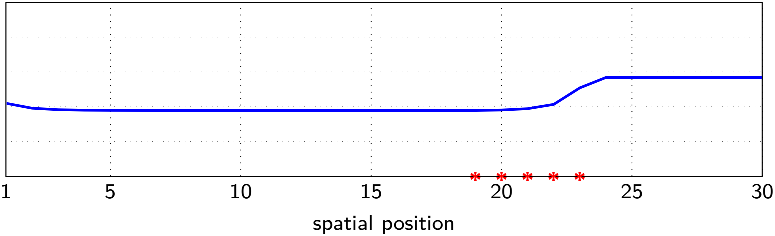


predicted BER per spatial position (optimized)

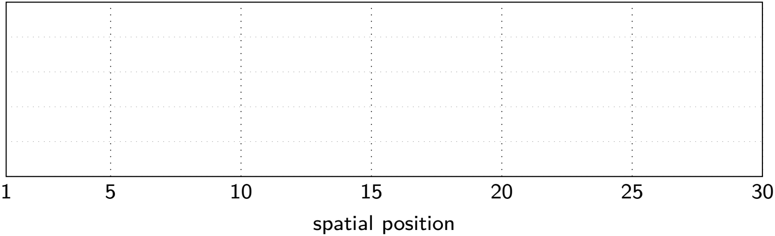


Optimization Result: Decoding Behavior

predicted BER per spatial position (baseline)

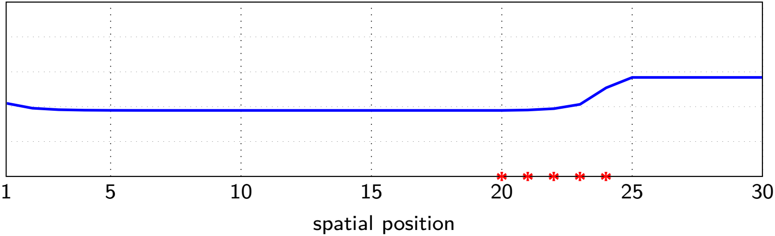


predicted BER per spatial position (optimized)

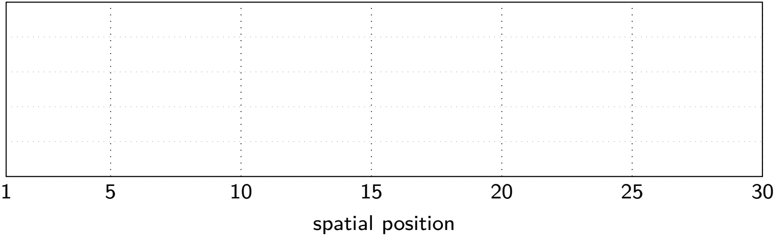


Optimization Result: Decoding Behavior

predicted BER per spatial position (baseline)

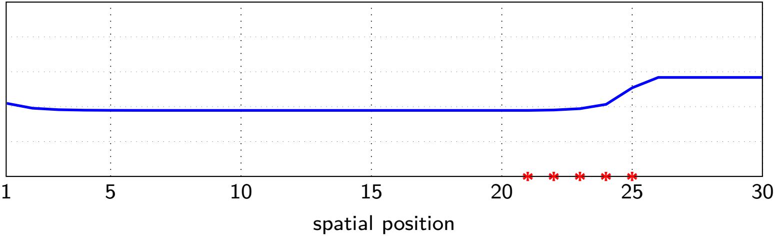


predicted BER per spatial position (optimized)

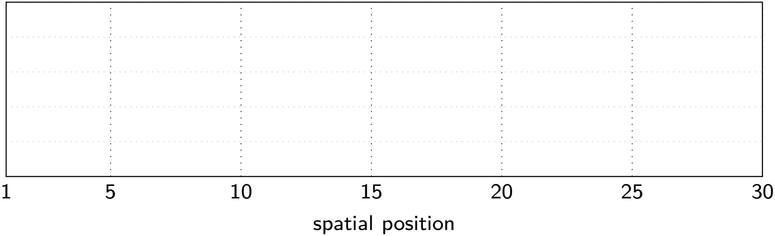


Optimization Result: Decoding Behavior

predicted BER per spatial position (baseline)

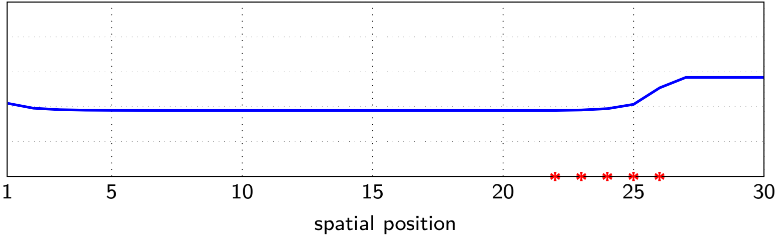


predicted BER per spatial position (optimized)

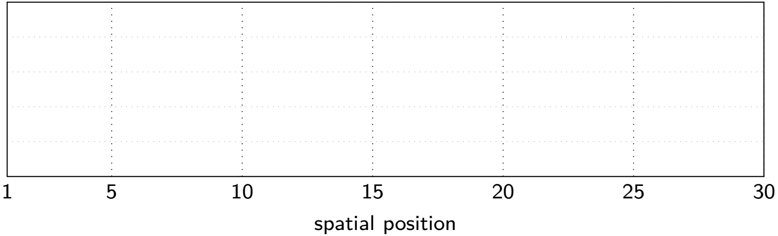


Optimization Result: Decoding Behavior

predicted BER per spatial position (baseline)

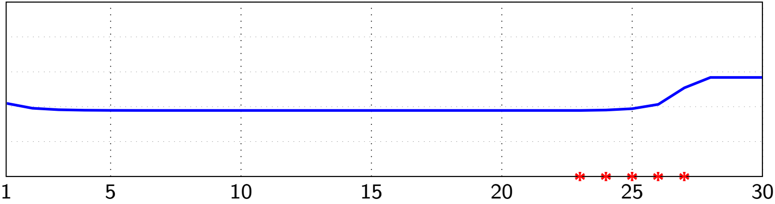


predicted BER per spatial position (optimized)



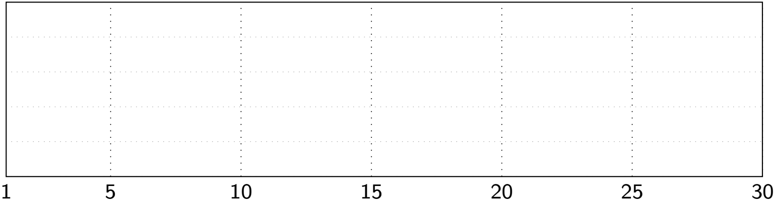
Optimization Result: Decoding Behavior

predicted BER per spatial position (baseline)



spatial position

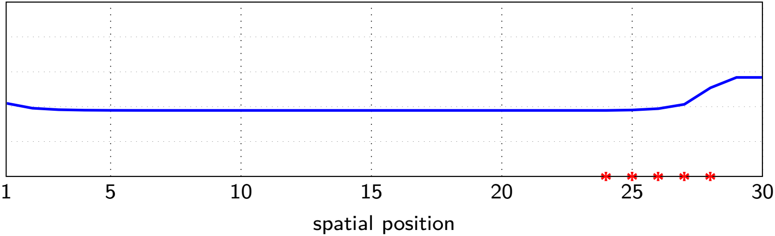
predicted BER per spatial position (optimized)



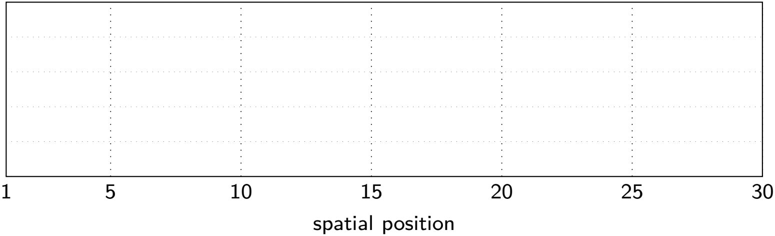
spatial position

Optimization Result: Decoding Behavior

predicted BER per spatial position (baseline)

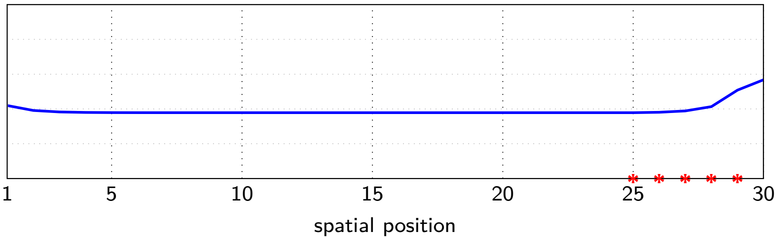


predicted BER per spatial position (optimized)

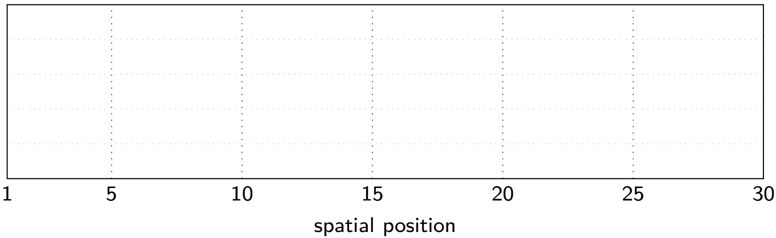


Optimization Result: Decoding Behavior

predicted BER per spatial position (baseline)

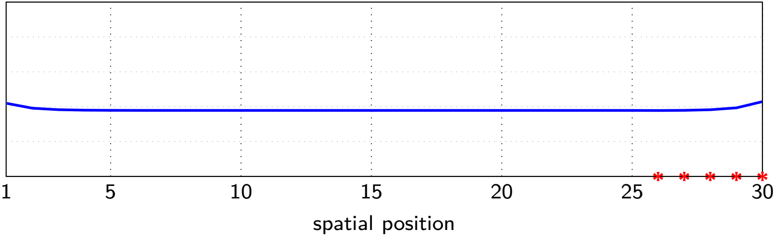


predicted BER per spatial position (optimized)

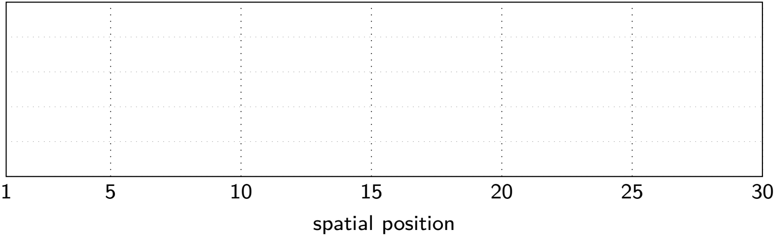


Optimization Result: Decoding Behavior

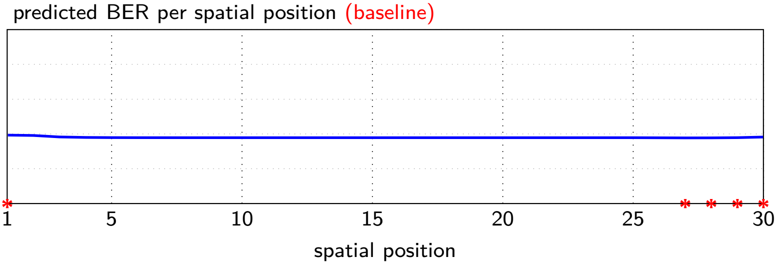
predicted BER per spatial position (baseline)

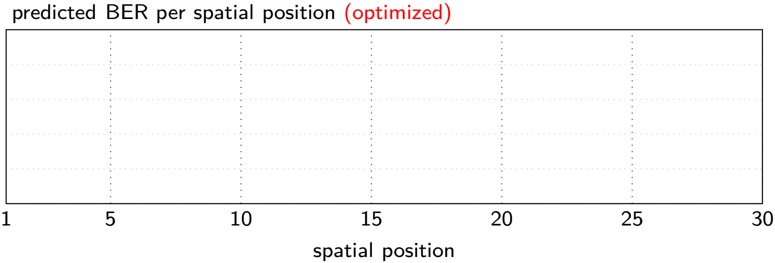


predicted BER per spatial position (optimized)

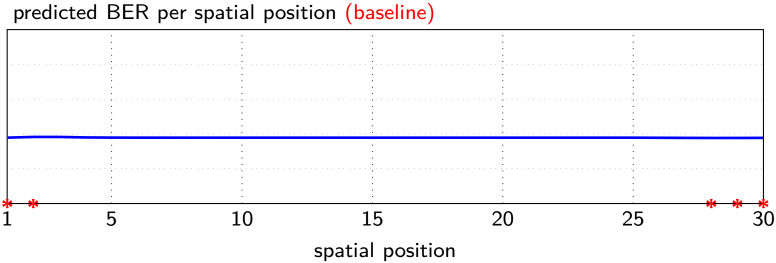


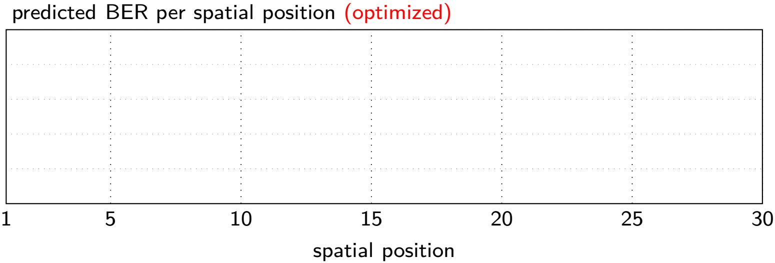
Optimization Result: Decoding Behavior





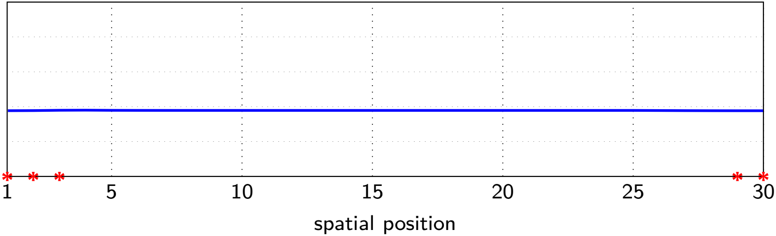
Optimization Result: Decoding Behavior



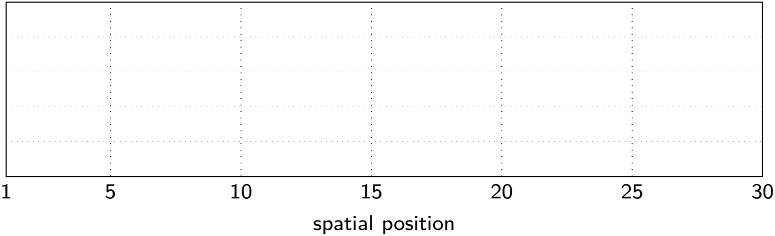


Optimization Result: Decoding Behavior

predicted BER per spatial position (baseline)

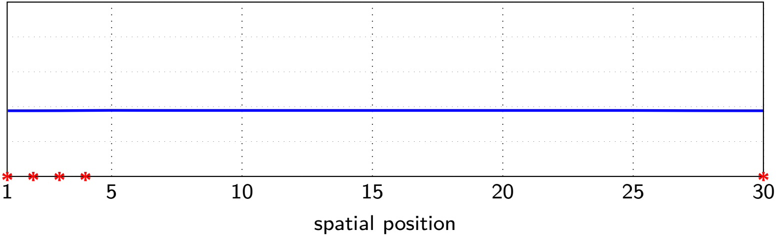


predicted BER per spatial position (optimized)

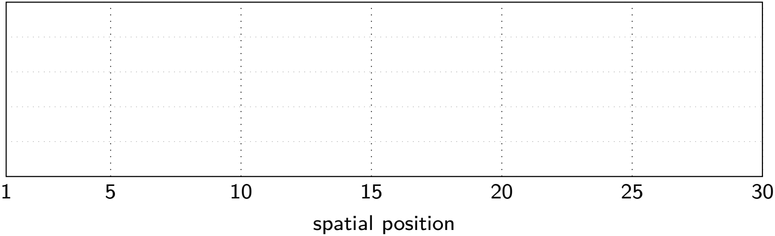


Optimization Result: Decoding Behavior

predicted BER per spatial position (baseline)

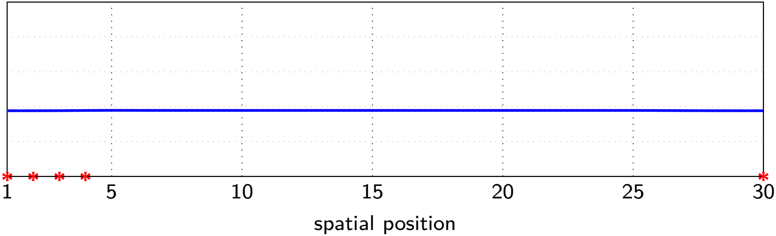


predicted BER per spatial position (optimized)



Optimization Result: Decoding Behavior

predicted BER per spatial position (baseline)



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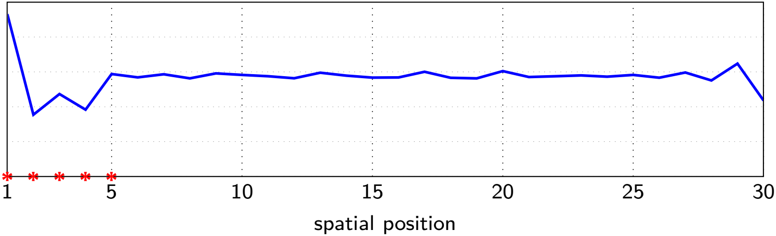
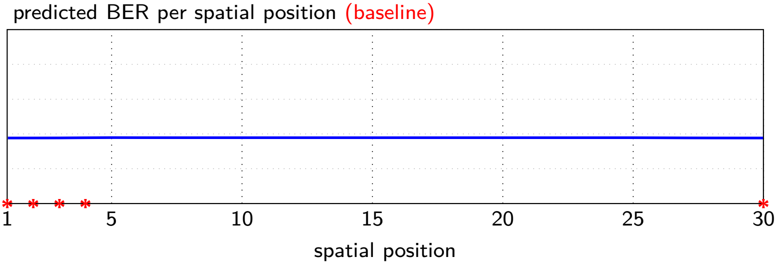
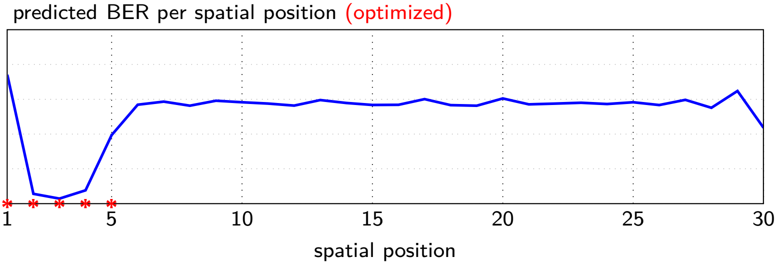
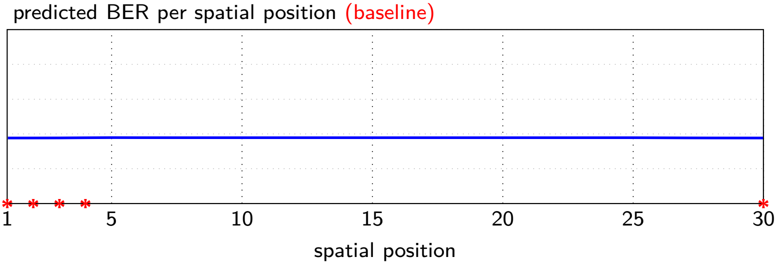


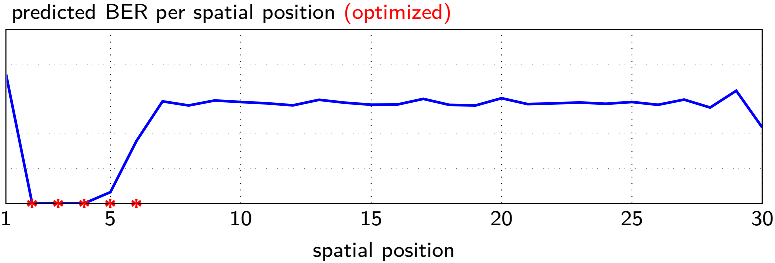
Optimization Result: Decoding Behavior



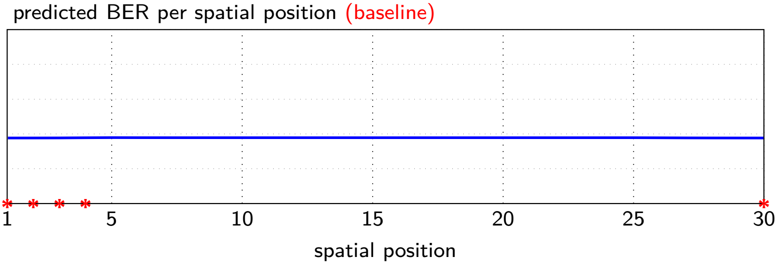


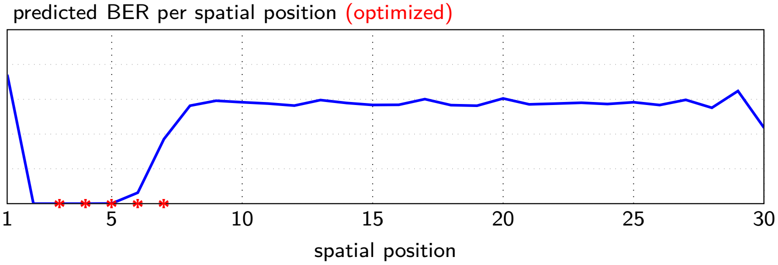
Optimization Result: Decoding Behavior





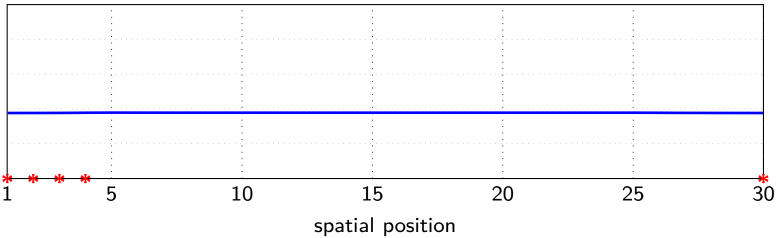
Optimization Result: Decoding Behavior



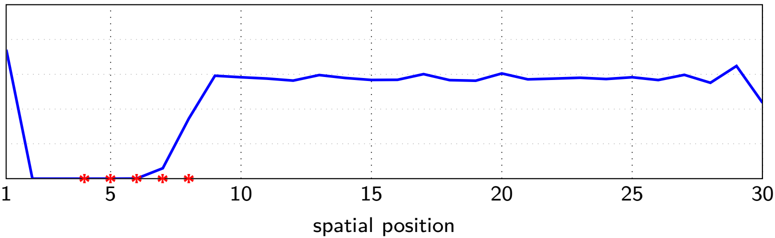


Optimization Result: Decoding Behavior

predicted BER per spatial position (baseline)



predicted BER per spatial position (optimized)



Optimization Result: Decoding Behavior

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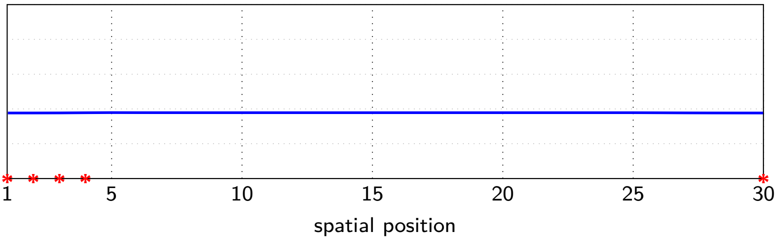

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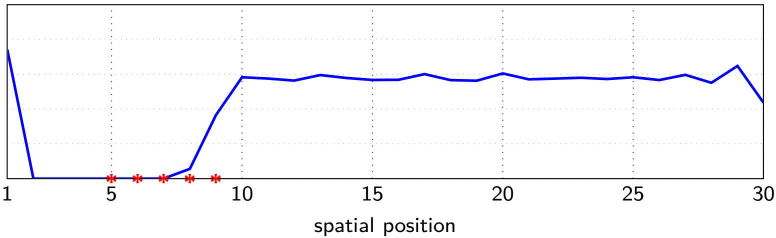
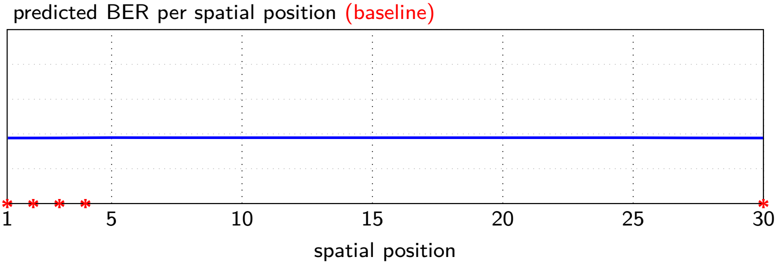
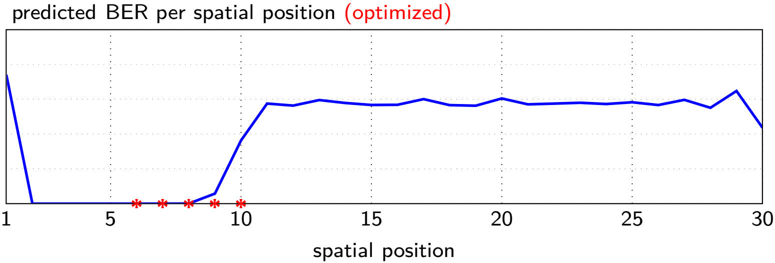


Optimization Result: Decoding Behavior





Optimization Result: Decoding Behavior

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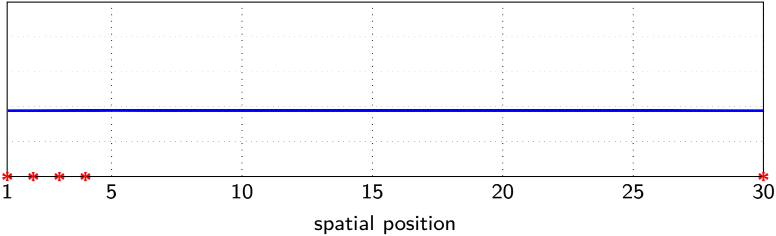

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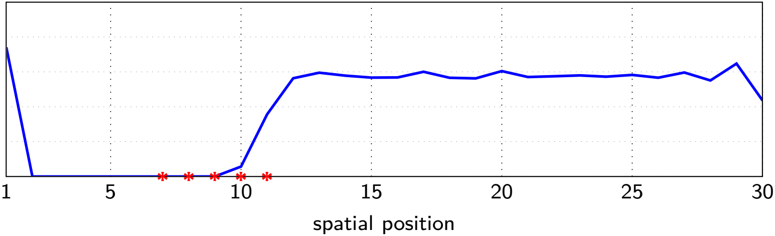
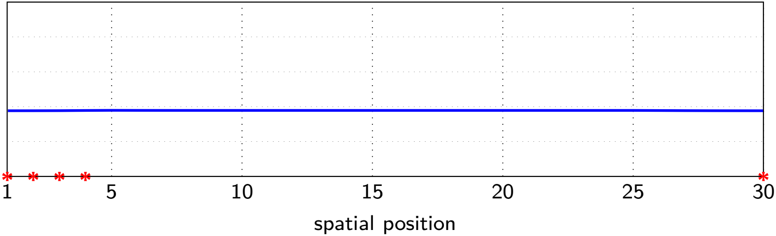


Optimization Result: Decoding Behavior

predicted BER per spatial position (baseline)



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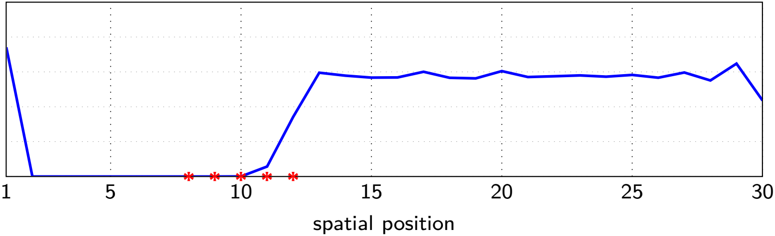
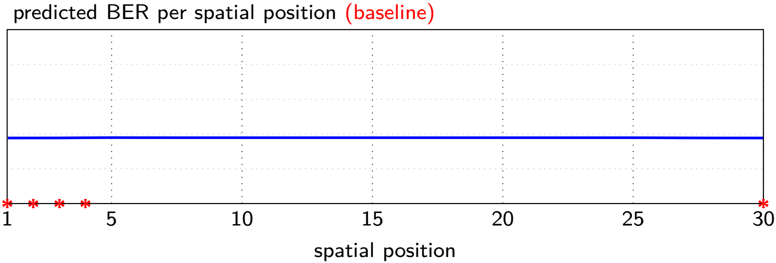
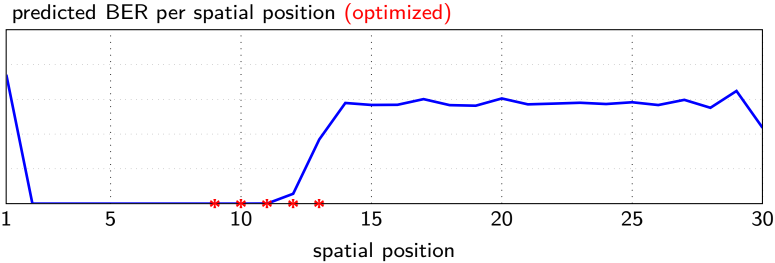


Optimization Result: Decoding Behavior





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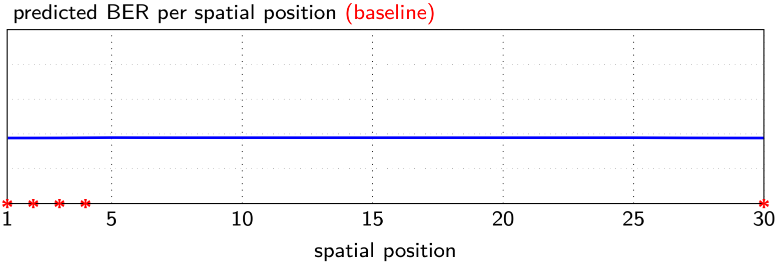
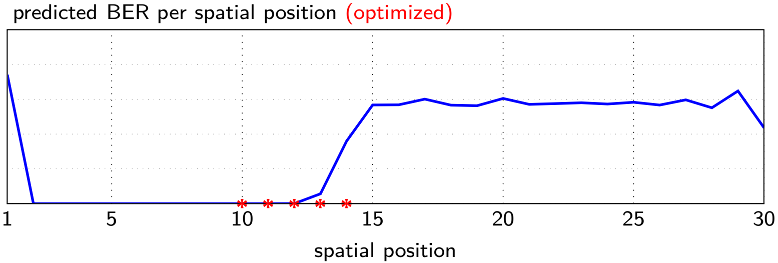
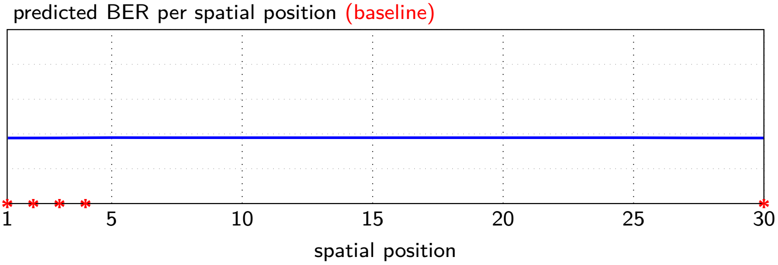
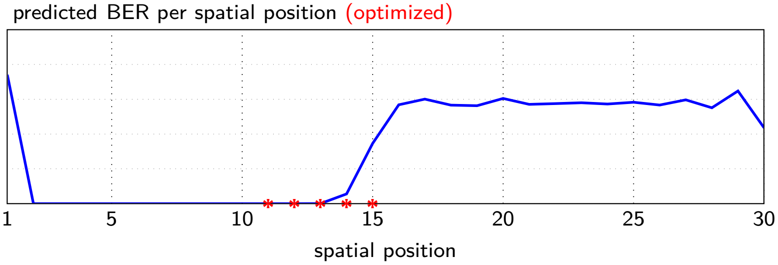




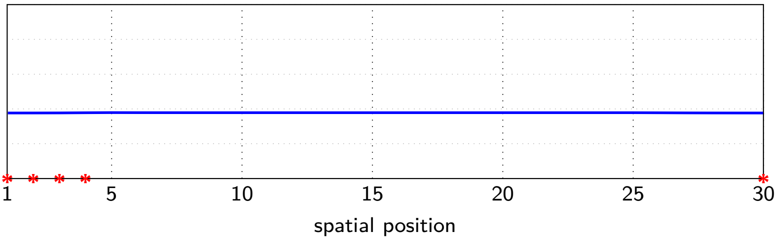
Optimization Result: Decoding Behavior



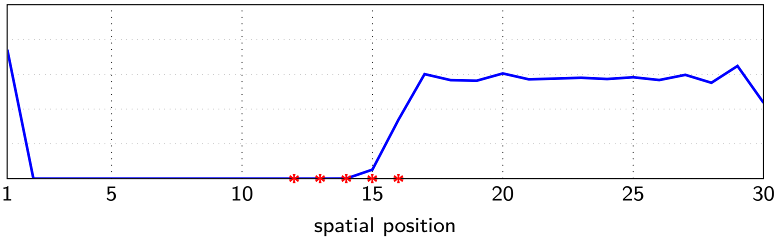


Optimization Result: Decoding Behavior

predicted BER per spatial position (baseline)

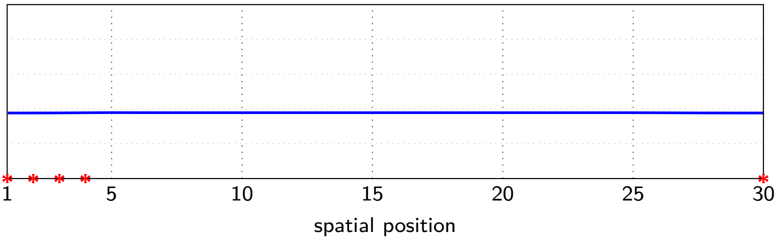


predicted BER per spatial position (optimized)

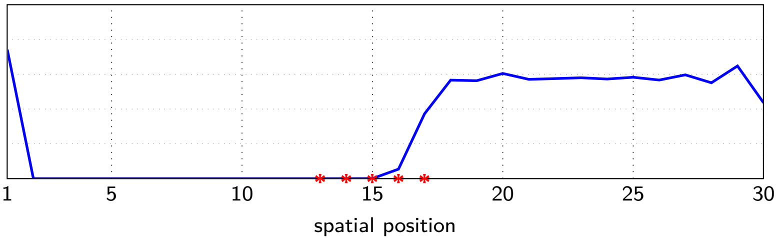


Optimization Result: Decoding Behavior

predicted BER per spatial position (baseline)

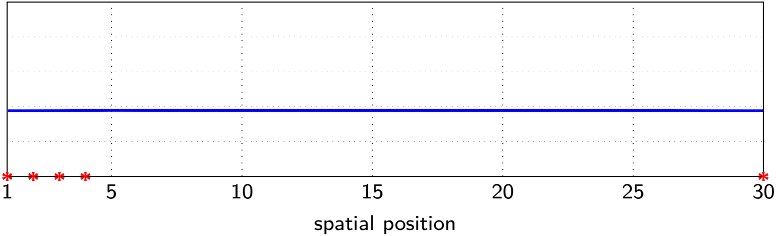


predicted BER per spatial position (optimized)

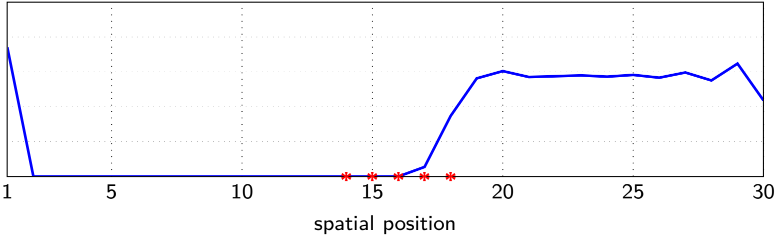


Optimization Result: Decoding Behavior

predicted BER per spatial position (baseline)



predicted BER per spatial position (optimized)



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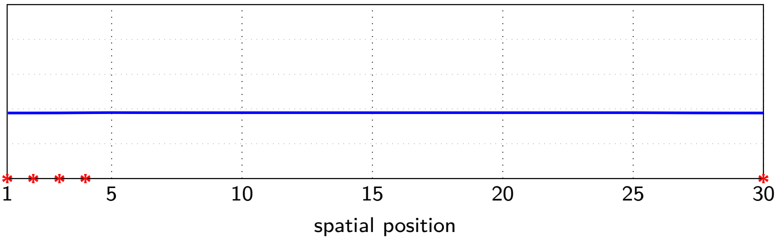

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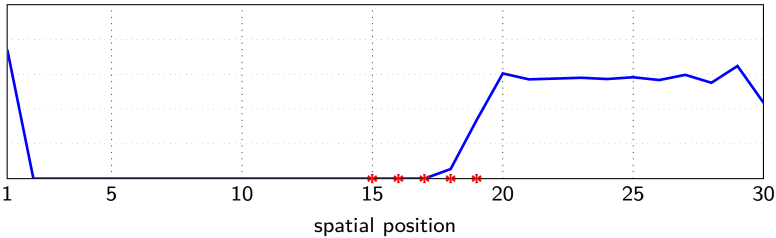
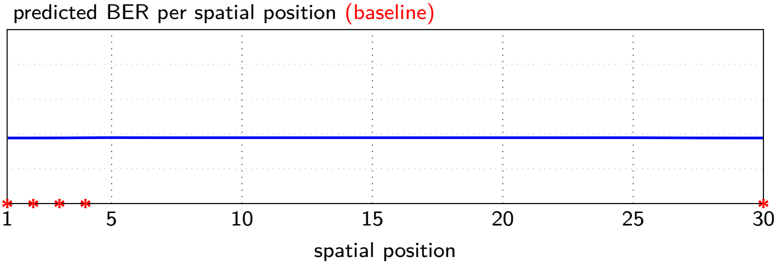
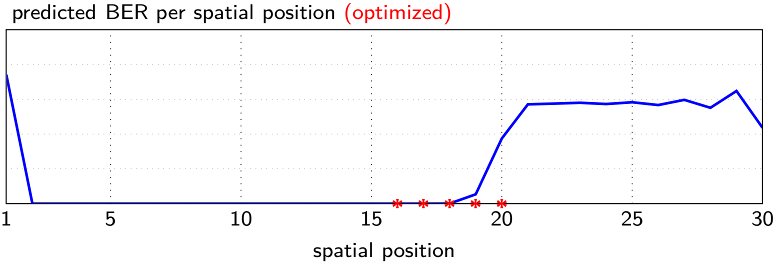
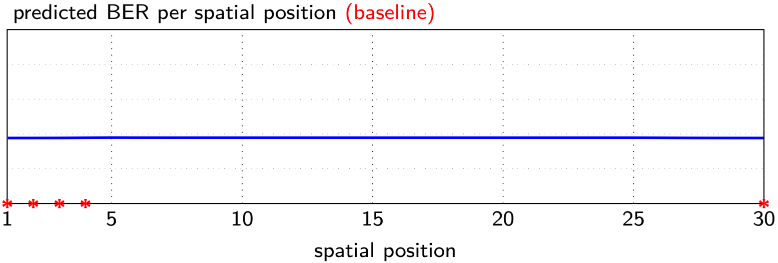


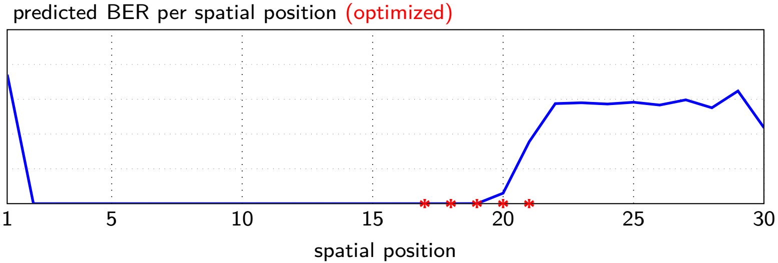
Optimization Result: Decoding Behavior





Optimization Result: Decoding Behavior





Optimization Result: Decoding Behavior

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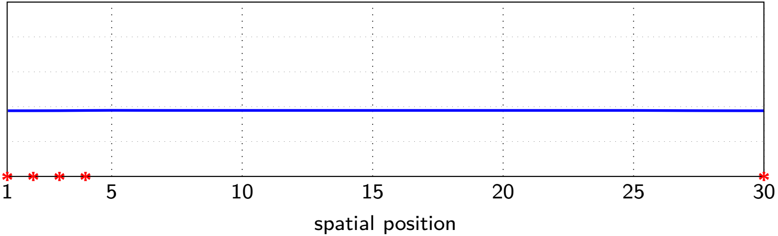
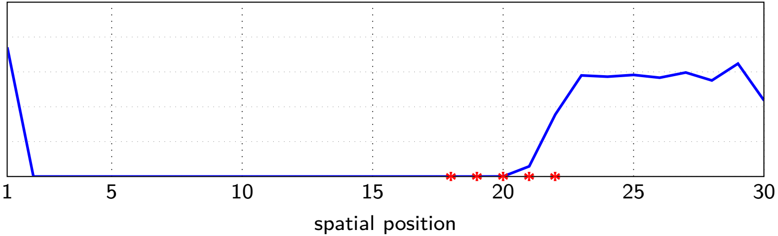
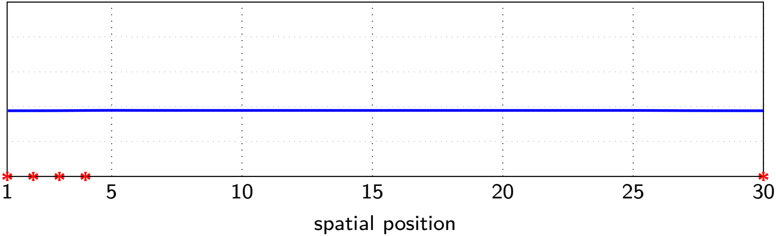


predicted BER per spatial position (optimized)

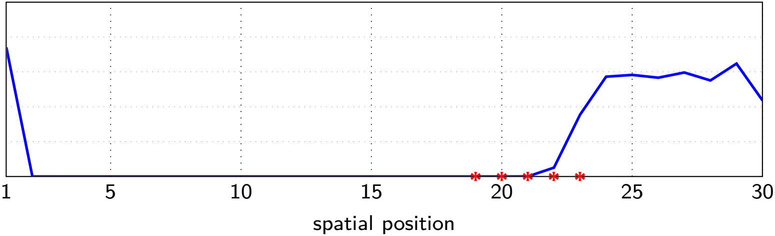


Optimization Result: Decoding Behavior

predicted BER per spatial position (baseline)

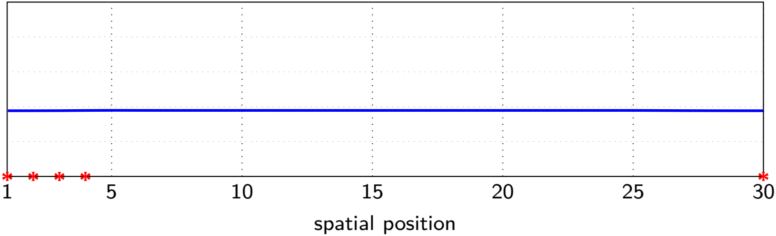


predicted BER per spatial position (optimized)



Optimization Result: Decoding Behavior

predicted BER per spatial position (baseline)



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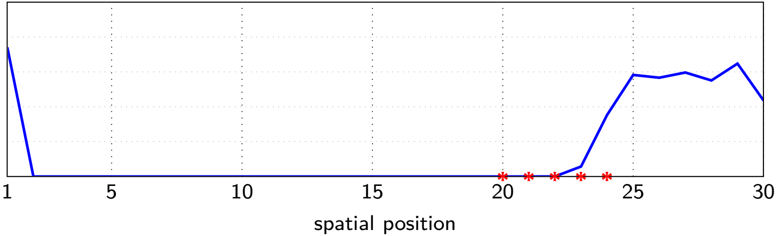
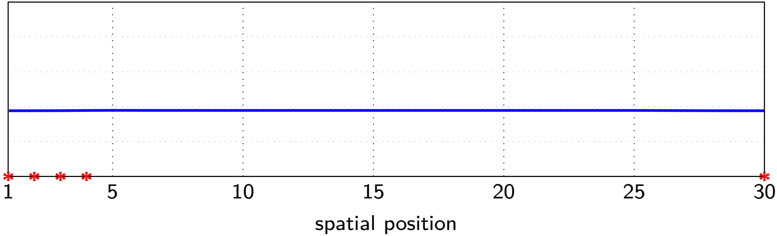


Optimization Result: Decoding Behavior

predicted BER per spatial position (baseline)



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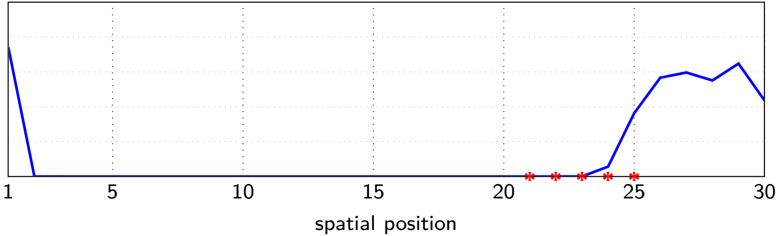

Optimization Result: Decoding Behavior

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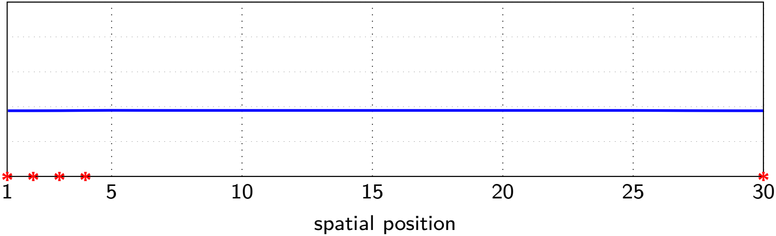
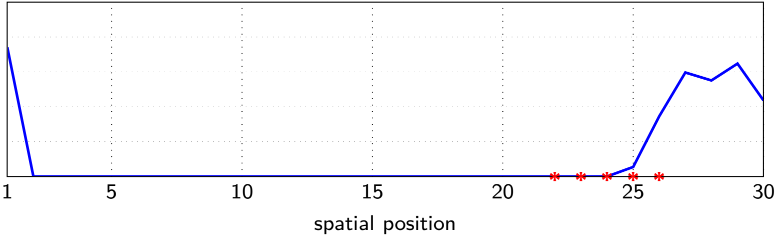
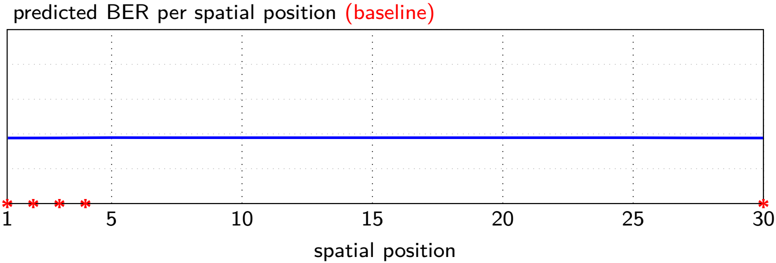
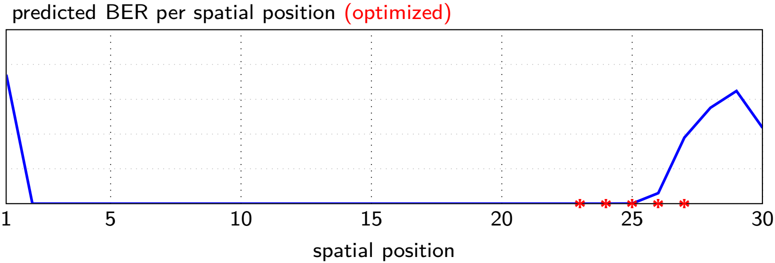


predicted BER per spatial position (optimized)



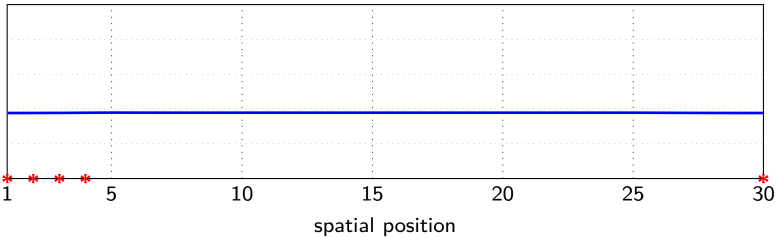
Optimization Result: Decoding Behavior





Optimization Result: Decoding Behavior

predicted BER per spatial position (baseline)



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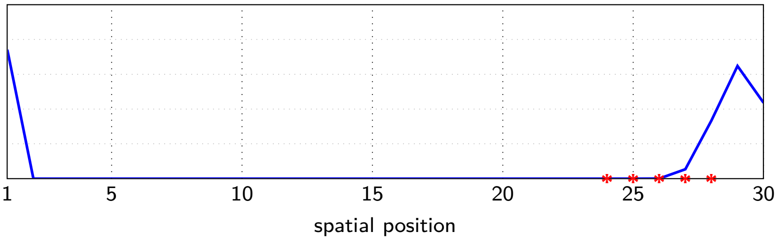

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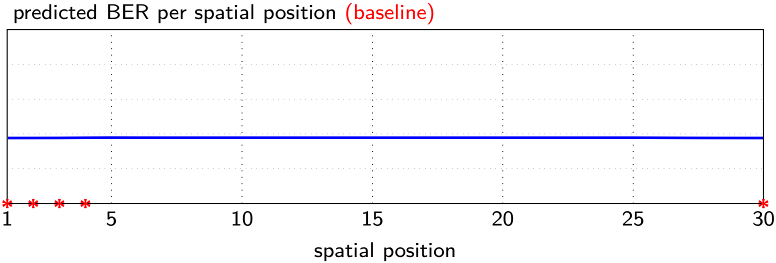

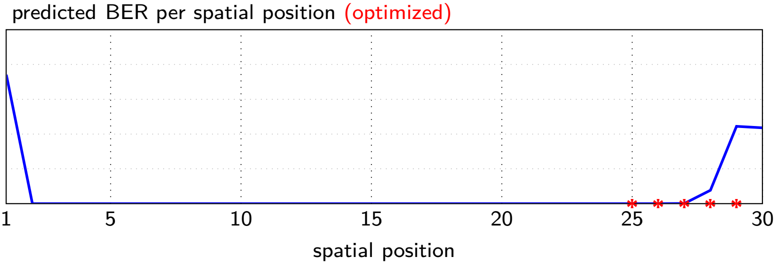

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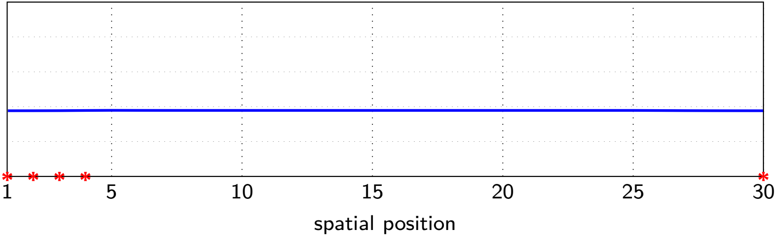

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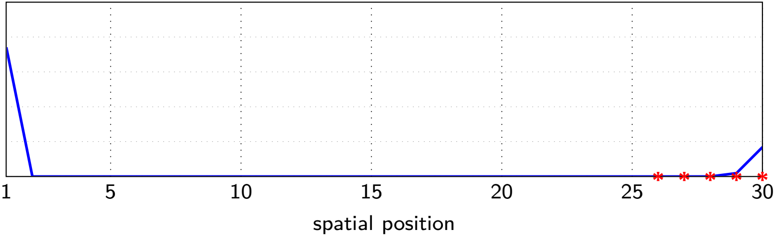

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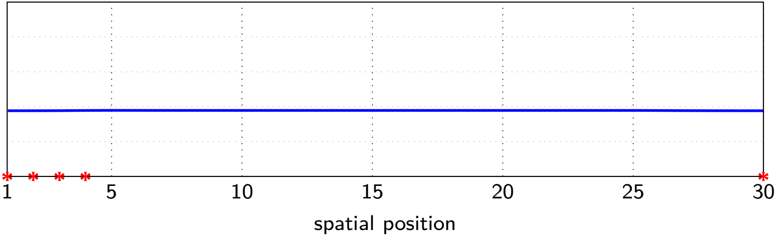

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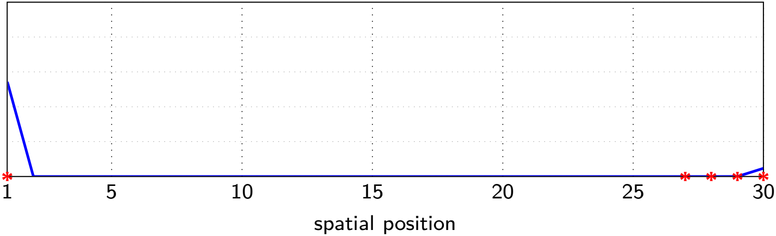

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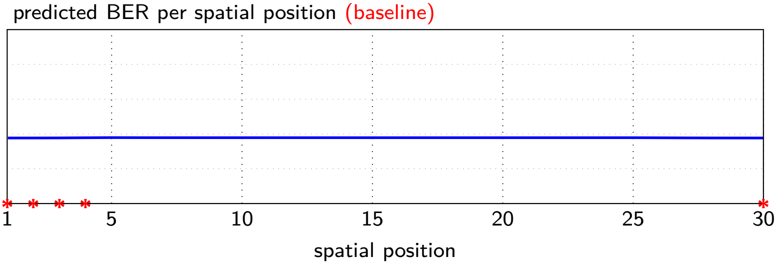

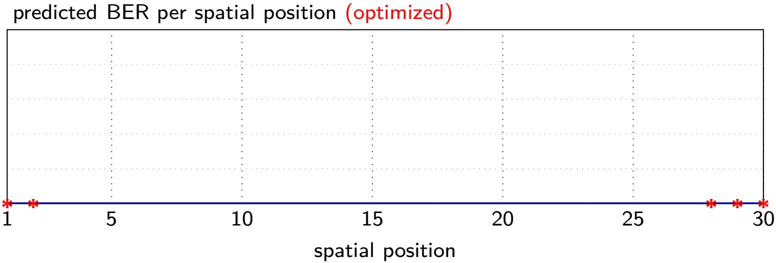

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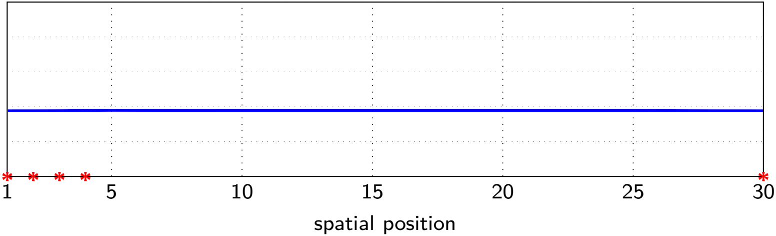

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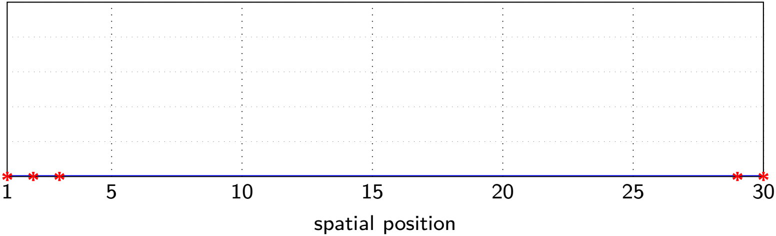
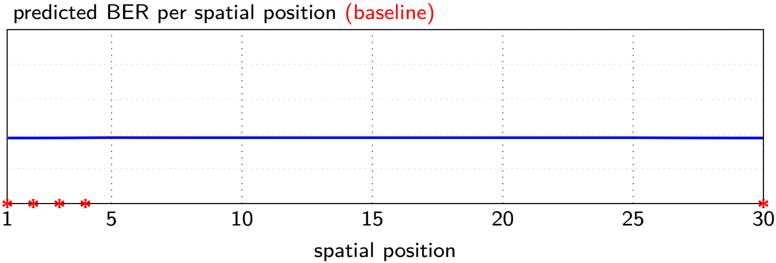
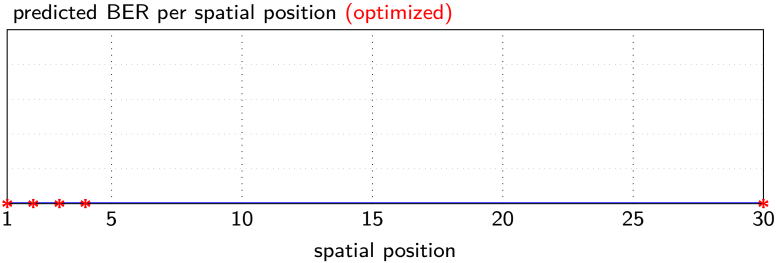


Optimization Result: Decoding Behavior





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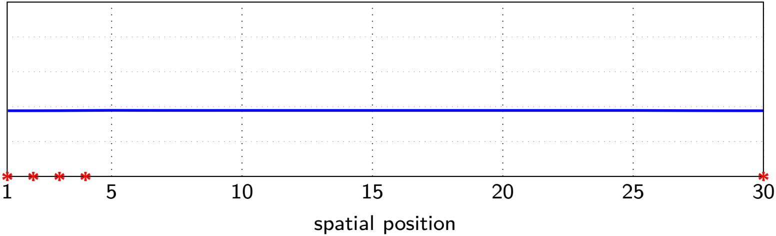

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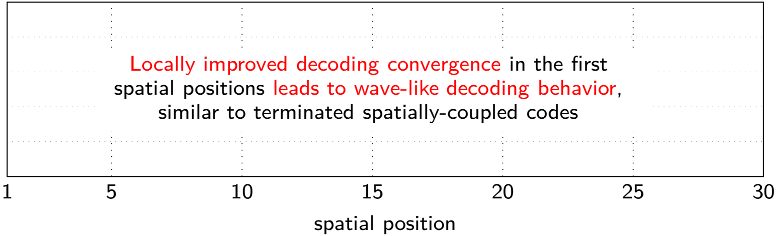

Locally improved decoding convergence in the first spatial positions leads to wave-like decoding behavior, similar to terminated spatially-coupled codes

Design of Spectrally-Efficient Fiber-Optical Systems

Summary

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Summary

- Spectrally-efficient communication with binary codes leads to the problem of bit mapper optimization
- Optimized bit mapper can offer significant performance improvements
- For tail-biting spatially-coupled codes, unequal error protection of a nonbinary signal constellation can be exploited to induce wave-like decoding behavior

Conclusions

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Conclusions

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Thank you!

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